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ABSTRACT

This volume presents different issues and views on research in mathematics education in Australia. Chapters include: (1) "An Introduction to This Review" (Kay Owens); (2) "Recent Research Monographs" (Kay Owens and editors of the monographs); (3) "Language Factors in Mathematics Education" (Nerida Ellerton, Ken Clements, and Philip Clarkson); (4) "Research in Problem Solving in Mathematics" (Steven Nisbet and Ian Putt); (5) "Technology-Assisted Mathematics Education" (Gary Asp and Barry McCrae); (6) "Research in Teaching and Learning Algebra" (Elizabeth Warren); (7) "Making Connections with Space and Measurement" (Tom Lowrie and Kay Owens); (8) "Early Arithmetical Learning and Teaching" (Alistair McIntosh and Shelley Dole); (9) "Undergraduate Mathematics Teaching and Learning" (Glenda Anthony, Ruth Hubbard, and Philip Swedosh); (10) "Issues in Mathematics Teacher Education" (Bob Perry, Beth Southwell, and Peter Howard); (11) "Gender and Mathematics: Changing Perspectives" (Helen Forgasz, Gilah Leder, and Colleen Vale); and (12) "The Politics of Mathematics Education 1996-1999" (John Horwood and Jan Thomas). Most sections contain references. (Includes an index.) (ASK)

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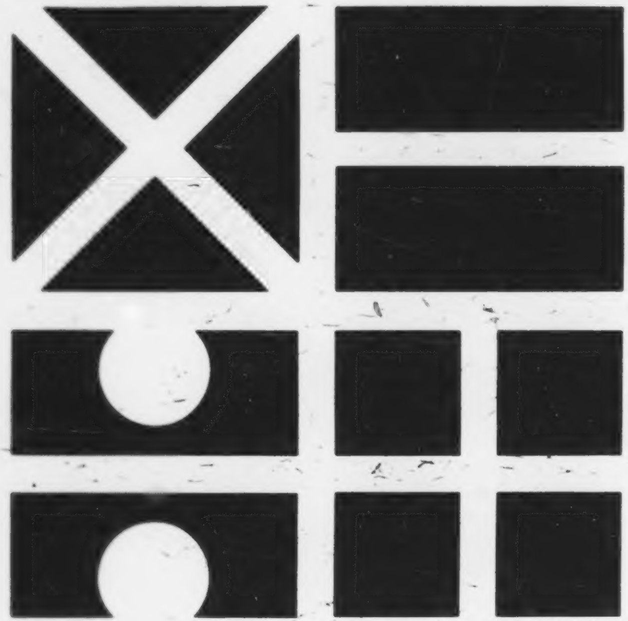
MATHEMATICS EDUCATION RESEARCH GROUP OF AUSTRALASIA

ED 441 691

Research in Mathematics Education in Australasia

1996 – 1999



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Disclaimers

The views expressed in the chapters of this review are those of the authors of the chapter and are not necessarily those of MERGA or the editors.

While authors have attempted to review all research in the field, this book cannot claim to be comprehensive. Authors and editors apologise for not referring to research which may have been difficult to source or which may have been overlooked. The editors recognise that large bodies of research have not been reviewed.

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1

Mathematics Education Research in Australasia

An Introduction to This Review

Kay Owens
University of Western Sydney
<k.owens@uws.edu.au>

Introduction[1]

The first MERGA-sponsored review of Australasian research in mathematics appeared in 1984, in time to be launched at the fifth International Congress on Mathematical Education (ICME-5) hosted in Adelaide. Its successful reception by the international mathematics education research community encouraged the production of regular four-year reviews to coincide with successive ICME conferences. Perusal of these volumes (Briggs, 1984; Blane & Leder, 1988; Atweh & Watson, 1992; Atweh, Owens, & Sullivan, 1996) illustrates the recent evolution and development of Australasian research in mathematics education generally.

The current review reports on mathematics education research reported in Australasia, or conducted by Australasians and reported elsewhere, during the past four years. The primary purpose of the review is to indicate the scope and focus of mathematics education research conducted in this part of the world, and to identify sources for researchers who may wish to examine particular aspects of the research in more detail.

This chapter summarises the objectives of MERGA and its major activities and indicates some contexts in which MERGA participates. It also presents an indicative summary of the content of the chapters, each of which reviews a distinct aspect of research in mathematics education in Australasia.

MERGA

MERGA is a professional association for those interested in mathematics education research, teacher education, and development. It has 300 personal members predominantly from Australia and New Zealand, but also from Brunei Darassalum, It has over 300 personal members predominantly from Australia and New Zealand, but also from Brunei Darassalum, Hong Kong, Korea, Japan, Samoa, Singapore, and Taiwan as well as from Brazil, Canada, Germany, Israel, Mauritius, Mexico, Norway, Oman, South Africa, Spain, Sweden, the United Kingdom, and the United States of America. In addition, it has nearly 70 institutional members whose libraries receive journals, conference proceedings, and reviews such as this one.

The aim of the Association is to promote quality mathematics education at all levels, particularly in Australasia, through:

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[1] This chapter draws on MERGA documents and the introductory chapter in the previous review written by Peter Sullivan, Bill Atweh, and Kay Owens.

- promoting quality research in mathematics education;
- disseminating information on recent mathematics education research and teaching developments;
- seeking to influence decision makers at all levels to take into account research findings in mathematics education;
- providing means for members to communicate with each other on matters concerning mathematics education; and
- maintaining liaison with other organisations with interests in mathematics education or educational research.

These aims were adopted by MERGA when it merged with a related organisation, the Mathematics Education Lecturers' Association in 1997. Since then, MERGA has focused on teacher education development as well as research as part of its scholarly work.

The main activity of MERGA is the annual conference, the venue of which has been in all states and the Northern Territory of Australia, and in New Zealand. Nearly 200 members have attended each of the recent conferences and about 100 papers have been presented in each of the four years covered by this review.

The following extract from the guidelines for presentation of papers illustrates the focus of the conferences:

> MERGA acknowledges the equivalent value of research and other forms of scholarly inquiry. As such, a broad range of research methodologies and their methods will be accepted. These will include, for instance, quantitative approaches such as experimental and quasi-experimental investigations, qualitative studies such as case studies or ethnographies, participatory projects and all styles of postmodernist inquiry. MERGA also acknowledges, and encourages, scholarship in mathematics education that draws on a wide range of theoretical paradigms and interdisciplinary analyses. MERGA acknowledges the importance of opportunities for sharing emerging ideas in mathematics education and teacher development.

Papers for publication in the conference books are critiqued by two reviewers selected from a panel of experienced researchers approved by the conference committee. This refereeing process provides a mechanism for blind peer review and critique, and so contributes to the overall quality of mathematics education research in Australasia. Early career researchers may submit their papers early so they can be reviewed and revised before the general conference review. In addition, MERGA encourages new researchers through the Early Career Awards, and support with conference and membership fees for full-time students.

The annual conference also provides a venue for Special Interest Group meetings, policy meetings, discussions on political issues, research symposia or workshops, and short communications. The friendly atmosphere encourages informal discussions on research and teacher development, and many research projects have been conceived at these venues.

MERGA publishes an internationally recognised scholarly journal, the *Mathematics Education Research Journal*, and much of the research reviewed in this book is from either the conference publications or the journal. Two special editions have been published in the last four years, *Gender and Learning Settings* in Volume 9 and *Learning and Teaching Mathematics: A Cognitive Perspective* in Volume 10.

The first of a regular series with a focus on teacher education, *Mathematics Teacher Education and Development 1*, reached fruition at the end of 1999. The content of the journals is listed at the end of this book. MERGA also commissions occasional additional publications such as *Children's Number Learning* (edited by Mulligan and Mitchelmore, 1996), and the book *Research and Supervision in Mathematics and Science Education 1998*. Our third research monograph will be the *Socio-Cultural Aspects of Mathematics Education: An International Research Perspective*, to be published in 2000. Summaries of the last two monographs are given in the next chapter and the first one is well covered in the chapter on "Early Arithmetical Learning and Teaching".

The Association also maintains a website and a directory of members, including information on research specialities and contacts. MERGA organises two awards for research, the Practical Implications Award, sponsored jointly with the National Key Centre for Teaching and Research in School Science and Mathematics, and the Early Career Award which is judged on presentations at the conference as well as on the quality of the written papers.

MERGA has a formal policy on mathematics education research, addressing the purposes of research in mathematics education, the components that should be present in such research, and the role of the Association in supporting research.

Details on submissions to the journal and other regular publications, membership, and the purchase of publications are given at the end of this book.

Support for Mathematics Education Research

The research reported in this review has generally not been funded at levels common in some other countries. Even within Australasia, in comparison with other disciplines, research in education generally, and mathematics education in particular, is poorly funded.

Each university in Australia and New Zealand has a system of competitive internal research grants that have supported some mathematics education researchers. There is a system of grants funded by the Australian Research Council, a number of which have been won by mathematics education researchers. Nevertheless, there are very real barriers confronting all education researchers in seeking competitive funding. More recently, large commissioned projects have been undertaken by researchers for government and non-government education systems. Some recent major curriculum documents and well-funded programs have been developed with some input from researchers, but usually these are severely constrained by political interests and boundaries.

There is a need for education researchers to explain to the academic community generally that sites for educational research are complex, with many direct and subtle influences operating. Clements and Sullivan (1991) argued that standard experimental designs are often unsuited to educational contexts, and contrived environments in which attempts to control variables are unlikely to yield data that can be meaningfully related to the complexities of normal educational settings. Often the most appropriate data are obtained through ethnographic methods, case studies, and participatory research. The dilemma is that decisions on research funding are often made by researchers from disciplines in which such methods are not practiced or valued. Increasingly, practitioner research methodologies are transforming educational settings in which mathematics educators work with practising teachers, but again this is not seen as pure or even applied research.

A major concern for all mathematics and mathematics education societies is the alarming decrease in well-qualified and competent mathematics teachers in primary and secondary schools as well as tertiary colleges and universities. Mathematics classrooms in schools are also limited by negative teacher attitudes, the domination of textbooks, and the impact of low-level national and state benchmarks and basics skills testing. Other negative factors include pressures on time as more and more demands are put on teachers and the curriculum, and negligible support for professional development in rapidly-changing times. Continuing inadequacies in funding to universities for mathematics and science programs is of major concern as it reduces interest in applicable mathematics.

Participation in Political Processes

In response to the low level of direct financial support for mathematics education research, and also because some educational developments were undertaken in isolation from research evidence and without input from researchers, MERGA has sought to participate more actively in political processes over recent years. Particular government initiatives in which MERGA has been interested have included the development of curriculum documents intended to guide teaching of mathematics in Australian and New Zealand schools towards greater emphasis on competencies and specific outcomes. MERGA and similar organisations have contributed to the debate surrounding these initiatives and subsequent developments. The chapter in this book by Thomas and Horwood not only reviews these processes but also makes a contribution to these debates.

As one of its main avenues of political activity for mathematics teachers, lecturers and researchers, MERGA participates in the Australian Mathematical Sciences Council (AMSC) which is a member of a political lobby group, the Federation of Australian Sciences and Technological Societies (FASTS). Members of the AMSC include the Australian Mathematics Society (AusMS) and the Statistical Society of Australia.

Scope of the Review

The term Australasia is used generally by authors to include Australians, New Zealanders, those from Pacific Islands, Melanesia, and some South-East Asian countries. It is noted that research published in the latter three regions is often difficult to source, and some authors have reviewed little research from these areas. The review does not cover all areas of research in mathematics education. The editors were dependent on MERGA members to prepare chapters, and some members with expertise and interest in particular fields were unable to assist. However, the index does allow some research articles on topics such as assessment to be drawn together from a range of different chapters. Unlike the previous review, some areas such as the sociocultural aspects of mathematics, and stochastics are hardly touched despite the fact that extensive research has been carried out in these areas. The next chapter on the content of the special monograph on sociocultural aspects indicates the extent of research in this area. Researchers in stochastics are particularly active, with well-developed projects. These include Jane Watson and Jonathan Moritz; John Truran and Kath Truran; Graham Jones, Ian Putt, and Bob Perry; Pamela Shaw; Robert Peard; and Jenny Way. There has, however, been a lack of research funds in stochastics education despite the growth in of the gambling industry, its use, employment, taxes, and gambling-related problems. Studies on constructivism and on cognition have been subsumed mostly in the chapter in this review on problem solving. Chapters not included in the last review but included in this review are on technology-assisted mathematics education and undergraduate mathematics teaching and learning. The previous review did, however, have a chapter on adult learners of mathematics and advanced mathematical thinking.

Authors have sought to provide a comprehensive guide to research in the respective fields although there may be instances in which relevant research has been overlooked or has been difficult to source. Authors consulted a variety of databases such as ERIC, AUSTROM, PsychAbstracts, hard copies of journals, books, conference proceedings, and collected papers. Authors have taken various approaches to their chapters. Some have used a framework to help readers through the diverse areas included in their chapter. Some have taken a critical stance to support their framework, while others have kept critical analysis of the research to a minimum.

Components of the Review

Issues in Learning and Teaching Mathematics

This review of research has ten chapters. The first part of the review contains six chapters on Issues in Learning and Teaching Mathematics.

Language Factors in Mathematics Education sets a broad context for language in teaching and learning. The authors, Nerida Ellerton, Ken Clements, and Philip Clarkson argue for a particular framework that states aspects of culture, mathematics, classroom discourse, sociolinguistics, psycholinguistics, curriculum theory, problem solving and natural language all impinge in educationally significant ways on mathematics classrooms and mathematics curricula. They consider language in specific content areas and in different cultural and political contexts. The section on bilingualism continues previous research interests in this area, as Australasia is rich in multilingual situations.

Attempts to describe concept develop in terms of levels or stages have sometimes led researchers to overlook the importance of language. Mathematics is not a natural first language and a student's development of a schematic model is important. Expressive modes, multimodal learning, use of natural language, problem posing, writing, oral discussion, and the opportunities to solve problems emanating from the classroom community are important factors in learning. Mathematics educators believe that students who understand mathematical concepts are likely to be able to talk about, explain, link, apply across contexts, and represent concepts well. Teachers, however, need support to change their classroom language, to encourage students' natural language, and to develop ways of supporting students with different ideas who are grappling with the different semantic structures associated with mathematical operations. Variance in semantic structures has been explored for all four basic operations, proportional reasoning and percentage problems, rates of change problems, and algebra problems.

From a contextual perspective, assessment of mathematics learning needs to incorporate multimodal language forms if it is to be consistent with the multimodal learning environments. Sociolinguistic research has shown that while bilingual and Indigenous language mathematics teaching is important, mathematical thought may need to be developed through new languages by Indigenous people. Code switching was also found to be likely when students were grappling with difficult word problems. Sociolinguistic research also showed that workplace conversations used mathematics concepts but the language was site-generated and did not conform to terminologies and language patterns taught in formal educational establishments. Patterns of discourse in mathematical classrooms are influenced by macro- and micro-cultural factors influencing factors such as who asks questions, what constitutes a mathematical investigation, and how students' feelings of uncertainty are resolved. Research in which teachers departed from traditional mathematics classroom discourse patterns revealed that students discussed more but covered fewer examples.

Steven Nisbet and Ian Putt show that *Research in Mathematical Problem Solving* continued during the second half of the 1990s, partly because of the emphases that social constructivist theories place on problem solving. Despite the recent highlighting of basic, often minimal, numeracy as a priority in government circles, problem solving continues to be described as an essential competency in school mathematics. Contexts for problem solving, especially real-world problems, are investigated, and the impact of assessment and the nature of problem solving on assessment are discussed.

The studies involving visual factors demonstrate the effectiveness of using imagery and various visual representations such as drawings, diagrams, and mapping techniques to assist students across many grades in their problem solving and mathematical conceptual development. Researchers discussed the different strategies used by students across all levels of schooling and one study considered how different problem-solving strategies involve different levels of cognitive demand. Studies on algebra considered problem-solving strategies, especially pattern recognition, suggesting that the solutions employed by students to solve any particular problem reflected the mathematical structure they imposed on it. Another study investigated the cognitive thinking employed by a research mathematician in solving a high-level problem.

Studies on the impact of collaborative learning, especially on problem posing, focused on language, roles, metacognition, strategies, and technology. Finally in this chapter there are a number of investigations of teaching problem solving in relation to special groups of students—young children, gifted students, students with hearing impairments, students at risk—and also in relation to gender differences. Several interesting methods such as the use of data maps have been used to carry out research into problem solving.

Gary Asp and Barry McCrae show in their chapter, entitled *Technology Assisted Mathematics Education*, that most teachers are in favour of all students using a calculator. However, only a little more than a third reported that they permitted students to use calculators often or to have complete freedom to use a calculator at any time. Teachers made almost no reference to using calculators in problem solving contexts. Those who reported using calculators as tools for learning and understanding mathematics were found to be more likely to be aware of research-based ideas on calculator use, or to have observed the benefits of calculator use arising from their own use of calculator activities. Students used calculators to perform calculations more often when the teacher was not present; so perhaps students' perception of teacher beliefs regarding the role of calculators influenced their choice of computational methods. Generally students used graphics calculators where they were expected to use them, but many students lost time and accuracy through inefficient use of calculators or data entry error. Calculators were still viewed as checkers of traditional methods by students.

Several studies have investigated the use of LOGO and dynamic geometry software to support constructivist learning. In general students' attitudes were found to be positive to the use of technology. While research into the use of the internet has not shown the results that teachers may be seeking, the use of computer algebra systems (CAS) has shown remarkable influence on learning. Several authors have identified how graphics calculators are used in assessment, and there have been some policy differences between Australian states here. For example, from the outset Western Australian examiners at Year 12 could set questions that required the use of a graphics calculator, but such questions were not allowed in Victorian examinations until 1999.

The chapter on *Research in Teaching and Learning Algebra* illustrates the issue of different approaches to teaching a particular area. Elizabeth Warren discusses how beginning algebra students move from arithmetic thinking to algebraic thinking. Two approaches are common in bridging this cognitive gap: (a) generalising the patterns in arithmetic, and (b) generalising the patterns found in functional situations such as number patterns, visual patterns, and tables of values. Students seem to have varying success with these approaches. The distributive law, the distinction between equation and expression, and the notion of variable continue to cause some students problems. Using the functional approach, one study investigated whether the symbolic language of algebra followed children's natural language development whereas another study showed that spatial visualisation of mentally manipulating, twisting and inverting stimuli was important in generalising from visual patterns.

A number of studies considered the development of understanding of algebraic concepts, in relation to a particular theory such as that espoused by Sfard (1991) as interiorization, condensation, and reification or encapsulation. Reasoning processes, language factors, affective factors, and problem presentation all play important parts in students' learning, their algebraic solutions, and their concept development.

The last part of the chapter on algebra focuses on the teaching of algebra. In particular, it reports on the use of computer algebra systems, the advantages of using two variables for word problems with beginning algebra students, and the teacher's role in developing appropriate language in algebra. There seems to be conflicting research with regard to the use of technology in supporting the development of conceptual and procedural knowledge in the algebraic domain; and this seems to be influenced by both the teacher involved and students' prior experience with technology in a mathematical environment.

Tom Lowrie and Kay Owens illustrate how concept development cannot be separated readily from either the learner or the context of learning. While the emphasis of their chapter on *Making Connections with Space and Measurement* is on concepts in Space and Measurement, nevertheless its focus on processes is relevant to other areas of mathematics. Representation of three-dimensional and two-dimensional shapes was a key issue in a number of studies. Representations showed how students were thinking, and could be used for encouraging learning.

Some studies have emphasised a particular theoretical perspective, with several studies critiquing van Hiele theory and making suggestions for extending it to represent more closely observed data. In this way, aspects of class inclusion, single and multi-property recognition, and relational understanding of properties for higher level students were examined. Another theoretical perspective was the idea of abstraction from physical contexts that represent a complex concept such as angle. A third perspective has been to consider the development of imagery in assisting students in primary school to develop spatial thinking. Several researchers have explored the links between visual and nonvisual processing in problem solving as well as the impact of affective variables, knowledge, and methods of responding to progress in problem solving. The ideas of conjecturing, solving and proving have been closely linked to studies investigating the use of drawings.

The contexts for learning including early-childhood play, computer-based experiences, and workplace knowledge have been shown to influence the learning and use of spatial mathematics. The development of the concept of length has been explored in a Geo-LOGO computer environment but most research in measurement has been on the development of the area concept. In particular, the use of drawings, concrete materials, discussions, and the links between quantity and area were found to be educationally significant in the development of the concept of area.

Alistair McIntosh and Shelley Dole review studies on *Early Arithmetical Learning and Teaching*. Research on whole number includes studies in early number, early development of operations and understanding of operations, children with learning difficulties, aspects of word problems, and assessment. Calculators were found to assist performance on a wide range of items including mental computation, negative numbers, place value and, most markedly, decimals. Some long-term research in multiplication and division has led to a framework, incorporating six developmental levels and assessment tasks, designed to help teachers in assessing and improving children's number knowledge.

Several studies focused on children with learning difficulties. Interestingly, another study of students' skills showed two different profiles: (a) strong in number but weak in problem solving, and (b) strong in both number and problem solving. The role of reasoning by analogy in children's solving of word problems was considered a key in understanding student learning.

Studies of the development of decimal understanding and misunderstanding indicate little change in students' understanding of decimals over the period of a longitudinal study. Students' misconceptions about decimals are a significant problem. One study explained this in terms of the interaction between available and accessible place-value and regrouping knowledge. In addition to the studies on children's cognition of decimals, there have been studies on how teachers can assist students to understand decimal numbers, and to reconstruct their ideas of whole numbers to include decimal fractions. The benefit of allowing students to work in pairs was noted.

Research into fractions included young children's notions of fairness and fair shares, fostering young children's fraction development in the classroom, the use of analogs in fraction teaching, using computer programs to promote fraction understanding, students' understanding of the connectedness of common and decimal fractions, and number sense associated with common and decimal fractions. For some students, their efficient strategies assisted in task completion, but such strategies would not assist students to abstract the particular quotient fraction construct. Another study (contrary to fraction literature that children's whole number knowledge interferes with fraction understanding) revealed the value of tapping into whole number knowledge through appropriate tasks to build fraction knowledge. The researchers raised the issue of requiring students to perform operations with fractions and decimals as set in school mathematics curricula when clearly many students have limited and limiting conceptual understanding of these topics. Other studies focused on understanding and learning about percentages as well as written methods that best assist students in proportional reasoning.

Almost all the recent research on children's mental computation methods has centred on addition and subtraction and the value of flexible computational strategies. Classroom practices designed to develop children's mental computation strategies and abilities have been studied. Visualisation was found to be a key to students' number sense. Children's representations of Cartesian products and rectangular area problems show why the concepts underlying multiplication are hard for students to grasp. An issue in dealing with number sense is assessment, especially in relation to paper-and-pencil testing.

Issues in Mathematics Education at the Tertiary Level

The second part of the book covers the tertiary level. For a long time the *Teaching and Learning of Undergraduate Mathematics* has been the poor cousin within the domain of mathematics education research. An increasing interest is most visibly evidenced in the burgeoning conference activity at the international level. Glenda Anthony, Ruth Hubbard, and Philip Swedosh focus on issues related to student participation in undergraduate mathematics courses, including the effects of policy changes, changing student profiles, and perceptions of mathematics and of mathematics learning and teaching. The authors also review research that focused on the nature of mathematics learning at the tertiary level and related implications for teaching and course design.

In Australia there is increasing concern about a weakening of university mathematics. It was found that tertiary students holding cohesive conceptions of mathematics perceived the learning environment as more satisfactory and fulfilling, and they achieved at a higher level than did students reporting fragmented conceptions; but many students did not particularly enjoy their mathematics studies. School leavers, in contrast to mature-age students, tended to think that mathematics was less useful and that it was too much work. Students expressed their valuing of a more personal approach. The understandings that lecturers hold about their students and teaching greatly influenced the content and form of their teaching. Collaborative learning is increasingly being used within university environments. Despite the encouragement of appropriate assessment methods, research studies were united in the view that changing the assessment alone would not be sufficient to orient students to deep approaches to learning.

Bob Perry, Beth Southwell, and Peter Howard summarise some *Issues in Mathematics Teacher Education*. Some challenges arise from the nature of and forces upon teacher education in general, and some are peculiar to mathematics. In primary schools in Australasia, mathematics is taught by generalist classroom teachers, and this creates challenges for teacher education. The authors consider teacher education research projects across all levels of learning—early childhood to adult. Studies cover beliefs, attitudes and values, and the mathematical knowledge of practising and prospective mathematics teachers; preservice mathematics teacher education; professional development in mathematics education; and mathematics teaching as a sociocultural construct. In the period 1996–1999, studies have been undertaken into the espoused beliefs of teachers and student teachers and their impact on all aspects of the planning, teaching, and learning of mathematics—and in particular on teachers' utilisation of problem solving.

Several studies have considered the impact of preservice courses on attitudes, mathematical self-concept, and attitudes to technology. The effect of students' thought chains of past experience, individual attitudes towards mathematics, limited subject-matter knowledge, perceived irrelevance, students' practicum experiences, reasons for studying in the teacher education program, and the lecturers' underlying philosophy towards mathematics can all act as constraints to the reform of mathematics teaching. Reflective practice and student teachers acting as researchers into their own and others' beliefs and attitudes impact on learning. Interestingly, teacher educators present a variety of philosophies of mathematics education.

There has been extensive work on practising teachers' use of manipulatives as well as their beliefs about mathematics, learning mathematics, and teaching mathematics. In particular, teachers varied on the factors of child-centredness and transmission teaching; and teachers of different educational systems, gender, and position (seniority) tended to vary on these two factors. Teachers' beliefs shifted as a result of practice, and a new curriculum had increased professional dialogue regarding the teaching of mathematics; but teacher confidence was perceived to be a major factor in classroom change. Little research has been reported in the area of values within mathematics teacher education, but they are seen as influential. Research supports the claim that students' perceptions of teacher-student interpersonal behaviour are not necessarily the same as their teachers'.

Another area of research has been that of preservice-teacher knowledge. In particular, it was found that there was a need for mathematics teacher education to stress number sense and problem solving in their courses. The cultural aspects of education have frequently been under-emphasised in mathematics education courses. The importance of continuing professional development and the importance of acknowledging the sociocultural contexts in which learning and teaching mathematics occurs suggest that the findings emanating from these studies will impact upon the lifelong teacher education of mathematics teachers at all levels.

The Context of Mathematics Education

The third part of the review revisits the Context of Mathematics Education although each chapter, particularly the language, space, and teacher education chapters, have made reference to the importance of context.

Helen Forgasz, Gilah Leder, and Colleen Vale begin their chapter by summarising the contributions made by Australasian researchers to major reviews, edited books, and special journal issues on *Gender and Mathematics: Changing Perspectives*. One book raised the issue of the gulf between the ideas enshrined in policies related to gender equity and their implementation. A biographical dictionary of Notable Women in Mathematics includes the lives of three Australian women, including Gilah.

Monitoring of males' and females' participation and performance rates in mathematics subjects has continued. Research addressed the question, "What factors, beyond gender, might be contributing to gender differences in mathematics education?" Lower participation rates by female students still continue to be a problem but research showed a more complex picture involving achievement, the nature of assessment, school setting, and affective variables. In particular, the influence of the affective factor Mathematics as a Male Domain and of classroom discourse were highlighted. Research also attended to tertiary students' participation rates, discrimination, and access to technology. Research in the field of gender and technology in mathematics education indicated that the use of technology has gender implications. Without doubt, in contemporary Australia, public debate about gender, mathematics, and technology persists.

Jan Thomas and John Horwood in their chapter on *The Politics of Mathematics Education 1996–1999*, note that there is little interest in an area that has such devastating effects on mathematics education. For non-indigenous minority language groups, issues surrounding language and culture in mathematics learning are largely ignored in curriculum policy in both countries. They note that the poor state of affairs with regard to teacher shortages and government control of research is likely to see a re-emergence of mathematics as a gatekeeper for privileged professions.

There has been an increasing shortage of qualified mathematics teachers. There are issues of recruitment and maintenance of students in science degrees and then of recruitment of mathematics graduates into education. Further, there are fewer tenured, full-time mathematics educators teaching more students with increased administrative loads including the supervision of sessional staff. The extensive use of short-term contracts, sessional staff and packaged curriculum is discouraging the development of a mathematics teaching profession in Vocational Education and Training. In New Zealand, the effectiveness of a project for mathematics graduates entering teacher education and experienced teachers of other subjects to become mathematics teachers has implications for improving the supply of mathematics teachers.

There was an emphasis on presenting results from the Third International Mathematics and Science Study with an over-exaggerated good light rather than carefully analysing what the results may say about how to improve mathematics education, the on-going shortages of mathematics teachers, or the practice of drafting of non-mathematics teachers into teaching mathematics at the junior secondary level. During the same period there was a considerable reduction in government support services and a move to casual consultancies. Professional associations protested that a recent government paper on research posed threats to basic research, both in the discipline of mathematics and in mathematics education. The drive towards accountability in Australian education systems has led to the continuing benchmarking of numeracy and attempts to define *numeracy for all* in a way that distances it from mathematics.

Reviews of advanced mathematical sciences in Australia and New Zealand showed that the mathematical base was fragile. Both Australia and New Zealand have shifted more of the cost of tertiary education to students, and the number of postgraduates in education and teacher participation in such courses has reduced. Jan Thomas and John Horwood conclude their chapter with the important point that the mathematical sciences in Australia and New Zealand need bipartisan support and long-term planning, not ad hoc decisions for short-term political gain.

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Recent Research Monographs

Kay Owens
University of Western Sydney
<k.owens@uws.edu.au>

Introduction

This chapter summarises MERGA's two recent research monographs. The areas of research covered by the two books would not otherwise be covered adequately in the review. The chapter has been compiled from the introductory chapters of the two books, mostly quoted verbatim.

Research and Supervision in Mathematics and Science Education

Edited by John Malone, Bill Atweh and Jeffrey Northfield
Published by Lawrence Erlbaum
1998

This book provides both researchers and supervisors with information regarding the essentials of planning, conducting, reporting, and publishing qualitative research, along with the characteristics of quality supervision. The book draws on recent research and practice to illustrate examples of approaches and strategies that have been used successfully with and by postgraduate students. It also has a strong future orientation in that it describes how postgraduate supervision and research will change as more and more students gain access to and make use of the vast array of technological resources now available. The book's practical approach, its creative future perspective, and the contemporary and wide generalisability of the thinking and ideas presented in each of its chapters are appealing.

Dylan Williams provides a framework for thinking about research in mathematics and science education. He discusses in terms of *text, context*, and *reader*, the various dichotomous dimensions (e.g., qualitative vs. quantitative) previously used to classify educational research. He then argues that the process of research defines the research but still requires community judgment.

Robyn Zevenbergen discusses the critical decisions that are made in carrying out ethnographic research in the classroom. John Schaller and Ken Tobin discuss the complexities of communicating classroom experiences and the nature of interpretive research. They broach the subject of using fictive stories as interpretive research. John Truran and Kathleen Truran discuss the use and practicalities of clinical interviews in research. They then discuss the analysis and interpretation of data, its presentation and preservation. The technical aspects of analysing verbal communication between interviewer and interviewee is a feature of Bob Bleicher's chapter. He uses interactional sociolinguistics to make sense of recorded classroom talk, and how to interpret comments made by interviewees thinking on their feet.

Peter Taylor and Vaille Dawson critically reflect on their own problematic student-supervisor relationship. The sense of disempowerment felt by the student during an action research study in which she was attempting to transform her classroom resulted from the demands to conform to normative expectations and standards. Gilah Leder, and her students Helen Forgasz, and Julie Landvogt discuss their interactions. In particular, the students discern that the intellectual and personal skills of the supervisor are critical while they themselves need to meet her demands. The role of *ideal supervisor* is critiqued. Loren White also describes the pressures put on the researcher, collaborator, teacher, supervisor juggling multiple roles. He addresses the various dilemmas posed by his work with colleagues in these different roles. Frank Crawley discusses issues relevant to collaborative action research studies. He documents the tensions experienced by a group of team members as they undertook the recursive steps of action research, from the problematising of existing teaching practice to reflecting on the outcomes of actions taken.

Supervision structures are examined by John Malone. In particular, he discusses distance-education and part-time students. He describes strategies for overcoming the absence of a host of supporting facilities. Wolff-Michael Roth and Michelle McGinn outline a conception of learning to do research that takes practice as its core theoretical notion. It discusses how best to use this apprenticeship model for developing theoretical research perspectives. Andy Begg, Beverley Bell, Vicki Compton, and Elizabeth McKinley describe another supervision structure. It focuses on avoiding undue duplication of effort while maintaining key components such as research methods courses and supervision support meetings.

Tom Cooper, Annette Baturo, and Leonie Harris discuss the basics of writing up scholarly work. They focus on the mechanics of writing, the flow of arguments, and the integration of the entire work. Jack Hourcade and Holly Anderson extend the discussion on the process of writing, the importance of writing, and increasing productivity.

The final chapters focus on the way that information technology (IT) can be harnessed to assist those engaged in research and supervision. David Squires discusses the use of IT for selecting a research area, developing a comprehensive awareness of the field, clarifying methodological issues, and checking accuracy of information. He also explains how it can assist student and supervisor in more quality interactions. Peter Rillero and Bee Gallegos discuss the efficient use of bibliographical databases and the resources on the internet.

Socio-Cultural Aspects of Mathematics Education: An International Research Perspective

Edited by Bill Atweh, Helen Forgasz, and Ben Nebres
Published by Lawrence Erlbaum
2000

It is widely accepted today that as general research in mathematics education evolved during the 1960s and 1970s, psychological paradigms, methodologies and research questions dominated the field. A few researchers then began to investigate factors related to the performance of under-represented and under-achieving segments of the school population. These research foci opened the gate for the consideration of social factors as critical in understanding how they effected the outcomes of mathematics teaching. Arguably, the majority of these early studies with a social perspective used mainly quantitative research methodologies that were prevalent at the time. In the 1980s and 1990s, mathematics education research witnessed a diversification of research thrusts and theoretical models coming from sociology, anthropology, and linguistics. Within this context, this edited collection of research articles represents the variety of mathematics education research questions raised from sociocultural perspectives, presents the wide scope of these theoretical perspectives, and reflects the current status of research questions and knowledge in this important area of mathematics education. Recent developments in more well established areas of research that are still prominent in the field—gender, social class, and ethnicity—are examined in some chapters. In other chapters, authors have explored topics that have received considerable attention during the past decade, such as ethnomathematics and mathematics education and culture, while some authors have discussed the more recently emerging issues such as post-modernism and globalisation.

Psychological, social and political perspectives. Stephen Lerman points out that the field of psychology and to a limited extent mathematics education have undergone considerable changes in the last decade or so, in line with developments in cultural studies, feminist research, and postmodernism. Stephen argues the case for a cultural psychology and for a move away from constructivism. He outlines a research program for mathematics education which is firmly based in cultural, discursive psychology. The author discusses theoretical issues, refers to some of the important literature, and presents some research in aspects of the program.

Paul Dowling argues that mathematics education as it is widely practised is a mythologising activity in that it regards other activities as themselves instances of or representable by mathematics. The myth of participation renders invisible its distinction between mathematics and non-mathematical practices. Effectively, this constitutes mathematics as a necessary component of, for example, everyday domestic practices. The myth of reference, on the other hand, privileges specialised mathematical knowledge. The nature and distribution of these and other mathematical myths reflects social and cultural divisions which relate more closely to an earlier phase of modernity than to the current global configuration. Many current state initiatives in the United Kingdom (and elsewhere) are tending towards a narrowly instrumental and fragmented interpretation of schooling. The challenge for mathematics education is to move beyond both its own traditions and the bureaucratic structures that are being imposed upon it.

Ole Skovsmose and Paola Valero observe that the connection between mathematics education and democracy is used as justification for including relevant mathematics education, but little systematic reflection has been given to this area by the international community of researchers in mathematics educators. Their chapter brings together different social disciplines from which the basic theoretical tenets of mathematics education for democracy can be stated. Relevant theoretically and practically oriented projects that have this connection as their central theme can be developed.

Finally in this section Uwe Gellert, Eva Jablonka, and Christine Keitel argue that the different conceptions of mathematical literacy are related to how the relationship between mathematics, reality and society is conceived. They use the term *mathematical literacy* as a metaphor referring to well-educated and well-informed individuals. In surveying some of the major conceptions designed to deal with the new demands for mathematics education, they look for essential components of a conception of mathematical literacy from a general education perspective. Then they consider *commonsense mathematics*. The conclusion connects the two topics by introducing the concept of *reflectiveness* as a necessary competence for mathematical literacy.

Issues in culture and globalisation. Questions need to be asked about how much of our students' mathematics education should be local to meet their needs to understand and control their immediate life and how much of it should prepare them to survive globally. We also need to inquire into the politico-historical development of mathematics education curriculum, and practices in different countries that enable rational reform in mathematics education.

Bill Atweh and Phil Clarkson claim that mathematics education is one of the most internationalised areas of higher education. This is evidenced in the similarity of the curriculum reforms happening around the world and the increasing number of international conferences and research activities. They discuss theoretical issues in globalisation and internationalisation of mathematics education with particular emphasis on cross-country collaboration in professional development.

Jan Thomas explores the complex issues that surround and influence policy decisions, especially at the government level, in mathematics education. In particular she explores possible trends towards diminishing the influence of expert opinion as governments seek rapid solutions in a global framework. Most nations now consider themselves to be democracies but, because education is so dependent on government funding, it is often difficult for individuals to participate actively in debate critical of government policies without being penalised in some way. The role of professional research-based groups is often at odds with their perceived role and purpose.

Khoon Yoong Wong, Zaitun Binti Hj Mohd Taha and Palanisamy Veloo, look at three ASEAN countries, Brunei Darussalam, Malaysia, and Singapore, which have a common British colonial heritage and national examinations influenced by the Cambridge 0-level examination. However, mathematics education has developed in a different way in each country, under the influence of situated sociocultural factors, namely, historical background, cultural mores, major political events, national education structure and aims, and language policy. The differences have led to distinct school systems and different experiences of student achievement. Their "Situated Socio-Cultural Model" shows the dynamic interplay between the different factors and their impact on school mathematics.

Bill Barton and Roslyn Frank argue that recent interest in how anthropology and linguistics relate to mathematics has led to recognition that mathematical thinking is a function of language in ways not previously recognised. Formal mathematics can be seen to have developed from metaphors deeply embedded in our languages. This raises the question of relativity in mathematics. Do different languages embody different types of mathematics? This chapter examines some emerging evidence in the grammar and syntax of Indigenous languages that are structurally very different from the Indo-European linguistic tradition. The educational consequences of the possibility of different mathematical thinking is briefly discussed.

The goal of the study conducted by Richard Kitchen is to understand some of the complicated and dynamic issues that affect the pedagogy of secondary-level mathematics teachers in Guatemala, Central America. Inappropriately adopted educational models usually did not fit the countries' actual needs, circumstances, and resources. Two mathematics teachers labouring in inner-city public schools in Guatemala describe the political and social contexts of their work. Poverty and lack of resources create a context where the students do not have textbooks. The teacher teaches from a 30-year old textbook and photocopies exercises for homework. Political unrest leads to many missed classes and limited teaching time. The study should help experts from developed countries understand the world of teachers in poorer countries. Before any new interventions are made, we should ask whether these can prosper in the desperate classroom situations in many developing countries.

Teachers, students, classrooms. Over the past few decades, mathematics education literature has begun to focus more on what is called the implemented curriculum, that is teachers, students, and classrooms. There has been a sense that past reform efforts have focused on curricular reform (the intended curriculum) and assessment of student achievement (the achieved curriculum) and that the limited success of these earlier efforts may be due to insufficient understanding of what actually happens in the classroom. There is a sense today that the heart of the problem, and also of the solution, lies in the interaction between teacher, students, and classroom. The chapters in this section invite us to look into (a) the way we do research on teachers, students and classrooms; (b) the relationship between home and classroom culture; (c) the factors that determine whether students continue in or drop out of mathematics; and (d) the way we should look at the relationship between teaching effectiveness and classroom resources.

Bill Atweh and Marcos Arias Ochoa argue that research and professional development programs that are empowering to the teachers are much more likely to sustain effective and continuous change in school mathematics. A case study is presented in which the research process operates under certain underlying beliefs that teachers are professionals, who can best promote their own professional development and that effective knowledge for school change should develop in and from the school. A theoretical basis is given, using Habermas' theory of *knowledge-constitutive* interests.

Jill Adler argues that the history of inequity in provision in South Africa means that numerous schools still do not have basic resources like water and electricity, let alone sufficient classrooms and learning materials. The solution is not to distribute resources but the author argues that a critical application entails a reconceptualisation of resources as a verb so as to shift the focus of attention in mathematics teacher education, for example, away from resources themselves and an image of the good teacher as one surrounded by resources, to the teacher using resources in context. The image here is of the resourceful teacher. Furthermore, such a reconceptualisation extends the notion of resources to include social and cultural resources like language and *languaging*.

Robyn Zevenbergen notes that the typical mathematics classroom has a specific culture and proposes that the consonance or dissonance between this culture and the home cultures of the students have an important impact on learning. In a comparison of a middle-class and a working-class classroom, one sees from the transcripts that there is an ease of interaction in the middle-class classroom whereas there is less success (interruptions and misunderstanding) in the working-class classroom. It seems clear that the typical language register of the mathematics classroom is familiar to middle-class students because it is similar to the linguistic patterns of their home environment. This is not so for working-class students, putting them at a disadvantage. Making these linguistic patterns more explicit can assist in helping students learn these cultural codes so they can participate more effectively. It may also lead to a reconstruction of classroom practice to make it more effective for a particular group of students. The theoretical framework uses the notions of *habitus* and *field* of Pierre Bourdieu.

The chapter by Xin Ma looks into the question of what internal and external factors are dominant in determining whether students in high school continue to take or drop out of mathematics courses. It looks at demographic factors; cognitive and affective factors such as prior attitude towards mathematics and influence of significant others at home; effects of school climate and culture.

Among the factors that emerge as most influential in determining whether students continue to take higher-level mathematics courses are: (a) their prior attitude towards mathematics, (b) support from the home and involvement of parents in the school, and (c) schools "protecting the academic core", that is, minimising the extraneous and disruptive effects that keep teachers from teaching and students from learning. The study is based on recent longitudinal studies and can help schools focus on the factors that are most likely to strengthen students' participation and performance in mathematics.

Many countries have become aware of the importance of their students' achievement in mathematics and of the situation of disadvantaged groups in their societies. This has led to looking more deeply into the factors that determine success in school mathematics and, in particular, the influence of sociocultural factors. Early research in that area has considered ethnic background of students as a critical factor associated with opportunity to learn mathematics and subsequent achievement in it. Two ethnic and racial groups studied in depth are the Hispanic and Afro-American populations in the United States. The last two chapters in this section deal with recent findings from research conducted in that context.

Luis Ortiz-Franco and William Flores begin by reviewing studies which show that over the past decades, the general family income levels of Latino students have decreased but their level of mathematics achievement has actually increased. This result seems to contradict the usual hypothesis that mathematics achievement and socioeconomic status rise together. However, the gap between mathematics achievement between Latino and White students remains large, particularly in complex problem-solving skills. Some factors are presented which may help understand these results. One is the parental factor, namely, studies have shown that the involvement of Latino parents in the education of their children was equal to, or greater than, that of White parents. Also, the educational level of Latino parents has gone up considerably over the past decades. On the other hand, Latino students' achievement in higher-level mathematics skills has not improved, and this may be due to the fact that they tend to go to poorer schools and they may not be exposed to the teaching of such skills.

Vivian Moody looks at the American setting and the different ways that African American students cope with the dissonance between their indigenous world and that of school mathematics. Critical Theory, African American cultural orientations and their application to the mathematics education of African American students are presented to provide a critical perspective for the understanding of the situation facing African American students in mathematics. Two case studies, both of successful students, are presented and analysed within this critical perspective. In one case, the student's experience can be aptly described as succeeding in situations of struggle against racial prejudices and peer criticism. The struggle is also that of moving back and forth between indigenous African American culture and succeeding within what is perceived to be the White culture of the mathematics world. The other student's experience is more one of facing challenges, in particular, the challenge of mathematics itself. Through these cases one comes to a deeper understanding of the personal and cultural obstacles facing African American students and their ways of overcoming them.

School mathematics and everyday mathematics. The authors of chapters in this section of the book have explored conceptions of school mathematics and its relationship to the lives of learners in a range of different contexts.

Betty Johnston and Keiko Yasukawa offer a critical reflection of the authors' journeys about numeracy and mathematics. By drawing on the theories that they have considered, and the continuing interrogation of their own teaching practice, they tell the tale of various milestones they have reached in their theory development, and connections they have made with theories and developments in other areas such as mathematics education, literacy education, and critical pedagogy. They also identify the current absence of some theories with which they hope to make connections in the future.

Swapna Mukhopadhyay and Brian Greer point out that the most common justification given for school mathematics is that mathematics has its roots in, and is usefully applicable to, real-world problems, yet research and documentary evidence show that most children regard mathematics as useless, uninteresting, and irrelevant to their lives. In this chapter, they propose to examine the reasons for this state of affairs through analysis of word problems. They study examples of word problems in different cultures such as India, China, Europe, and across many centuries, in the light of their sociocultural context and the explicit and implicit purposes for which they were designed. They also propose an alternative framework within which word problems are characterised as exercises in modelling aspects of the real world. Starting from this perspective, they go on to argue that when the focus is on the modelling of social phenomena, mathematics can become a tool for justice in the tradition of Moses, Freire, Frankenstein, and D'Ambrosio.

Gloria Stillman and Jo Balatti examine some of the suggested approaches for incorporating ethnomathematics within classroom mathematics in the context of an upper secondary school classroom. A comparison is made of the ethnomathematics of the financial planning industry and the ethnomathematics of school mathematics illustrating that two ethnomathematics coexist in, and are produced by, the same capitalist culture.

In the chapter by Joanna Masingila and Rapti de Silva, the authors argue that students need mathematically meaningful contexts through which to learn mathematics. They discuss some historical examples from social and cultural situations in which people sought to make sense of things from within contexts and in so doing developed mathematical ideas out of these contexts. They then tie this in with research on how middle school students use mathematical ideas in a variety of out-of-school situations, how these situations supported the students' classroom learning, and how students made sense of ideas mathematically. They end the chapter by proposing how a learning theory (Saxe's "Emergent Goals Framework") and an instructional theory ("Realistic Mathematics Education") might be merged with a team approach to teaching and learning to address the issues they raise in this chapter.

Recent directions in gender and mathematics education. The psychological exploration of *sex differences* gave way to a recognition that *gender differences* were social, rather than biological, in origin. Early studies focused on identifying the factors contributing to females' observed disadvantage portraying women as deficit but alternative perspectives were postulated. It was also argued other social factors including socioeconomic status, ethnicity, race and religion also needed to be considered to untangle and understand the complex interactions that contributed to observable gender differences.

Helen Forgasz and Gilah Leder re-examine gender equity issues from males' as well as females' perspectives. Findings from four studies involving participants of different ages illustrate changing patterns of beliefs and differences among ethnic groups. The extent to which boys, rather than girls, are now perceived as educationally disadvantaged is critically discussed.

Some of the early criticisms of research on single-sex schooling were addressed in the unique setting in which Joanne Rossi Becker reports on the evaluation of a *single gender academy*. Ethnographic methods were used to determine if the single-gender academy provided an equitable setting for the young women and men. The study examined whether teachers structured curriculum and instruction differently for the two groups, and if females played a different classroom role than that exhibited in many studies of coeducational classrooms.

Research on gender issues in mathematics at the tertiary level is fairly limited. Leone Burton reports the results of an interview-based study with 35 male and 35 female practising research mathematicians. The mathematicians' epistemologies and the impact of them on their professional lives were explored. The study documents the effects the sex of these mathematicians had on their understanding and practise of their discipline, whether the female and male mathematicians were differently distributed among the disciplinary areas of mathematics and, whether the career experiences of the men and women differed. The findings are related to the learning of mathematics in schools where, the author argues, homogeneity reflects a discipline which has been dominated by male styles, choices and expectations.

Particularly in more affluent nations, the use of technology within education is receiving increased attention. Naturally, the wide adoption of the technology and its early association with what was perceived as male domain subjects such as mathematics, has raised questions related to gender and technology. Nicola Yelland describes findings from a research project in which pairs of children (girls, boys and boy/girl) interacted with computer-based LOGO learning tasks embedded within a mathematics curriculum. The strategies used by the pairs of children as they solve the problems were examined. Differences evident in the tactics deployed and the interpersonal exchanges among the various dyadic pairs are presented. The author also discusses the use of higher order thinking skills in problem solving and the ways in which teachers can scaffold children's learning to support the use of these strategies.

Concerns about equity in mathematics classroom are central to many reform initiatives around the world. Drawing on previous research on gender and mathematics and various feminist perspectives, Joanne Goodell and Lesley Parker develop a detailed operational definition of the essential features of the ideal "Connected Equitable Mathematics Classroom" (CEMC). Students, classroom practices, and the curriculum form the three focal elements. Research evidence is presented which highlights factors facilitating or preventing a group of teachers from implementing particular features of the CEMC in their mathematics classrooms.

Finally, it is noted that, despite efforts to obtain them, the voices of women from non-Western countries are not heard in this book on the issue of sociocultural aspects of mathematics education.

Issues in Learning and Teaching Mathematics

Language Factors in Mathematics Teaching and Learning

Nerida F. Ellerton
University of Southern Queensland
<ellerton@usq.edu.au>

Philip C. Clarkson
Australian Catholic University
<p.clarkson@mercy.acu.edu.au>

M. A. (Ken) Clements
Universiti Brunei Darussalam
<clements@ubd.edu.bn>

Framework for this Chapter

Like the chapter on language factors in the last four-yearly MERGA review (Ellerton & Clements, 1996a), this review is loosely structured around Ellerton's (1989) framework which depicted a cross section of a three-dimensional model of factors influencing mathematics teaching and learning. That framework proceeded from the assumption that aspects of culture, mathematics, classroom discourse, sociolinguistics, psycholinguistics, curriculum theory, problem-solving and natural language all impinge in educationally significant ways on mathematics classrooms and mathematics curricula. The framework implies that language aspects associated with mathematics classrooms and curricula might profitably be analysed and interpreted in terms of issues raised in the intersecting literatures associated with the various categories.

It is recognised, of course, that no framework can capture *all* the relationships, theories and practices that different scholars associate with the term *language and mathematics*. Bauersfeld (1998), for example, contended that Ellerton's framework did not pay enough attention to the Continental hermeneutical approach as summarised by, for example, Mason and Waywood (1996), and Brown (1997). Ellerton (1999a) and Ellerton and Clements (1998b), on the other hand, contended that Bauersfeld (1998) was unable to extricate himself from the constraints of the interactionist framework that he and a number of other Continental educators have developed. Although that framework has proved to be an important tool for researchers, it could be argued that because its main focus is the semiotics of classroom interactions, it does not take seriously, indeed completely neglects, a number of important aspects of language which influence mathematics learning.

Adler (1998) contended that Ellerton's framework derived from an unduly narrow view of *culture*. To this, Ellerton (1999a) replied that not only did her framework embrace a robust, broad conception of culture, rather than a restricted one, but it also implied that cultural factors should be central in any discussion on the influence of language on mathematics learning. As Southwell (1997) has pointed out, culture and language are inextricably linked and it is the role of

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schools to provide for both the acquisition and transmission of the beliefs and values in culture.

Ellerton's framework does not explicitly make links for the reader, and indeed, it does not specifically refer to aspects of language such as textual analysis, genre, semiotic signifier/signified distinctions, technology, bilingualism and code switching. However, comments on each of these themes and on other important areas are included under one or more of the major headings depicted in the framework (see Ellerton & Clarkson, 1996). Each of the separate literatures contains a large number of themes and sub-themes which can be separated and subjected to intensive study. Thus, for example, Ellerton (1998), in discussing the communication component of her framework, pointed out that communication comes in many forms (e.g., verbal, written, non-verbal, pictorial, imagined), and can be either expressive or receptive. Further, communication occurs within many contexts (e.g., work, school, home, sport, etc.).

The broad sweep of Ellerton's framework has motivated writers to interpret the idea of *language factors in mathematics learning* in more holistic ways. Nevertheless, this review chapter should not provide detailed comments on all research studies that come within the ambit of Ellerton's framework, because some are only peripherally relevant to the main theme of language factors in mathematics teaching and learning. For example, the problem-solving component of the framework would clearly be informed by articles which deal with the role of imagery in the processing of problems, but the role of imagery in problem solving is a specialist area in its own right. On the other hand, imagery is an important *inner language* through which a problem solvers communicate with themselves, then one can justify its inclusion within a review on language factors in mathematics teaching and learning. Australasian researchers have researched the role that mental imagery and other components of long-and-short memory play in identifying problem structure and in influencing solution presentation (see, for example, Leeson, Stewart & Wright, 1997; Lowrie, 1996, 1998; Lowrie & Hill, 1996; Mulligan & Mitchelmore, 1997; Mulligan, Mitchelmore, Outhred, & Bobis, 1996; Mulligan, Mitchelmore, Outhred, & Russell, 1997; Mulligan & Thomas, 1998; Owens & Clements, 1998; Owens & Outhred, 1997, 1998).

It should also be mentioned here that frameworks other than Ellerton's have been developed and put forward by Australasian researchers. For example, Hill (1997), writing in the context of early literacy, presented a 5 x 6 two-dimensional grid to summarise major aspects of language development. One dimension accounted for driving theories, dominant research paradigms, major theorists, research studies, and goals. The other dimension listed key psychological and philosophical perspectives (maturational readiness, developmental readiness, connectionist, emergent, social constructionist, and critical theory). Issues raised in this chapter could be considered in the light of Hill's framework or Bauersfeld's interactionist framework (1998), or the hermeneutical perspective put forward by Brown (1997), or the semiotic emphases of Pimm (1995) and Chapman (1997), or Brousseau's (1997) theory of didactical situations.

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In her model of language shifts in mathematics education, Chapman (1997) argued that school mathematics is a social practice which comprises the interrelated dimensions of social context, culture, and language. Her analysis suggested that language, in any or all of its forms, must contribute to both the interactional and thematic development of any mathematics lesson. Text-connecting practices in thematic development are especially important and need to be seen as "transformative practices which are essential to 'critical' mathematics learning" (p. 172), even though they are often implicit rather than explicit in classroom practice. Chapman's (1997) model has two dimensions represented as two continua; one extends from "the metamorphic form to the metonymic form" and the other from "low modality to high modality" (p. 172). According to Chapman, a shift towards more mathematical language, which involves a correlational shift towards the metonymic form and towards high modality, is a requisite of mathematics learning" (p. 172).

Curriculum Theory and Language Considerations

At the beginning of the new millennium there is an increasing tendency to attempt to link discussions on *literacy* and *numeracy*, and to consider the role of these concepts in a theory of curriculum more suited to an inclusive philosophy of education which embraces the concept of lifelong learning. We have decided, therefore to dedicate a section of this review to certain aspects of the language of contemporary curriculum development, especially in relation to mathematics education in Australia and New Zealand. This is consistent with the framework adopted because curriculum development was one of the named categories (Ellerton, 1989).

Special attention in this section of the review will be paid to five issues:

1. How is numeracy defined, and in particular, what is its relationship to literacy?
2. Variations in definitions of mathematical terms.
3. To what extent is mathematics itself a language?
4. How is the concept of *levels* defined, and in practice how are levels (in any particular area of the curriculum) validated?
5. What are the roles of analogues and metaphors in the communication of ideas, especially mathematical ideas?

Numeracy, Literacy, and the Politics of Mathematics Education

An important aspect of language which has the potential to impinge in educationally significant ways on mathematics classrooms and mathematics curricula is the definition and image of mathematics explicitly and implicitly projected by teachers and by those who develop mathematics curriculum documents. During the period under review the issue of what constitutes numeracy (or numeration), and the relationship which numeracy has to mathematics and literacy, became a subject of debate. According to FitzSimons, Jungworth, Maaβ and Schloeglmann (1996), the term "numeracy" was first used in the Crowther Report released in the United Kingdom in 1959. In that report, numeracy was taken to mean "the minimum knowledge of mathematics and scientific subjects which any person should possess in order to be considered educated" (FitzSimons et al., p. 756). However, that definition is obviously a slippery one. What is minimum knowledge? Does the term numeracy really embrace scientific knowledge? And what is the meaning of the term educated person?

FitzSimons et al., who recognised that numeracy is "a many-sided concept" (p. 757) and discussed the issue of what kind of mathematics is really necessary for most people, and in which contexts. They could not reach any firm conclusions on that matter because, they said, much too little is known about how mathematics is used or might be used in trade and professional occupations, and indeed in all areas of life.

The Political Rhetoric

During the period under review, politicians and education bureaucrats associated with government and non-government systems of education attempted to develop the notion of numeracy as the mathematical correlate of literacy. Much of the education rhetoric emanating from governments over the period insisted on the need to identify basic literacy and numeracy skills in terms of student outcomes, and then to establish nationally agreed-on grade- or level-related benchmarks. This, it was claimed, would enable literacy and numeracy *standards* to be operationalised, and annual government-controlled pencil-and-paper testing regimes could be used to establish initial performance levels. Thereafter, progress towards the achievement of higher standards could be monitored. The following passage, from Wilson (1998), is typical of the rhetoric:

> The success of the early school years can be judged most effectively through the surrogate of measures of literacy and numeracy. We can be confident that the child who achieves essential knowledge and skills in those areas in the first three years has a necessary but not sufficient basis for education worth having.
>
> That is why literacy and numeracy benchmarks are being developed for Years 3 and 5. They are like a judgment on the first line of a story. They tell us whether the critical first step has been taken. ... A child who is not successful in literacy and numeracy in the early years has damaged chances of succeeding in school in general. (p. 3)

Statements like this raise several questions: What is numeracy? How is it related to literacy? How is it related to mathematics? How should it be measured? Can it be taught? What needs to be done in schools to promote number sense? (McIntosh, 1996).

Australasian Attempts to Define Numeracy

The Autumn 1998 issue of the Curriculum Corporation's periodical, *EQ Australia*, was dedicated to the theme "Literacy and Numeracy." The first two articles in a section on numeracy were titled "What is mathematics?" (Holton, 1998) and "What is numeracy?" (Lovitt, 1998a). It is significant, perhaps, that the word numeracy did not appear once in the three-page article by Holton, a professor of mathematics (except in a preamble to the article, written anonymously, which stated that "What is mathematics?" is a different question from "What is numeracy?").

Lovitt (1998a) argued that although numeracy is an ill-defined term most educators believe that it has something to do with "key ideas which underpin what is needed by adults to be full participants in their community, and the lack of which would render those adults disadvantaged" (p. 12). According to Lovitt, numeracy is more than numbers, but embraces space, measurement, statistics and logic. From a school perspective, numeracy is more than mathematics in the sense that responsibility for its development should not rest solely with the mathematics department (see also Hogan, Jeffrey, & Lewis, 1998).

Lovitt (1998b) recognised that "the search for an established definition and place for numeracy in the Australian curriculum continues within each State and Territory" (p. 13). He expressed concern about the possibility that some States seemed to be determined to monitor levels of numeracy and literacy through centrally-developed pencil-and-paper tests. These, Lovitt (1998a) maintained, "conjure up notions of a minimalist position which is not at all what many advocates for finding a shared meaning are attempting to create" (p. 12).

If one reads the special issue (Autumn, 1998) of *EQ Australia*, which was dedicated to the theme "Literacy and Numeracy," one is struck by the lack of substantial research findings offered by those supporting government policies on numeracy. One is also struck by the generality of the terminology. The terms numeracy and literacy seem to be inextricably linked to political agendas. On the other hand, who would deny that it is important for all school children to acquire the mathematical skills needed to survive with dignity in contemporary society?

As Sträßer and Zevenbergen (1996) have pointed out, what is the point of students acquiring a mathematical skill if they do not recognise everyday situations when the application of that skill would be useful? Newman (1983) error analysis research (see Clements & Ellerton, 1996a; Ellerton & Clements, 1996b; Rasidah Junaidi & Wong, 1998; Whitting, 1998) has repeatedly demonstrated that in traditional school mathematics regimes, many students who pass pencil-and-paper mathematics tests do so by regurgitating memorised knowledge and processes. They do not know which skills to use when confronted with word problems or real-life situations in which the skills are needed. From that perspective, if the effectiveness of numeracy profiles is to be measured by pencil-and-paper tests then there is likely to be a serious validity problem.

The first three letters of the words of numeracy and number are the same, and an unfortunate likely development associated with the increasing popularity of the term numeracy is that the widely-held view that mathematics is mainly to do with numbers will be further emphasised. McDonaugh (1998), who investigated primary school children's beliefs about the nature of mathematics, found that the children did not refer to space and chance and data concepts. Emily, one of the students interviewed, believed that the only tasks which should be labelled as mathematical were those for which numbers were involved.

Variations in Definitions of Mathematical Terms

From a semiotic perspective, much mathematical learning depends on learners being able to link concepts (the signified) with symbols (signifiers or signs). Thus, for example, Wright's (1998) research has revealed that many 5- to 7-year-old children find it difficult to learn the numeral and number words for the abstract concept signified by the English word "twelve". In fact, much of school mathematics is concerned with helping students to link signs, symbols, words, abstract concepts, skills, rules, and real-life situations. That, presumably, is the stuff of numeracy, and the aim is to develop applied *number sense*.

Confusion over the meaning of terminology can hinder conceptual development, and from that perspective it is interesting to realise that sometimes the curriculum documents of different education systems use the same word to denote different mathematics concepts, and sometimes they use different words to denote the same concept. In the period under review, several authors drew attention to differences in definitions of mathematical terms that exist not only from school system to school system, but also from textbook to textbook, and from one examination board to another. Thus, for example, Noridah and Clements (1999) pointed out that an examination question on composite functions on a 1998 Cambridge (UK) A-level examination assumed a definition of composite functions which did not appear in the Cambridge guidelines. Furthermore, the assumed definition differed in significant mathematical ways from definitions presented in the majority of textbooks available in nations which presented students for the Cambridge examinations. Noridah and Clements (1999) reported interview data showing how this had caused confusion with students in Brunei Darussalam.

Noridah and Clements (1999) also reported data which indicated that differences in definitions of mathematical concepts, such as a trapezium, were likely to confuse students. Their paper revealed that some curriculum documents define a trapezium as a quadrilateral with "exactly one pair of parallel sides," but others prefer "at least one pair of parallel sides" (p. 244). Furthermore, data were presented showing that even experienced teachers were confused by the fact that different textbooks use different terms (e.g., trapezium and trapezoid) for the same concept (see also Clements, 1999b).

The formation of appropriate cognitive links depends on whether a person not only knows the necessary linguistic agreements, but is also able to associate this with previous episodes and internalised imagery. For instance, someone whose mental image of a square is a quadrilateral with horizontal and vertical sides is unlikely to believe that a square is still a square if it is rotated through, say, 45° (Whitland & Pegg, 1999). Similarly, someone who believes that a trapezium is a quadrilateral with one pair of parallel horizontal sides is likely to have difficulty accepting that a rhombus with four oblique sides could be a trapezium. That difficulty is likely to remain even if they are told that many mathematicians prefer to define a trapezium as a quadrilateral with at least one pair of parallel sides (rather than exactly one pair of parallel sides). Chinnappan, Lawson, and Nason (1999) used teachers' concept maps to show that even experienced secondary mathematics teachers may not be fully aware of the relationships among quadrilaterals. They suggested that this could adversely influence the quality of their students' learning.

During the period of the *New Math*(s) there was much debate about how an angle should be defined, and whether symbols like 90° referred to an angle or to the size (or magnitude) of an angle. Is an angle a set of points (the union of two rays with a common end-point) or is it the amount of turning from one ray (or line segment) to another? Although debate on such matters can easily become overly academic, research tends to suggest that we do need to link concepts, with appropriate language and applications. Furthermore, the language and applications that we choose to emphasise can create word patterns, visual images, and memories of episodes which will influence students' conceptual development. Thus, Mitchelmore and White (1998a) who investigated relationships between concrete experience and the angle concepts of 144 students in Years 2, 4, and 6, found that "dynamic similarities were only reported between moveable situations, whereas static similarities were reported between all situations" (p. 271). This led Mitchelmore and White (1998a) to recommend that the main focus in the initial teaching about angle should be on identification of the arms of an angle in situations, rather than on the turning aspect.

Is Mathematics a Language?

The extent to which mathematics can be regarded as a language was taken up by Jourdan (1999) and by Galligan (1997a, 1997b). Jourdan provided examples to support his argument that, like any natural language, mathematics has its own vocabulary, its own terminology, its own notational systems, and its own grammars (which consist of rules and conventions that shape the way it is written). Jourdan also recognised, however, that mathematics is not a natural language in the same sense as, for example, English and Arabic because there is no group of people for which mathematics is the first language.

In a cross-cultural study comparing the different structures of the language of mathematics in English and Mandarin, Galligan (1997a, 1997b) reported results which suggested that although language appears to be an important factor in processing mathematics word problems, the selection of schematic models is also important. One of her findings was that the structure of word problems in written Mandarin appears to advantage Chinese readers, in comparison with English-speaking counterparts (see also Galligan, in press).

Levels and Stages

In the 1990s the concept of a level has been widely used in government-sponsored mathematics curriculum documents in Australia and New Zealand (and elsewhere) and in research. Neyland (1998), after documenting the widespread use of the concept of levels, drew attention to the inherent contradiction between the language of many contemporary competency-based mathematics curriculum documents and arguments by philosophers that formal mathematics knowledge does not have "an overall and unique hierarchical structure" (p. 68). Neyland quoted Ernest's (1991, p. 237) statement, "If the mathematics curriculum is to reflect the discipline of mathematics faithfully, it must not represent mathematics as having a unique fixed hierarchical structure." However, if mathematics is not hierarchical, then what can we say of curriculum documents that place mathematics knowledge, and tasks, in levels, and interpret children's responses to tasks as being "at" particular levels?

Noridah and Clements's criticise the popular van Hiele's framework for geometrical development which assumes five levels of development. According to the theory, at Level 1 children do not realise that there is a relationship between squares and rectangles, but at Level 2 they come to know that every square is a rectangle and every rectangle is a parallelogram. Noridah and Clements (1999) pointed out that such differences, however, could be attributed to linguistically-based definitional differences rather than to developmental differences.

To put the matter bluntly, many primary and secondary students believe that an equilateral triangle is not an isosceles because their teachers have emphasised that a triangle with three equal sides is an equilateral triangle and *not* referred to it as an isosceles triangle. If these students report that "three sides means it is not isosceles" (Currie & Pegg, 1998a) then it would be difficult to argue that such data can legitimately be interpreted in terms of stages or levels. As Whitland and Pegg (1999) have asserted, it appears to be the case that important clarifications are needed if the van Hiele model "is to be relevant in describing the development of young children's geometric understandings" (p. 553).

The evolution, in 24 secondary students (Years 8 to 11), of cognitive linkages between types of triangles (scalene, right-angled, isosceles, and equilateral) was investigated by Currie and Pegg (1997, 1998a), who interpreted their data in terms of van Hiele levels and the SOLO model. In another study, Currie and Pegg (1998b) investigated the evolution of cognitive linkages between different classes of quadrilaterals (square, rectangle, rhombus, parallelogram, kite and trapezium), and these results were also interpreted in terms of van Hiele levels. The studies appeared to assume that all students were familiar with definitions which were based on principles of class inclusion (e.g., Every square is a rhombus because ...).

Warren (1999) interpreted data from her study into lower secondary students' conceptions of a *variable* in terms of Küchemann's (1981) six stages of development for the concept of a variable. Warren gave a written algebra test to 379 students aged between 12 and 15 years, and interviewed some students after the test, for the purpose of probing students' views of the concept of a variable. According to Warren, it was difficult to interpret many of the students' responses to questions on the pencil-and-paper test. Thus, for example, pencil-and-paper results suggested that many students thought of a letter as representing an object, but the interview data seemed to indicate that this assumption was simplistic.

Mitchelmore and White (1998b) adopted the language of stages in their proposed framework for research into angle concept development. They defined three stages: (a) situated concepts, (b) contextual angle concepts, and (c) abstract angle concepts. Their research sample consisted of 36 students from two Grade 4 classes, and they concluded that, between Grades 2 and 4, most children form contextual angle concepts of turns, crossings, slopes, corners, bent objects, and bent paths. In that sense, they have moved well into Stage 2. Mitchelmore and White (1998b) questioned why currently "abstract angle knowledge develops largely as a result of secondary schooling" (p. 23). (See the chapter on "Making Connections with Space and Measurment" for more details of their research.)

After a two-year study of primary school children's probabilistic thinking, Jones, Langrall, Thornton, and Mogill (1999) described the development of such thinking in terms of levels. They constructed a 4-level framework that covered development of thinking about the concepts of sample space, probability of an event, probability comparisons, and conditional probability. They labelled their four levels, subjective, transitional, quantitative, and numerical, and linked their theory with Biggs and Collis' neo-Piagetian models of general development in which sensorimotor, iconic, concrete-symbolic, formal and postformal modes of functioning were assumed to exist, as were four cognitive levels (prestructural, unistructural, multistructural, and relational). The language and thinking of individual children were interpreted according to this complex framework. Thus for example, the "puzzling" conceptions of one girl (who, according to Jones et al. [1999], tended to function in an iconic way), were classified as being at the prestructural level, because "the pervasiveness of her subjective judgments essentially conflicts with her other judgments about whether all outcomes could occur" (p. 505).

We would comment that, in general, it is difficult to validate stages and/or levels of development which are defined and used as data classification devices in research studies. That is especially the case if development is regarded as mainly an age-related psychological or biological phenomenon. Even if a stage/level classification scheme recognises that development occurs as a result of many factors, it may not take account of the unique features of many students' past experiences, and especially their interpretations of events and discourse which have transpired in classrooms in which they have been present.

Furthermore, the association of children's responses with particular stages or levels, although popular with those espousing outcomes-based curriculum frameworks, can be restrictive and even problematic (Boulton-Lewis, 1999). The attention of teachers and researchers may focus on framework-related considerations, but other events, which someone else with a different framework might have regarded as important, can go unnoticed. In that sense, the danger of *fitting data to the theory* is real. This points to the need for research designs to build in checks on reliability of classification and interpretation of data, and to articulate assumptions which underpin the research (e.g., assumptions about students' past experiences, their understanding of mathematical language and definitions).

If a group of students has not experienced direct teaching of particular concepts or symbols, then the students are likely to respond to stimuli introduced by a researcher in terms of what they want the researcher to know about *their* interpretations of events, people, symbols, and signs surrounding the stimuli. The selective attention of researchers, on the other hand, can be influenced by the research framework (e.g., van Hieles' levels, or SOLO categories), which they are imposing on the situation while the framework is unknown to the students.

In particular, students' responses to set tasks (of the pencil-and-paper variety, or otherwise) involving mathematical concepts can represent little more than the students' knowledge of agreed linguistic conventions and understandings, rather than indicators of various stages or levels of development. Because the nature of the tasks has to be communicated to the students, it follows that language must be involved, one way or another. If there is a mismatch between the language in which the task is presented and the student, or between the language and expectations of the classifier and those of the student, then classification by levels can be misleading. Furthermore, there is the possibility that a different set of researchers could prefer to interpret the data according to a different research framework.

Some years ago, Ellerton and Clements (1994) commented on the twin difficulty of (a) validating levels for education research, and (b) ensuring that allocation to levels (by teachers or by researchers) is done reliably. In this review we draw attention to four dangers associated with levels classification. The first is that there is always the possibility that the person classifying data does not have access to enough relevant data to be in a good position to classify accurately. A second danger is that the classifier does not fully understand the system of classification. A third danger is that the levels may not be carefully defined and technically validated, in which case reliable and meaningful classification is impossible. And the fourth danger is that the selective attention of those carrying out the classification into levels can be so focused on the system of classification that the exercise becomes a self-fulfilling prophecy (whereby data are made to fit a theory).

To illustrate the danger, we shall refer to an example used by Pillay, Wilss and Boulton-Lewis (1998), who described the development of students' algebraic thinking in terms of four levels, namely inappropriate, arithmetic, pre-algebraic, and algebraic. These researchers asked students to solve the equation $3(x + 7) = 24$. The most efficient way to proceed (even though one could easily proceed using an algebraic process to such a task in the following way, "Three eights are 24, so x must be 1." Pillay et al. would probably classify such a response as at the pre-algebraic level.

With respect to van Hiele levels classification, Lawrie (1999) felt that there was a need to give "a sharper quantitative edge to what had been primarily a qualitative procedure" (p. 216). Her response to this perceived need was to apply the Rasch model in the *deeper* analysis of test items (in the context of elementary geometry) used for classifying 60 preservice primary teachers' responses to van Hiele levels. Although a sample of 60 is usually regarded as too small for Rasch analysis, the idea of establishing levels-theory on a more solid, less subjective, foundation, is noteworthy.

After postulating the existence of a five-level hierarchy of thinking with respect to multidigit number sense Jones, Thornton, Putt, Hill, Mogill, Rich and Van Zoest (1996) carried out studies which were intended to validate the hierarchy. In a report on their study, Jones et al. (1996) did not refer to any of the standard classification schemes for addition and subtraction word problems (e.g., Change, Combine, Compare, Equalise), yet the problems they presented to the students involved counting, partitioning, grouping and number relationship tasks which could be classified according to standard systems. The researchers' focus was on their five-level hierarchy (pre-place value, initial place value, developing place value, extending place value, and essential place value), yet a major factor influencing student performance could have been the semantic structures of tasks.

Swafford, Jones and Thornton (1997) found that although the van Hiele levels of 31 of 39 primary teachers were at the first three levels, at the conclusion of a four-week course, 29 of the 39 were in the top two levels. They concluded that for adults either the progression to higher van Hiele levels can be rapidly facilitated by instruction, or the tests of van Hiele levels are very sensitive to recall in geometry and perhaps not reliable for people who have been away from geometry for an extended period. We think *both* possibilities are likely to be true. With respect to the second possibility, we would comment that adult teachers are likely to have well-developed language skills. If, for example, they are told, in a professional training program, that any equilateral triangle is also an isosceles triangle because it has *at least two* congruent sides, then they are likely to understand and remember what has been said. However, if a Year 4 primary pupil is told that any equilateral triangle is also an isosceles triangle because the expression "*at least two* congruent sides" is satisfied by a triangle with "three congruent sides," then they might not grasp the meaning of such a semantically complex statement.

Analogues, Metaphors and the Communication of Mathematical Ideas

A longstanding issue in mathematics education is the extent to which children can be assisted to learn through the use of concrete representations or other analogues which, from the perspective of a mature mind, seem to be easily connected to a mathematical concept (Boulton-Lewis, 1999; Kanes, 1996). Thus, for example, *base-10 blocks* are widely used in primary schools for the purpose of assisting children to develop numeration and computation (English & Halford, 1995). Many teachers intuitively believe that, if students are given the opportunity to manipulate a set of wooden cubes with dimensions 1 x 1 x 1 cm^3, 1 x 1 x 10 cm^3, 1 x 10 x 10 cm^3, and 10 x 10 x 10 cm^3, then this will improve their place-value concepts (Price, 1998). The teachers' confidence is based on their recognition that the volumes of the blocks, in cm^3, correspond to the natural numbers 1, 10, 100, and 1000, respectively. Furthermore, they are trained to teach children to manipulate the blocks in particular ways in the expectation that this will assist them to understand the rationales for standard place-value notation and for pencil-and-paper algorithms for the four arithmetical operations. The blocks can also be linked with fraction concepts and operations on fractions.

Price (1998) compared the effectiveness of Year 3 children using base-10 blocks and a computer software-generated tutorial program, for revising two-digit number concepts and teaching initial three-digit number concepts. Results indicated that, although some participants improved their place-value concepts, the improvement was by no means uniform. Price concluded that the use of base-10 blocks will only be effective if it is based on careful planning. Of particular importance was his conclusion that "a teacher of young children cannot assume that students are making sense of number representations in the same way that the teacher is" (p. 459).

The reservations about base-10 blocks expressed by Price (1998) echoed English and Halford's (1995) comment that the effectiveness of the blocks "will be limited if the children do not form the correct mappings between the analog representations and the target procedures" (p. 106). English and Halford argued that the effectiveness of analogues like the base-10 blocks is likely to be enhanced by verbal links made by the teacher during the learning sequence, and by the learners' own verbal explanations in which they interpret what they are manipulating in numerical terms. English and Halford also emphasised the importance of multimodal representations (e.g., when money is introduced, or when place-value charts are used in association with base-10 blocks and verbal explanations).

A multimodal teaching strategy developed by Wong and Veloo (1996, 1997) recommends that students' representations of mathematics concepts, and their mathematics problem-solving efforts, be framed by six modes of mathematical representation: (a) the real thing, (b) a story, (c) a diagram, (d) appropriate terminology, (e) appropriate numbers, and (e) other appropriate mathematical symbols. Although we would applaud the idea of expanding the modes of communicating in mathematics lessons, we would question the need to ensure that, for any particular concept, all students go through each of the modes.

English (1998a) examined the analogical reasoning of 11-year-old children who were attempting to make sense of, and solve, complex multiplication and division problems. English promoted a three-level framework (problem text, problem situation, and mathematical) for studying the mental models formed and used by the students. Her conclusion that analogical transfer is unlikely to occur unless the child is able to recognise common structures provides, according to one commentator, "the key to developing students' abilities to learn" (Kehle, 1999, p. 472). Like many other cognitive psychologists, English (1998a) paid particular attention to the role of metaphor in providing a scaffold to extend and support student learning.

A similar position was taken by Gervasoni (1999), who reported on the work of an Early Years Research Team, in Victoria. Case studies of three teachers generated data which suggested that early childhood learners develop better number sense when problem-solving activities are not only used, but linked to community events. Furthermore, opportunity needs to be consistently provided for students to use oral language through discussing ideas with fellow pupils and with the teacher. Opportunities for interaction through games which encouraged estimation and prediction—in other words, active learning—were also deemed to be important.

Ellerton (1996a) warned against the development of over-specialised languages in mathematics education research "which are used to communicate with and between only a narrow range of people who use the same genre" (p. 2). In a subsequent paper, Ellerton (1997) likened the tensions and dilemmas which face mathematics educators to a "series of roads, criss-crossing different lives and providing choices for the individuals who traverse them" (p. 373). Ellerton then elaborated upon dilemmas concerning mathematics classrooms, content, curriculum and culture, and accountability, professional development and research, by representing each dilemma by two sides of a triangle, with the third side suggesting the need to bridge the dilemmas via an appropriate solution. The bridges are unlikely to be built, however, if those on each side have not learned to speak the same educational language.

Expanding the Modes of Communication in Mathematics Classrooms

Research on Rich Learning Environments

It is now received wisdom in mathematics education that mathematics will be best learned in environments in which a range of both receptive and expressive modes of communication are encouraged. Mousley (1999), for example, reported that mathematics educators believe that students who understand mathematical concepts are likely to be able to talk about, explain, link, apply across contexts, and represent the concepts. By contrast, if students are restricted to one mode of communication then the development and application of concepts and skills are likely to be impoverished. Thus, for example, Baturo, Cooper and McRobbie (1999) reported that Karen, a Year 6 student who appeared to learn fraction concepts extremely well through a computer-based tutorial program, was found, in an interview situation, to have a much weaker understanding of fraction concepts than was expected. Similarly, Baturo and Cooper (1999) reported that Year 6 and Year 8 students "had incomplete, fragmented, or non-existent structural knowledge of mixed numbers and improper fractions," even though these had been dealt with comprehensively in class.

In recent years, the view that students' constructions of knowledge are correlated with the quality of the articulations that they can express and the cognitive links that they can demonstrate has received additional emphasis because of the increasing attention paid by educators to socio-cognitive influences on learning. Australasian researchers and teachers have increasingly tended to argue that schools need to provide rich environments which will assist individual learners to make, and articulate, appropriate *curriculum-relevant* constructions (Clarke & Clarke, 1998; Ellerton, 1996c).

Furthermore, if assessment of mathematics learning is to be authentic, it needs to incorporate multimodal language forms if it is to be consistent with the multimodal learning environments (Ellerton, 1997). In other words, single-mode pencil-and-paper tests and examinations should not be more than a part of an overall multimodal scheme of assessment (Beesey, Clarke, Clarke, Stephens, & Sullivan, 1998; Clements & Ellerton, 1996b). There is evidence that this has been achieved far more often in primary schools than in secondary schools (Clarke, Gronn, & Clarke, 1998).

Studies on the value of diagrams and multimodal learning in learning and teaching are reviewed in the chapter "Making Connections with Space and Measurement" in this review book. They include studies by Diezmann (1997, 1999a, 1999b), Owens and Outhred (1997, 1998), and Outhred and Sardelich (1997, 1998).

Thomas, Mulligan and colleagues (Mulligan, Mitchelmore, Outhred, & Bobis, 1996; Mulligan, Mitchelmore, Outhred, & Russell, 1997; Thomas, Mulligan, Goldin, 1996) studied representations of counting displayed by children in Grades K–6. They explored how primary school children's internal representations for numbers develop towards becoming autonomous systems. The internal systems were classified as *verbal/syntactic* (using mathematical vocabulary, developing precision of language, and offering self-reflective descriptions), *imagistic* (non-verbal, non-notational mental models such as visual and kinaesthetic representations), and *formal notational* (using notation, relating notation to conceptual understanding, creating new notations). A strong conclusion of the Thomas et al. (1996) study was that the active processing of images plays an important part in the development of children's understandings of numeration. This led to the recommendation that an expanded set of modes of communication be facilitated by teachers, with children's mental images being "described, drawn, compared and discussed in class" (p. 312). In addition, although children's external representations, both static and dynamic, may not correspond to conventional mathematics, the view should be taken that they reflect each child's unique internal constructions.

A developmental framework and multimodal instructional program was proposed for nurturing multi-digit number sense by Jones et al. (1996), and was designed to cover number development up to, and including, mental addition and subtraction of numbers to one thousand. The study adopted what might be termed a Vygotskian sociocultural approach, with the teachers effectively trying to extend students' thinking within their zones of proximal development. The researchers presented data which suggested that not only did the students progress between levels but also the teachers involved believed that they themselves had gained a deeper appreciation of the concept of number sense and how this might be developed in young children.

Hunting (1996) is another to have emphasised the need for teachers to assist students to integrate language, action, and emerging numerical concepts. Pepper and Hunting (1998) also argued that the modes of communication do not develop simultaneously and, in particular, young children can develop conceptual understanding of addition and subtraction before formal written notations are introduced. Further details of the Mulligan and colleagues' studies, Jones et al. studies, and Hunting studies can be found in the chapter on "Early Arithmetical Learning and Teaching" in this book.

Outhred and Owens (1998) noted that children's attention often needs to be drawn, by teachers, to certain features of shapes, the use of composite shapes (e.g., a row of tiles), and to possible fractional units. Owens and Clements (1998) and Owens, Perry, Conroy, Geoghegan and Howe (1998) suggested that mathematics education researchers need to give more attention to certain less obvious, but nevertheless crucially important, variables that influence the quality of children's representations: two such variables would appear to be *selective attention* and *responsiveness* to learning environments and learning tasks.

According to Menzel and Clarke (1999), in a *negotiative* process whereby teachers mediate between content and student, the responsiveness evident within the mediation is likely to facilitate student learning. In this process, teachers acknowledge student interpretations and consequent mathematical constructions, and they also elicit representations of the students' knowledge. This enables teachers to adapt, modify, or elaborate the initial representations in ways which promote so-called correct mathematical interpretations and constructions by students.

Teachers often encourage students to think, and to create written representations of concepts, in particular ways. Sometimes, a student's use of a particular mode of communication, such as writing or drawing a diagram, is more related to the teacher's preferences than to the student's preferences. Obviously, many children feel most comfortable when they use the mode of expression expected of them by their teacher. There are, however, some general rules about preferences for representations which can be gleaned from research. In his doctoral study, Lowrie (1996) found that when Year 6 children were encouraged to use whatever mode they preferred on a set of tasks which included familiar and unfamiliar tasks, the students preferred to use more verbal analytic approaches when tackling familiar tasks, but more visual approaches when attempting to solve unfamiliar problems.

Writing to Learn Mathematics

Waywood (1996), who, in the late 1980s, carried out a four-year longitudinal study into secondary students' writing about mathematics and their reflections on mathematics lessons, theorised the writing-to-learn phenomenon in an article aptly titled "The Beginnings of a Phenomenology of Writing-to-Learn." According to Waywood, one needs to look beyond the writings of Vygotsky, Piaget, Dewey, Halliday and Bruner if one is to establish an adequate theoretical base for the movement. For Waywood (1996), analytical, critical and phenomenological philosophy has broken the link between language and the inner world. Thus, for example, the language expressed in a piece of writing and the inner world of the writer reflects a "manifestation of the resources available for saying available within the culture" (p. 604). It also reflects a choice "to say or not to say" (p. 605).

Waywood's (1996) paper is complex, and the interested reader is referred to Mason and Waywood (1996) for a more complete exposition. Mason and Waywood (1996) commented that the hermeneutic tradition which developed from the interpretation of sacred-text *reading* has been extended to take almost any thing or act as a *text*. Thus, Mason and Waywood asserted, "language is like the air we breath or the water that surrounds a fish" (p. 1078). However, the patterns of language use, the discourse, presupposes and formats what can be said. Does thought require language or does language emerge through thought?

Towards the end of his paper Waywood (1996) commented:

> Phenomenologically, writing to learn mathematics can be a "leaping in" of the word of schooling or a "leaping forth" as a word of learning, it is the dialogue of these words that is meant by attending or intending becoming. While the dominant metaphor of schooling is to attend it is difficult for students to become learners. (p. 611)

A writing-to-learn text is a communication about becoming a learner. What one reads in a text does not represent the writer's inner self, for that "is never expressed but is the silent frame" (p. 611).

Reflective writing can also assist mathematics teachers and teacher education students to begin to question traditional teaching practices, and this is especially likely to occur if the teachers are encouraged to move beyond mere description to analysis and explanation. Sullivan and Mousley (1998) used their interactive multimedia package, which permits users to carry out an intensive study of a mathematics lesson (Mousley, Sullivan & Mousley, 1996), as a stimulus for reflective writing, small group discussion, and interviews—all obviously language-based mediums—with teacher education students. Sullivan and Mousley (1998) reported "a noticeable improvement in the quality of the language of thinking associated with the interpretation of teaching incidents" (p. 111). Zevenbergen (1997b), who reviewed the interactive multimedia package, commented that "the resource enables 'participants' to study in depth the processes involved in the teaching/learning process" (p. 113).

Withers (1996), arguing from a metacognitive perspective, urged teachers of mathematics to assist students to write creatively about the mathematics they are studying. In stressing the value of *process writing* in mathematics, Withers stated that such writing benefits the students, for it encourages them to reflect on mathematical relationships. Teachers also benefit by gaining access to the problems students pose, and to the ways they solve problems. This can reveal the students' interests, their conceptual links, and their preferred modes of thinking, which can assist teachers in lesson preparation, in lesson delivery, and in dealing with individual students.

However, despite such potential benefits, many teachers seem to think that getting students to write about mathematics and to reflect on their learning of mathematics, are in the too-hard basket. Thus, for example, Bicknell (1999) reported that although senior secondary mathematics teachers were enthusiastic about the value of getting students to express explanations and justifications in writing, they recognised that there were "distinct disadvantages for students weak in language skills" (p. 79). Bicknell reported that the students, for their part, were positive about the writing process, recognising that being required to give written explanations and justifications made them think about what they were trying to say. Nevertheless, all of them found it difficult to write adequate justifications for their reasoning, and were concerned that they did not really know what constituted a quality response. Bicknell (1999) recommended that writing should become a normal part of mathematics lessons, and that students be provided with exemplars of the kind of writing that was expected of them.

Shield and Swinson (1996) introduced what they called the *link sheet* to both primary school students and teacher education students. This device is simply a page divided into four rectangular sections, one section each for a mathematical example, an everyday example, a diagram or picture (or graph), and an explanation. Shield and Swinson claimed that link sheets can be used as a pre- or post-instruction aid to assist students to develop helpful conceptual links in long-term memory. The onus is placed on the student to construct overall meaning by using four different modes of communication.

Although many writers have advocated the use of reflective writing tasks in mathematics learning environments, there is not much hard evidence to support claims that the practice has had long-term educational benefits. Shield and Galbraith (1998), in a major review article on "Student Expository Writing in Mathematics," argued that students' writing in mathematics has been constrained by the models of mathematics and the modes of communication of mathematics experienced by children in schools. They asserted that this makes it extraordinarily difficult for teachers to move most of their students towards higher levels of thinking about mathematics.

Shield (1996, 1998) and Shield and Galbraith (1998) provided a coding scheme with five main descriptors: approach, recount, generalise, link, and integrate. Shield and Galbraith (1998) emphasised the value of nurturing students' ability to write generalised statements of procedures, to demonstrate procedures, to link procedures with prior knowledge, and to demonstrate how the procedure fits with other aspects of mathematics. They claimed that if teachers were able to separate and reward such features in children's writing, then this would be likely to stimulate deeper mathematical thinking and understanding by the children.

Language Factors and Research Based on SOLO Taxonomy Categories

Callingham (1998) used the SOLO taxonomy to analyse the responses of 136 students, and of preservice and inservice teachers to tasks in which graphical displays required interpretation. She concluded that the preservice and inservice teachers gave a range of "responses in iconic and concrete symbolic modes," thereby "demonstrating multimodal functioning" (p. 205). She noted that when the teachers were faced with more difficult tasks, they changed from "responses in earlier modes ... to those demonstrating higher levels of understanding" (p. 221). Callingham urged teachers to develop learning environments which encouraged multimodal responses from students and thereby facilitated "movement from the iconic to the concrete symbolic path" (p. 222). In another study, Callingham (1999) once again employed the SOLO taxonomy, this time to analyse responses of three Year 8 classrooms on tasks which could be done at various levels of sophistication. Although unistructural-multistructural-relational (U-M-R) cycles were identified, some tasks did not attract any relational responses, and Callingham concluded that further work was needed, "particularly in relation to large-scale testing" (p. 141).

Chick and Watson (1998) used the SOLO taxonomy in classifying cognitive behaviours of groups of students working in triads on tasks involving the interpretation and representation of given data. Chick and Watson (1998) developed a four-category classification system for the level of collaboration within groups. Categories ranged from no collaboration between group members, to considerable useful collaboration within a generally harmonious social environment. Chick and Watson were particularly interested in the extent to which collaboration directly affected mathematical outcomes. Not unexpectedly, they found that on particular tasks the SOLO levels were different for different members of the same group. The researchers reported that they "had a definite feeling that the groups often behaved more as three individuals than a genuinely collaborative team" (p. 159).

Mulligan and Watson (1998) used a multimodal version of the SOLO taxonomy as the basis for their research into the formation of images "at the prestructural and iconic level" by Years 2 and 3 primary school children. They reported that in an initial interview, 20% of the children gave prestructural responses to word problems which invited multiplication and division responses. Mulligan and Watson stated that when the children learned to interpret the semantic structure of the problems, and to represent equal-sized groups through physical or concrete models, "they began to focus on numerical aspects of the problem in the iconic mode" (p. 79). For example, the students responded to problems like "Twelve toys are shared equally among the children. If they each had three toys, how many children were there?" For many early primary school children the semantic structure of a question such as this would be complex, and it is hardly surprising that Mulligan and Watson found that semantic structure was a major hurdle for many children. This could have had a pronounced effect on the SOLO classification of pupils' responses. More will be said concerning semantic structure and mathematics word problems later in this review.

Multimodal Teaching Strategies

Wong and Veloo have worked together in Brunei Darussalam throughout the period covered by this review on a series of studies aimed at providing a research base, and materials, for expanding the modes of communication in mathematics teaching, learning and assessment (see, for example, Veloo, 1996; Veloo & Wong,1997; Wong & Veloo, 1996; Wong, Lim-Foo & Veloo, 1998; Wong & Rose Dalilah, 1999). They have developed a Multimodal Teaching Strategy (MMTS) which emphasises the need for learners to translate flexibly within and between six different modes of representation, and to develop divergent thinking.

The six different modes are summarised on a Multimodal Think Board, and each mode is related to one key action verb, namely, real thing → do, word → communicate, number → calculate, symbol → manipulate, diagram → visualise, and story → apply. The main idea is to encourage students to use all of the modes of communication when grappling with key concepts in all topics in school mathematics curricula (see, for example, Wong and Rose Dalilah [1999] for details of a MMTS approach to teaching and learning percentages) in much the same way that Shield and Swinson (1996) suggested for their link sheet. A major teacher education application is the development of linked materials and lesson plans. In addition, tertiary mathematics teacher educators worked with teachers in classrooms in introducing to students different modes for the same task. Wong, Lim-Foo and Veloo (1998) summarised the largely positive (although there were some negative) reactions of teachers, students, and parents to MMTS developments. A large amount of developmental work has been carried out, and a cooperative research program, between mathematics educators in Brunei Darussalam and Singapore, is planned.

Problem Posing

During the period under review both mathematics curriculum development authorities (see, for example, Maguire, 1999) and Australasian mathematics education researchers have increasingly recognised the potential of problem posing for generating expressive modes of learning in school mathematics learning environments. Significantly, each researcher cited in this section of the review acknowledged the relevance of language factors, especially the influence of semantic structure, in problem-posing situations. Ellerton (1997) indicated that if classroom teachers of mathematics merely strive to teach students to generate correct answers to set problems, then the ability to solve problems is unlikely to be significantly enhanced. Paradoxically, if the metacognitive processes, especially the awareness of the semantic structure and mathematics relationships, of learners are encouraged by asking students to pose problems, then this is likely to have a strong, positive effect on their problem solving.

Obviously, students engaged in the process of posing problems are required to construct questions, and therefore are likely to go beyond merely receptive modes to adopting more creative expressive modes of communication. Stoyanova (1998a), in her doctoral research into the problem posing of capable secondary students, developed a classification framework which assumed that every problem-posing situation can be classified as free, semi-structured, or structured.

Stoyanova regarded a problem-posing situation as free if students are asked to generate a problem from a given, contrived or naturalistic situation. Thus, for example, students might be asked to make up problems that might be suitable for a mathematics competition, or problems which they themselves would find difficult to solve. A situation is semi-structured if students are given an open situation and are invited to explore the structure and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences. For instance, a primary school student might be asked to make as many problems as possible in which a given sequence of numerals and operation signs (e.g., $3 \times 25 + 15 \div 5 - 4$) would be used. And a situation is structured if problem-posing activities are based on a specific problem. Stoyanova and Ellerton (1996) offered the following example of a structured problem-posing situation:

> Last night there was a party and the host's doorbell rang ten times. The first time the doorbell rang only one guest arrived. Each time the doorbell rang after that, three more guests arrived than had arrived on the previous ring.
> Ask as many questions as you can. Put them in a suitable order.

Stoyanova's research revealed that capable children can be highly creative in their problem-posing, making full use of most of the common modes of expressive communication (see Stoyanova, 1998a, 1998b; Stoyanova & Ellerton, 1996).

As part of a three-year study into the problem-posing performances of seventh-grade students, English (1997a, 1997b) monitored the development of each of 29 students, 23 of whom participated in a three-month classroom problem-posing program (and the other six served as controls). The program involved the students in (a) problem exploration and reflection, (b) statement of problem preferences, (c) problem sorting, (d) modelling new problems on existing structures, (e) creating new problems on existing structures, and (f) creating a new problem from problem components. In addition, open-ended situations invited the students to transform given problems into new problems. English reported that although the 23 students showed educationally significant improvements in their abilities to pose problems (in comparison with the six control students), the program made little difference to some of the students' willingness to pose problems involving semantically complex language structures.

In an intervention study in which Year 7 students became involved in a Thought-Revealing Problem Program, English (1999) found that students improved in their ability to pose problems from open-ended situations. They also learned to identify problem structures, and became better problem solvers and, apparently, more divergent and flexible thinkers. From a language perspective, their expressive language developed within the context of problem situations. Not only did they learn to respond to given problems, but they also developed in their ability to create related problems.

English (1998b) also carried out research with eight-year-olds into four issues concerned with problem posing:

> 1. Do children pose a variety of problems for the standard symbolic addition and subtraction sentences? That is, do they recognise the formal symbolism as representing a range of problem situations?
> 2. Do children generate a broader range of problem types for informal number situations in which there is an absence of formal symbolism?
> 3. How do children with different profiles of achievement in number sense and novel problem solving respond to problem-posing situations in symbolic and non-symbolic contexts?
> 4. Does participation in a problem-posing program lead to a greater diversity in problem types created in the two contexts and how do children with different achievement profiles respond to such a program? (p. 86)

In English's (1998b) research, the eight-year-old children were presented with a range of formal and informal stimuli in one-to-one interviews, and were invited to pose word problems. Further interviews were conducted after the students had participated in sixteen 45-minute problem-posing sessions. In interviews conducted before and after the problem-posing sessions, English found that the addition-subtraction word problems that the children posed were all of the Change variety. This, coupled with the observation that the children had difficulty solving problems with more complex semantic structures (e.g., those of the Compare variety), once again drew attention to the fundamental importance of semantic structure in primary school mathematics. English (1998b) also reported that the children in her study had much greater difficulty creating a range of problems in formal contexts than in informal contexts, and that those children with different achievement profiles displayed different response patterns, especially with respect to mathematical structures and operational complexity.

Further discussion on problem posing is presented in the chapter "Research in Problem Solving in Mathematics Education" where studies such as those by Lowrie (1999) and by Outhred and Sardelich (1998), and Maguire (1999) indicate the significant role of language and the difficulties some students faced. In the chapter on "Issues in Mathematics Teacher Education", Southwell (1998) shows how problem posing verified preservice teachers reflections on their own problem solving. Maguire (1999) identified four issues which needed to be investigated:

1. If children work collaboratively, do they generate semantically more complex problems?
2. Does experience in problem posing lead to semantically more complex problems being posed?
3. Does exposure to stimuli (such as photographs) result in semantically more complex problems being posed?
4. What kind and degree of support do teachers need if they are to become more successful in incorporating problem posing into the mathematics curriculum?

Sociolinguistics

Adler (1998) called for Australasian researchers to interrogate meanings of culture used by researchers in the field of mathematics and language. As Bauersfeld (1998) has pointed out, many and varied meanings have been given to the noun *culture* and to the adjective *social*, and that as early as 1952 at least 400 attempts to define culture could be located. In this section, the following description of culture originally offered by Mousley and Clements (1989) will be assumed:

> The term "culture" generally refers to social heritage—those characteristic behaviours which are transmitted from one generation to the next. While the notion of culture includes collective mental artifacts such as symbols, ideas, beliefs and aesthetic perceptions (and mathematics has recognisable forms of each of these), it also embodies the distinctive forms of discourse, behavioural norms of common patterns of the organisation of institutions, groupings and rituals. (p. 398)

This description appeared in an article entitled "The Culture of Mathematics Classrooms," but clearly in mathematics education our use of the term culture is not restricted to classrooms. That is why we lean towards the above description of "culture," which embraces all contexts and situations. For as Cox (1999) has shown, learning acquired outside the classroom can render much of what transpires in the classroom nonsensical if the mismatch between the experiences and language of the teacher and those of students are so great that, in fact, neither is likely to communicate to the other even if they are trying to talk about the same thing.

In the following discussion it will be assumed that anthropological investigations are, essentially, concerned with identifying and describing culture. Furthermore, since this is a review on language factors, this section on sociolinguistics reports on research which has explored links between language, culture, and mathematics education. Also, in view of Adler's (1998) critique, we would wish to reiterate that the framework on which this review is based assumes that in our quest to identify and understand factors which influence teaching and learning of mathematics, cultural factors need to be seen as pre-eminent (Ellerton, 1996b;1999a).

Barton's (1996) review "Framework for Discussing Anthropological Perspectives and Mathematics Education" located writing and other modes of commentary on or about mathematics and culture along a continuum. The continuum was intended to represent the extent to which the subject of the writing is "inside or removed from the internationalised version of 'capital-M' Mathematics" (p. 1037). In particular, Barton placed the following seven categories of mathematical writings along his continuum: (a) Pure Mathematics; (b) Applied Mathematics; (c) Mathematical Studies (e.g., articles on design, or navigation); (d) Mathematics in Cultural Settings (e.g., weaving systems, counting procedures, sporting statistics); (e) Descriptions of the Culture of Mathematics (e.g., history of mathematics); (f) the Cultural System of Mathematics (e.g., accounts of how mathematicians think about the nature of mathematics); and (g) Mathematics as a Cultural Phenomenon (where mathematics is compared with, say, religion, or science, or art).

Barton argued that although most of the work linking mathematics and language has been done by mathematics educators, nevertheless, the weak version of the Whorfian hypothesis implied that the mode of language used in any mathematical discourse must affect the thinking about the inherent qualities of the mathematics of those who interact with the discourse. In other words, the *world view* of students of mathematics will be linked to the world view embedded in the mathematical discourse.

From that perspective, Barton (1996) drew attention to three cultural interfaces that affected mathematics and mathematics education in Australasia (and other parts of the world) in the 1990s:

1. The increased migration of Chinese, Koreans, Vietnamese, and Malaysians to Pacific Rim countries;
2. The leap in mathematics exchange through the Internet (see, also, Michener, 1997); and

3. The renaissance of individual peoples through educational initiatives.

According to Barton, these three situations "draw attention to the role of language in mathematics" and challenge the general perception of the subject as the most free of linguistic influence" (p. 1045).

Barton's second major contention was that the importance of bilingual and Indigenous language mathematics teaching is likely to increase, rather than decrease, adding that the way people mathematise is part of their heritage "and enculturation depends on the use of first language" (p. 1050). As Ellerton (1997) pointed out, in school mathematics the issue of "single language policy" versus a policy which allows the language used in instruction in a mathematics classroom to correspond to the first language spoken by the majority of learners, represents a major dilemma for education policy makers at the beginning of the new millennium. It will not go away (Clements & Ellerton, 1996a; Ellerton & Clements, 1998a).

Learners' Language(s) and the Language of Instruction in Mathematics Classrooms

In commenting on the situation with respect to Maori language and school mathematics in New Zealand, Barton, Fairhall, and Trinick (1998) argued that both language and mathematics were being acted on by opposing forces. On the one hand there was "the risk of becoming fossilised," and on the other, the danger of being corrupted" (p. 3). According to Barton et al., the issue of whether mathematics should be taught in indigenous languages was not a serious one for speakers of languages like English, French, Russian, or Mandarin, for in those languages mathematics and its applications can be discussed at its highest levels. However, "for small indigenous groups, this is a critical question" (p. 3). They pointed out that the issue had been attended to in different ways in different places. In Israel, for example, the language of instruction is Hebrew, but in New Caledonia a world language (French) is used in mathematics classes, and in Brunei Darussalam, English is used although the language of everyday communication is Bahasa Brunei. English is used in secondary schools in Papua New Guinea, despite the fact that it is not the first language of most of the learners (or of most of the teachers).

According to Barton et al. (1998), the debate is particularly strong among indigenous groups emerging from a colonial past and/or during a cultural renaissance, because "language is a carrier of culture, and without such a renaissance is difficult" (p. 3). Barton et al. examined aspects of the experience in New Zealand over the past 20 years. They pointed out that it took 15 years for Maori mathematics vocabulary to change from intermittent primary use in primary classrooms to its use in curriculum documents up to the end of secondary school. They argued that as a result of the experiment a Maori mathematical discourse had emerged with distinct characteristics.

Like most forms of mathematical discourse, Maori mathematical discourse is "conceptually dense, multi-directional and jargon-filled" (p. 6). However, other characteristics are peculiar to Maori mathematical writing "and may be attributed to vocabulary and grammatical developmental processes" (p. 6). The use of the passive mood, for example, is widely used in Maori mathematical discourse because the passive mood is more common in Maori than in English (altnough it is common in mathematical English). Also gerunding--the creation of nouns from other parts of speech—which is a traditional Maori language construction, is frequently used in Maori mathematics discourse. However, Barton et al. (1998) argue that use of the passive voice and gerunding is so common in Maori mathematics discourse that this is affecting the way Maori school children speak Maori. In other words, the mathematics discourse is contributing to fundamental changes in the language.

Barton et al. (1998) argue that it was a wise decision to have mathematics classes in the Maori language, and to have classes where two languages are used and code switching is encouraged. There are numerous benefits for all concerned including mathematicians and educators who are being forced to "push out the boundaries of the definition of 'mathematics' to include some culturally-based activities or concepts which were not previously regarded as part of the subject." However, there are dangers. In the search for cultural identity, it sometimes happens that old ways are mistakenly adopted. "Apprenticeship learning, rote learning, and algorithmic teaching have all been suggested as culturally appropriate for Maori" (p. 8).

Barton et al. (1998) closed their article with some important questions that are worth reproducing here:

> Mathematics, as well as Maori language, may be caught between similar poles. Internationally, mathematics is being communicated and conducted increasingly in the English language. Is it being fossilised into modes only expressible in that language? Are new avenues of mathematical thought being cut off by not doing mathematics in new languages? Is it possible that a new generation of mathematicians, brought up through an indigenous language, may be some of the most creative mathematical thinkers of the next century? (p. 8)

As Barton et al. (1998) comment, such questions are "intriguing" (p. 8). A teacher-linguist, Roberts (1998), wrote on mathematical registers in Australian Aboriginal languages. Roberts makes many of the same points for Aboriginal languages and mathematics as Barton et al. (1998) make for Maori language and mathematics. Roberts (1998) maintained that because mathematical language registers are a new development in Aboriginal languages, there are presently no expressions which *sound right*, but rather only different potential ways in which to express the information. If communities make explicit decisions about how they want to express themselves mathematically, these decisions will determine those expressions which will come to "sound right." (p. 10) Even so, there are about 90 Aboriginal languages still spoken in Australia—some are spoken by several thousand people, but others are spoken by only a few hundred. For all of these languages, Aboriginal speakers have been able to discuss mathematical ideas, using appropriate vocabulary and grammatical structures. However, Aboriginal cultures do not use mathematics in Western ways, and therefore their languages structure and sequence indigenous mathematical concepts in unique ways.

Roberts (1998) identified three possibilities for the development of mathematical registers in Aboriginal languages:

- Only use English to discuss Western mathematics;
- Elaborate Aboriginal languages, so that they incorporate some English grammatical structures, in order that some mathematical concepts are more easily able to be discussed;
- Integrate the Western mathematical concepts into Aboriginal languages, so that traditional Aboriginal grammatical constructs are used to discuss these Western mathematical concepts. (p. 11)

Roberts added that "no alternative is inherently better than the others" (p. 11), and invoked Cummins' threshold hypothesis when pointing out some of the virtues associated with the second and third of these possibilities. The adoption of the third strategy, in particular, might assist Aboriginal people to develop a mathematics register which reflects the world view embodied in their language. It might also be the best strategy for generating greater understanding of Western mathematics in Aboriginal communities.

In a report on an interview with an Aboriginal Education Assistant teaching in a geographically remote primary school, Howard (1996) stated that the Assistant believed that Aboriginal children were good at mathematics. However, "it was the language in the form of long words, students' lack of understanding of the words that are used when they are writing that makes mathematics hard for them" (p. 301). The Education Assistant specifically drew attention to the language used in mathematics tests. "That," he said, "was a major reason for Aboriginal children failing" (p. 302).

Of course, the above assumes that everyone decides that the *game* of school mathematics is worth playing. Both Bishop (1999) and Zevenbergen (1997a) discussed Bourdieu's notions of *language game*, and of *dominant* and *dominated* groups. Zevenbergen (1997a) used quotations from teachers to emphasise the notions of social expectations and *cultural capital*, and the conservative agendas influencing the work of those teachers who want to have more socially just mathematics education programs. Bishop (1999) asked readers to interpret the following quotation in terms of the first group being teachers and the second, learners. Usually, the teachers, as the dominant group, believe most completely in the illusion of the game. They have no disjuncture of experience that would bring the game into view. The dominated groups, on the other hand, are in one of four positions:

> First the dominated group fails to see the situation as a game, takes it as a naturalized ideology, and consequently aspires to "better" jobs, ways of talking, etc. Secondly, the dominated group knows it is a game which it wants to participate in by adhering to the rules of the game as given, thus shifting its

> position and contributing to the status quo. Thirdly, the group works with the game, in order to appropriate the rules to its own advantage, and thus takes an active role in changing some of the rules over a period of time. Finally, the dominated group can always resist the dominant ideology by exposing the game itself and unmasking the rules which the dominant group uses to shield itself against any potential rupture. (Roberts & Sarangi, 1995, p. 369)

This is a powerful quotation, and capable of wider interpretation in the context of mathematics education than Bishop suggests. Reflect, for example, on the possibilities if the game is curriculum reform, the dominant group is taken to be bureaucrats and politicians, and the dominated group comprises teachers of mathematics.

Code Switching

The extent to which code switching (that is to say, switching between the language of instruction and a student's first language) occurs in mathematics classrooms is increasingly being recognised as an important issue in mathematics education. Clarkson (1996), in a study of 80 Year 4 Vietnamese and 32 Year 4 Italian background students in Melbourne, found that a large amount of code switching occurred within mathematics classes (in which English was the language of instruction). Code switching was especially likely to take place when students were grappling with difficult word problems. This was the case even though all of the students had completed all of their schooling in Australia. In fact, about ten percent of the students inwardly translated all of the mathematics word problems that they were asked to solve into their first language. About half of the Vietnamese students, and a quarter of the Italian students occasionally codeswitched. The data on Italian-background students surprised their teachers because many of these students were first generation Australians.

The same patterns reported by Clarkson (1996) were also reported by Clarkson and Dawe (1997), in an extended study which involved Year 4 students in Melbourne and Sydney. Clarkson and Dawe found that students with high competence in both English and another (first) language did better than other students on a mathematics test which included standard manipulations and on a test with mathematics word problems, but the performance differences were not statistically significant. Once again, it was found that students were most likely to codeswitch when attempting difficult word problems.

In a study which examined attitudes of non-Anglo-cultural-background (NACB) and Anglo-cultural-background (ACB) students in lower secondary schools in Melbourne, Bishop, Leder, Brew and Pearn (1997) found that many NACB students reported that the standard of school mathematics expected of students in countries from which they emigrated was higher than that in Australia. Brunei Darussalam, where one of the authors (Clements) of this chapter works, Singapore (Clements, 1998), and indeed quite a few nations in Southeast Asia cover many mathematics topics earlier than in any Australian state or territory or in New Zealand.

It would be wrong, however, to assume that this implies that Australasian curricula need to be made more difficult. Teachers of mathematics in Brunei Darussalam, for example, complain that their students are simply not coping with the mathematical and language demands of what they are required to teach (Clements, 1998). There is considerable evidence to support their assertions on this matter (see, for example, Burns & Charleston, 1997; Chamberlain, 1998; Clements, 1999a; Ellerton & Clements, 1998b; Norida & Clements, 1999; Rasidah Junaidi & Wong, 1998). With curricula, more (or more difficult) is not always better.

Classroom Cultures and Discourse Patterns

In the 1990s there was a surge in interest in what actually transpires in classrooms, and Ellerton (1997) called for researchers to provide guidance on the kinds of bridges which need to be built to overcome the weaknesses inherent in teacher-dominated classrooms and in classrooms where teachers do not do enough. Different bridges will be needed in different places, because cultural factors mean that different societies have different images of the role teachers should play. Ellerton and Clements (1998a) called for teachers and researchers to work together on action research projects aimed at identifying the right blend of instrumental learning and relational learning, and on ways of expanding modes of communication in mathematics classrooms in culturally appropriate ways.

Certainly, results of research reported by Mousley and Sullivan (1996), in which classroom interactions in a Year 6 class were videotaped and subsequently analysed, emphasised the need for interventions to be culturally appropriate. Although the impression of persons who observed the lesson was that the teacher had been *inclusive* with the students, Mousley and Sullivan raised difficult issues. Among these issues were: (a) Should students be encouraged to take a relatively equal part in overt exchanges in mathematics classrooms? (b) Should teachers ask more questions than students? (c) When is silence a good thing (from an educational perspective)? Mousley and Sullivan (1996) indicated that they were struggling with these and other related issues, and they called for others to become engaged in that struggle.

The struggle will not be made any easier by the fact that teacher and student preferences for various mathematics classroom discourse patterns are influenced by macro- and micro-cultural factors. Taplin and Kwok's (1996) paper provided an excellent illustration of how a determined secondary teacher in Hong Kong was able to introduce more discussion into his mathematics classrooms over a one-year period. At first, the students, school authorities, and even the teacher himself, were concerned that the experiment was not working. It was at odds with school policy on having quiet classes, and there was some concern that it would adversely influence the students' chances of success on examinations. However, after about five months a more positive response to the experiment became evident.

The multiplicity of personal and group forces operating within and between mathematics classrooms affect both teacher and student expectations of what behaviour is appropriate in the teaching and learning of mathematics. In essence, collectively, they operate to define cultures of classrooms. The use of the word cultures in the plural is deliberate, for each classroom context is different. Different rules frame the behaviours that occur in different classrooms. The same teacher may not teach in the same way with a different class, and indeed may interact with different students in the same classroom in quite different ways. Furthermore, the same teacher can change the way he/she teaches the same class, and learners in that class can choose, even learn, to behave in different ways in the same mathematics class, with the same teacher. As Waywood (1996) noted, in the classroom there are two senses of dialogue to be considered. First there is the public dialogue, and second, the inner dialogues of individuals within themselves.

Even within the same school, teachers from what may appear to be reasonable heterogeneous cultural backgrounds may go about teaching the same topic in quite different ways. This fact was illustrated by Mousley (1996) who asked eight primary teachers in three schools in a lower-middle-class area of the same city to give a lesson on the following:

> From a given piece of cardboard make a regular shape which holds one cup of birdseed. Make a similar shape which is twice as big.

Mousley made notes in individual interviews with each of the eight teachers immediately after a teacher was given the task, and made further notes in pre-teaching interviews. Data from videotapes of the lessons, and audiotapes and observations of discussions by groups of students in the lessons, were also obtained. Each teacher was shown snippets of the videotape of his/her lesson, and asked to comment on their intentions with regard to particular teaching actions. In her brief analysis, Mousley (1996) commented on how the data illustrated how socio-historical forces influenced both the overall structure of lessons and the choice of particular strategies and procedures within the lessons. In other words, discourse patterns that manifest themselves in mathematics classrooms do not just happen; they are shaped by individual and corporate expectations, and do not occur within a historical vacuum.

Although discourse patterns in mathematics classrooms are likely to differ from teacher to teacher—and even, for the same teacher, from class to class—there are recognisable similarities in the structures of, and texts generated in, most lessons. Tomazos (1996) studied the pedagogical and content registers reported by 12 students for their Year 7 (upper primary) and Year 8 (junior secondary) mathematics classes in Western Australia. She observed striking similarities in the students' verbal reports, and concluded that the students believed there was a standard format for mathematics lessons. In particular, there was always a task-orientation stage, a time when students were engaged with an actual task, and an endpoint. In students' words, a topic was "done". No student referred to "using" fractions, or algebra,. The words "constructing," "investigating," "planning," or even "calculating" were never used by the students. The words "addition," "subtraction," "multiplication," and "division," were often used. According to Tomazos (1996) the messages about mathematics and mathematics learning the students received from their teachers were different from those intended by their teachers.

Clements and Ellerton (1996a) pointed out that as teachers become more aware of the interests and abilities of students in the classes they teach, they can be inwardly challenged with respect to the language and teaching approaches they should use. Should they attempt to match the language that they use to a particular group of students, or should they attempt to adopt some neutral form of language which, they expect, would have a better chance of being understood by most students? That question confronts every teacher, even though an explicit formulation of it may not occur at a conscious level. Teachers' classrooms actions—including the words they use, the students to whom they choose to speak individually, and the teaching approaches they adopt—provide data which attest to the inner workings of their minds during lessons. The discourse patterns are also influenced by the teachers' values (Seah, 1999), which influence the ways they plan and conduct lessons and shape their responses to pressures exerted by significant others (e.g., school principals, parents, and textbook writers).

What kinds of factors can influence teachers to seek to change the ways they teach mathematics? Clarke (1997) reported that the two factors which seemed to influence most the professional development of a Grade 6 teacher whom he studied were: (a) the daily opportunity to reflect on classroom events in conversations and interviews; and (b) the provision of innovative materials. From a language perspective, it seemed that in the Clarke (1997) study, the fact that the innovative materials were presented in final form generated within the teacher a degree of faith that agreement had been reached that such materials can be effectively used in classrooms. Yet, the teacher felt a need to talk about the materials, and how they might be incorporated into the classroom. Regular discussions with Clarke seemed to persuade the teacher that not only were there good reasons for the suggested changes, but that also it was possible for the teacher to achieve desired changes.

Clarke (1996) argued that one pathway to knowing mathematics is via the resolution of uncertainty, and that the process of resolution is fundamentally negotiative. And because negotiation is mediated by language, subjective interpretations of knowledge and events inevitably play an important role in an individual's negotiations and renegotiations of meaning. According to Clarke and Helme (1997), negotiation is a cyclic process of refraction (construal), reflection, and representation, the ultimate goal being consensus.

Clarke and Helme (1997) studied 28 mathematics classroom episodes in which one or more students expressed uncertainty, and efforts were made, sometimes unsuccessfully, to resolve the uncertainty. They noted that uncertainty could exist in relation to factual knowledge, procedures, meanings of terminology or representations, and the correctness of someone's statements. They also noted that the process of resolution could be achieved in different ways—sometimes, it came from appeal to, and acceptance of, empirical evidence; sometimes it was based on the authority of a knowledgable person or well-regarded text; and sometimes, it came from appeal to prior experience.

The idea of negotiation was also central to the *landmark* metaphor used by Ellerton (1998, 1999b) in her analysis of discourse patterns in a Year 8 lesson on the area of a triangle. Ellerton likened mathematical understanding to a destination, and the teacher to a navigator who guides learners along their journey. It is the navigator's role to give directions which link with what the learner already knows, and to guide rather than take absolute control. The aim is to enable the learner to negotiate the journey on future occasions, accurately and confidently and without the navigator being present. Ellerton's (1999b) analysis of the Year 8 lesson on the area of a triangle revealed that the teacher used landmarks which were not familiar to the learners, and so all subsequent explanations were virtually meaningless for the students. This can, as Shield (1999) noted, result in teachers resorting to "telling the students the way to do [problems]" (p. 443).

Barnes (1998, 1999) was particularly interested in how power relationships between members of collaborative groups influence the content, discourse patterns, and cognitive and affective outcomes, of the group, as a whole, and of group members individually. Barnes's (1999) framework for analysing talk within groups included notions of *disputational talk*, *cumulative talk*, and *exploratory talk*. The effect of the presence of a teacher was also considered, and it was found that if a teacher was present then the dynamics of group interaction were likely to be completely changed from what occurred when the teacher was not present. With the teacher present, students were likely to compete for the teacher's attention, and to try to impress the teacher with what they know. Barnes (1999) found that in cumulative talk, students usually achieved the successful resolution of uncertainty, and confirmed and validated one another's statements. Exploratory talk did not result in resolution of uncertainty. Indeed, it sometimes contributed to the generation of negative attitudes.

Macmillan (1997, 1998b), whose research was with preschool and first-year-at-school children, adopted Halliday's discourse analysis theories and techniques in her analysis of the effects of power relationships within groups on knowledge and concept acquisition and on attitudes. She explored links between oral genres and socio-cognitive processes, paying special attention to the roles and the effects of the following in class or small-group discourses: (a) logical and clarifying explanations; (b) procedural explanations; (c) reports and recounts; (d) arguments and discussion; and (e) evaluations. She found that in early childhood contexts, genres which relied heavily on logic were rarely productive because awareness of what might be a good reason was only beginning to emerge for most children.

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She concluded that the most prevalent genres of the children's self-initiated discourses, namely clarifying and procedural explanations and reports, were precursors to genres involving consistent rationality, order, and coherence (recounts, arguments, and evaluations).

Macmillan (1997, 1998b) reported that although small-group situations allowed the teacher to attend to, observe and monitor children closely, it also allowed for concealment of uncertainties. In addition, power relationships within a group had vitally important effects on learning and attitude formation. Whether a child received support or challenge from other group or from other class members was contingent upon the individual's status. It was also dependent on the child's interest in sharing ideas and impressions. Although some children were powerful models (even surrogate teachers) to other children. others were not well regarded by a group. The latter often struggled to gain acceptance, and if they failed they were likely to acquiesce by not attempting to engage socially or cognitively in group tasks.

Another paper concerned with power relationships was that by Brown and Renshaw (1999). In their analyses of a Year 7 mathematics class they employed a Vygotskian framework and Bakhtin's concept of *voice*. The latter "provides a means of identifying processes of appropriation and resistance as the teacher and students ventiloquate, revoice or challenge each other during collaborative activities" (p. 115). The particular class they investigated had been using a form of collective argumentation often used by teachers who encourage group-based learning. The students were asked to represent the task or problem alone and then compare and justify their representations within a small group of peers. Those in the group were expected to reach agreement within the group, and then present the group's ideas and representations to the class to test their acceptance by the wider community of peers and the teacher.

According to Brown and Renshaw (1999), their analyses suggested that students learned to accept the need for ideas to be meaningful (personally understood, not just copied), communicable, and testable. Students were even willing to challenge the authority of their teacher, and this was accepted by the teacher; although this required a joint effort from the teacher and the students "to resist old practices and to regulate each other's adoption of the more symmetrical roles" (p. 120).

Galbraith, Renshaw, Goos, and Geiger (1999) also used a neo-Vygotskian framework and Bakhtin's concept of voice in an analysis which provided a sociocultural perspective on learning in secondary classrooms where technology was integrated as a central resource. Galbraith et al. (1999) theorised four roles for technological devices, such as calculators and computers, in which different voices may be heard:

1. Technology as master ("What is it allowing me to do?")
2. Technology as servant ("Tell the thing what to do!")
3. Technology as partner ("How can we do this together?")
4. Technology as an extension of self ("Come fly with me!")

According to Galbraith et al. (1999) these four roles constituted a hierarchy. When technology is master, the student is subservient to the technology, but at the highest level, the efforts of student are merged with the power of the available technology.

Examples of research in which teachers consciously sought to depart from traditional mathematics classroom discourse patterns were reported by Buzeika (1999) and Forster (1999). In Buzeika's (1999) study, two lower primary teachers in Auckland specifically aimed at assisting their students to make sense of multi-digit computation by asking them to use and to record summaries of the ways they used a range of mental methods. Opportunities for discussion were provided at all times, all contributions were openly valued. Recording formats which reflected the descriptions of the children were encouraged.

According to Buzeika (1999), both teachers believed that although the children completed fewer examples than in more traditional mathematics lessons, they spent more time discussing their work with the teacher and with other students. However, although the question "Does anyone have a different way of working this out?" was routinely used by both teachers, both teachers recognised and responded to parental pressure to show the students standard algorithms. The teachers thought that a school policy needed to be established, making it acceptable for algorithms to be invented by students (and to be then represented symbolically). The existence of such a policy would have made it easier for them to justify, to themselves and to significant others (e.g., parents, other teachers, the school principal), the more expressive approach which they had used.

In Forster's (1999) study, the concept of class participation was investigated using a "multiple-paradigmatic research methodology" (p. 207) in which a "social constructionist" viewpoint was assumed. Referring extensively to an episode in a Year 11 mathematics classroom in which vectors were expected to be used in solving a three-dimensional problem, Forster explored the extent to which meaning was shared between the teacher and a student. Among the conclusions Forster reached were: "participation is negotiation," "participation is constituted by and constitutes cognition," "participation is interpreting language," "participating is persevering," "participation is assuming/ relinquishing authority," and "participation is being competent" (pp. 210–211).

Atweh, Bleicher, and Cooper (1998) also used Halliday's approaches to discourse analysis. Proceeding from the premise that mathematics classrooms are places where teachers and students socially construct interactive environments, usually with the primary goal of promoting learning, they studied the construction of the social contexts in two contrasting secondary classrooms. One of the classes was in an old, highly regarded, academically-oriented boys' school and the other in a girls' school in a working-class suburb. Although both teachers used teacher-driven, teacher-centred methodologies, analyses of the language they employed revealed that they had quite different expectations of their students. With the class of boys, the teacher believed that most of his students were bound for the university and, accordingly, provided rigorous definitions and made comments on cultural and historical factors, as well as everyday events that might be associated with topics being considered. The teacher of the girls' class, on the other hand, used less formal, more personalised language, and explanations tended to be aimed at helping the students remember *rules of thumb*.

Atweh et al. (1998) analysed the differences in the tenor and mode of discourse in the two classes. Interactions in the boys' class were often characterised by sarcastic remarks, by both teacher and students, and ironically this tended to foster a sense of equality between the teacher and the boys. The boys were active participants in the development of mathematical knowledge, taking responsibility, for instance, for checking their own answers. The girls, on the other hand, responded in matter-of-fact ways to their teacher's polite and gentle characteristics. At all times they allowed the teacher to be in control, and, also ironically, this meant that they acquiesced in a situation which suggested that it was the teacher, and not themselves, who was responsible for their learning.

Zevenbergen (1999) studied interactions between a teacher and a class of upper primary school boys and girls. Over a series of lessons, the boys were always asked substantially more questions than girls and, in particular, girls were rarely asked a question in the introductory phases of lessons. In her paper, Zevenbergen (1999) reproduced and analysed a transcript of a lesson in which she claimed the pattern and content of the discourse contributed to the "construction of masculinity" (p. 353). She added that some of the boys, as well as many of the girls, were effectively "marginalised from the study of mathematics" (p. 357).

The paradox that "the more teachers teach mathematics, the less mathematics is taught" was raised by Kanes (1999, p. 283). He questioned the adequacy of Shulman's typology of teacher knowledge (consisting of *content knowledge*, *curriculum knowledge*, *pedagogic content knowledge*, and other kinds of knowledge relating to the needs of students and characteristics of the context of teaching), and suggested that mathematics teachers' pedagogic content knowledge might not assist mathematics learning. According to Kanes (1999), Shulman may have been mistaken when he placed this body of knowledge at the centre of teachers' work, and insisted that expertise in this body of knowledge was at the heart of teaching expertise.

Kanes (1999) went on to raise the possibility that the language used in mathematics classes "has become entangled" and "precision has been lost, and this has given rise to meanings contrary to intentions" (p. 283). He reproduced transcripts of a Year 11 mathematics class on complex numbers which he interpreted as demonstrating that Shulman's pedagogic content knowledge can overshadow mathematical knowledge in actual teaching situations. Proceeding from a Vygotskian epistemological perspective, Kanes's theoretical framework included such concepts as *knowledge base*, *knowledge artefacts*, and *functional knowledge claims*. He interpreted his analysis as suggesting that the teacher's way of teaching, and her functional knowledge within her pedagogic content knowledge base, were "built around the dramatisation or staging of her mathematical knowledge base" (p. 288), as she combined the roles of observer, critic, coach, and commentator. Kanes (1999) concluded that the mathematical knowledge stood "in a peripheral relation to the lesson studied," and wondered whether "mathematics learning takes place because of the peripherality or despite it" (p. 290).

Like Kanes, Klein (1999) was concerned with the question whether current teaching practices in mathematics foster or inhibit numerate behaviours in students. Such is the strength of tradition, she argued, that even when a teacher tries to engage students actively in the learning process, the students may struggle to cope with a style of teaching which is unfamiliar to them (see, for example, Tomazos, 1997). Klein (1999) analysed practices in three primary mathematics classrooms in which group work was employed and students appeared to be actively engaged in their learning. She reported that, in fact, the learning environments were not *enabling* because they did not encourage exploration of patterns and relationships, they did not foster conjecture, and no communication of student-initiated mathematical ideas was observed. Rather, the emphasis was on *service mathematics* where the children were expected to learn facts and skills, and to learn to give right answers to explicit questions. All the activities were initiated and tightly controlled by the teacher. For Klein, the classes were not "conducive to the construction of a sense of agency so necessary for lifelong learners of the new millennium" (p. 306).

At an even more fundamental level, Zevenbergen (1996), questioned whether radical constructivism, an education philosophy that was accepted during the 1990s by many Australasian mathematics education researchers (Dengate, 1998), has helped to generate more democratic discourse patterns in school mathematics.

Vygotsky and Mathematics Education

Vygotsky's views on the role of language, and in particular the importance he attached to social processes, have had a significant influence on the thinking of many mathematics educators in Australia and New Zealand during the period 1995–1999. According to Vygotsky (1978), *psychological tools* such as language, counting systems, mnemonic techniques, algebraic symbol systems, writing, diagrams, and maps, mechanical drawings, and all kinds of conventional signs, are used by learners to influence others and to interact with them. Bickmore-Brand (1997), Gordon (1998), and Macmillan (1997) accepted Vygotsky's premise that the use of such tools influences not only choice of strategies in problem-solving situations, but also choice of categories for organising experiences. For Vygotsky, people's choices are intimately related to the linguistic, cultural, and social factors which frame their daily lives, and in that sense, language and other semiotic systems transform thought in ways that connect learners to their social environments.

Macmillan (1997, 1998a) studied texts and discourse patterns generated by children in early childhood settings, and found that the quantity and quality of mathematics learning was profoundly influenced by the social dynamics of classroom settings, especially power relationships among and between children. Gordon (1998), on the other hand, studied adult students as they attempted to learn elementary statistics. She concluded that learning statistics involves the whole person and is inseparable from the cultures and histories which surround them.

Bickmore-Brand (1997, 1998) developed the Vygotskian notions of Zone of Proximal Development (ZPD) and scaffolding, in her study of the effects of interactions between teachers and learners on mathematics learning and attitudes. In particular, she employed an anthropological research design to study the effects on the mathematics learning of the children in the two upper primary classrooms—one taught by a teacher who adopted learner-centred approaches, and the other by a teacher who used more content-driven methods. The two teachers held different beliefs about what they should do when teaching mathematics.

Bickmore-Brand's (1997, 1998) data supported her conclusion that children's thinking about mathematics, their valuing of mathematics, and their ability to link mathematics to unfamiliar but nevertheless appropriate real-life situations, were all affected by classroom climates, and especially teaching methods employed within classrooms. She argued that her study moved beyond constructivist approaches to embrace the more social, semiotic perspectives offered by Vygotsky. According to Bickmore-Brand, the teacher in her study who adopted the content-driven approach experienced problems in servicing the learning needs of many of her students. Those in the learner-centred classroom, by contrast, felt that students were regularly encouraged to express themselves freely and creatively. This, they believed, not only helped them to learn the mathematics, but also generated a feeling of academic well-being, even empowerment.

Galbraith et al. (1999) interpreted their data on the ways learners used technology in mathematics classrooms in terms of Vygotsky's ZPD, scaffolding, and communities of practice. Galbraith et al's analysis showed students using the *teacher's voice* both in giving explanations and in creating atmosphere for serious discussion. Dialogue was seen to be driven by the desire to solve and also to understand, and a graphics calculator was assigned a *servant* role. According to Galbraith et al. (1999), a classroom culture of enquiry developed and acted as a ZPD "for the cultivation and nurturing of mathematical maturity and expertise" (p. 229). Goos, Galbraith, and Renshaw (1996) analyse the student talk in order to evaluate when talk becomes collaborative mathematical discussion. Quality depended on students' orientation towards the task, their relative task-specific expertise, and the degree of challenge the task presents.

In contrast to the above studies, all of which concentrated on the analysis of oral classroom discourses, Goos (1999) studied the extent to which Year 11

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students were able to gain an understanding of elementary aspects of chaos theory by reading explanations and worked examples in a textbook. The teacher asked the students to read passages from the textbook, and encouraged them to talk to each other about what they read. The teacher then led discussions by asking questions such as "What happened here?" and "Is that right?" Goos sought to identify the metacognitive function of the students' dialogue, and to analyse this through a coding system. It was concluded that although many teachers are concerned about the possibility of students becoming over-dependent on textbooks, in the analysis presented the mathematical text mediated social interaction between students in a way that transformed their relationship with mathematical knowledge. Rather than treating it "as a pre-packaged object containing facts to be absorbed, the students interrogated, critiqued and appropriated the text by speaking its patterns in their own language" (p. 245).

Stillman and Galbraith (1998) also carried out a study which focused on metacognitive aspects of the problem solving of senior secondary students. They focused on the students' methods for processing problems, including the metacognitive activities that led to the processing. They used a response-map technique to identify characteristic response types in students' written scripts. Students were also videotaped working in pairs on problems, and analysis revealed a cyclic processing approach which involved information gathering, representing, processing, and validating. These steps rarely occurred in a linear progression, as "students cycled back and forth between them through the activities as they attempted to progress" (p. 183). The most successful groups tended to spend less time on problem orientation, and more time on problem organisation, and monitoring of progress (especially evaluation). According to Stillman and Galbraith (1998), problem-solving performance is likely to be enhanced if students are encouraged to make full use of their available cognitive and metacognitive resources. Helping students to learn to use their inner voice is important for problem solving.

Psycholinguistics

Distinguishing Between Semantic and Syntactic Structures

The difference between semantic structure and syntactic structure in mathematics word problems is not a simple matter. For example, at the 1997 MERGA annual conference, held in Rotorua, some Australasian researchers questioned the claim by the Hong-Kong-based researcher, Lopez-Real (1997), that the following two statements differ essentially in their *syntactic* structure:

p and q are numbers. p is 6 more than q. (1)

p and q are numbers. 6 more than q is the same as p. (2)

Lopez-Real (1997) claimed, in discussing his Hong Kong data, that there is no clear dichotomy of semantics and syntax, "but rather the mental models constructed by children are likely to be a combination of semantic and syntactic processing" (p. 317). At the same Rotorua MERGA conference, Galligan (1997a), following McGregor and Stacey (1993), used the term "syntactic translation." She reported data that suggested Taiwanese students use syntactic translation more than Australian students, pointing out that the English expression "the number x is 5 less than the number y" "does not translate syntactically directly but the Chinese version does" (p. 180).

Many of those who attended Lopez-Real's (1997) and Galligan's (1997a) presentations at Rotorua maintained that it was more accurate to say that the statements in the last paragraph differed in *semantic* structure rather than *syntactic* structure. They cited Resnick's (1982) definition of syntactic knowledge, as that involved in the correct performance of mathematical procedures, and semantic knowledge, as that involved in the understanding of meaning of meanings and procedures (see Dole, Cooper, Baturo & Conoplia, 1997). The view was expressed that, when the main issue is *overall meaning*, rather than making sure agreed-on rules of expression are being followed, then the term semantic structure is more appropriate than syntactic structure. Thus, if persons attempting to understand statements needed to grasp the *overall meanings* of word sequences which made up the statements, then semantics rather than syntax was involved.

In this chapter, the argument expressed at Rotorua that statements such as (1) and (2), above, differ in semantic structure (and not syntactic structure), will be taken as correct. Similarly, students who translated the statement "x is 2 more than y" as "$y = x + 2$," and thought that the statement meant that "to get y you had to add 2 to x" had failed to grasp the *semantic structure* of the original statement. On the other hand, the authors of this chapter would claim that students who, for example, believed that any letter must stand for 1 (because, for example, they had misinterpreted their teacher's statement that "x by itself means 1 times x") would be making a *syntax* error. MacGregor and Stacey (1996a) reported data which suggested that from 25% to 50% of middle-secondary mathematics students who had been studying algebra for four years were not aware of key syntactical aspects of elementary algebraic notation.

Pedagogically speaking, though, researchers' arguments about whether an error is due to syntactic or semantic structure are probably less important, than assisting teachers to recognise the likelihood of certain types of errors being made on certain types of questions. Teachers also need help in working out methods for helping students avoid, or at least, overcome common language-based errors. Thomas (1997) recommended that mathematics teachers, textbook writers, and examiners should link their presentations of concepts to contexts familiar to learners and avoid difficult linguistic constructions. There have been numerous attempts, in recent years, not only to link common errors with common practices in mathematics classrooms and mathematics textbooks (e.g., MacGregor & Stacey, 1996a; Stacey & MacGregor, 1999), but also to develop intervention procedures which will enable children to overcome persistent weaknesses (e.g., Pawley & Cooper, 1997).

The prototypical example of an apparently simple word problem which generates a high proportion of incorrect responses is the "Professors and Students" problem:

> Write an equation for the following statement: There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors. (Quoted from Pawley & Cooper, 1997, p. 320).

Many researchers, in Australasia and elsewhere (e.g., Pawley & Cooper, 1997) have reported that even a high proportion of upper secondary and university students make the reversal error (i.e., they respond $P = 6S$, rather than $S = 6P$). At issue is why the problem is more difficult than would be expected. In the 1990s, MacGregor and Stacey (e.g., MacGregor, 1991; MacGregor & Stacey, 1993) have made significant contributions to the literature on this issue, which has been investigated by researchers in many countries.

Respondents who write P = 6S for the professors and students problem and justify this response by saying that it means that "for every one professor there are six students," are semantically correct (and arguably correct according to their own constructed symbol usage). However, they would be regarded as making a syntax error by those knowledgeable in how symbols are used in algebra. Whether the error is to be regarded as syntactic or semantic, is beside the point, however, for there can be no escaping from the fact that, unless it is a careless error, it is a language-based transformation (which some writers call *translation*) error.

MacGregor and Stacey (1996a, 1996c) found that 90 students (in mixed-ability classes in Years 9, 10, and 11) experienced major difficulties in formulating algebraic equations to relatively straightforward word problems (e.g., "The three sides of a triangle are different lengths. The second side is 3 cm longer than the first side, and the third side is twice as long as the first side. The side lengths add up to 63 cm altogether. How long is the first side?"). The data indicated that almost all of the students comprehended the problems, for they could solve them by arithmetic means. However, in Newman (1983) error-analysis terminology, they were unable to cope with the Transformation process. MacGregor and Stacey (1996c) concluded that most of the students were not familiar with the concept of an equation, and they did not seem to regard an equation as a statement about relationships. In the context of responding to a word problem, these students thought of an equation as a summary—an abbreviated way of writing down the terms of the problem.

In another study (MacGregor & Stacey, 1998) asked 268 students in Years 9, 10, and 11 to solve three pairs of problems which had the same meanings but different semantic structures. One of the pairs was:

> A: Jan has $\$x$. Mark has \$5 more than Jan has. Altogether they have \$47. How much does each person have?
>
> B: \$47 is shared between two people, Mark and Jan. Jan gets $\$x$, Mark gets \$5 more than Jan gets. How much does each person get? (p. 54)

The other question pairs had semantic structures corresponding to Versions A and B, but were set in different contexts.

For students in Years 9, 10, and 11, a correct answer for Version A was obtained by 61%, 83% and 82% of the students, respectively, and for Version B 51%, 82%, and 72%, respectively, obtained correct answers. Similar performance differences occurred for the other two pairs of questions. MacGregor and Stacey (1998) concluded, after using the chi-square test, that statistical differences were not significant. We would argue, however, that this conclusion depends on the form of the null hypotheses and in particular on the statistical test employed. For example, if a null hypothesis was based on a binomial distribution, with nine trials, and probability of Version A getting a better result than Version B equal to 1/2, then with nine successes in nine trials, the results certainly would have been statistically significant. From our perspective, semantic structure affected the ability to formulate and solve an equation in an educationally significant way.

Word Problems With Additive (or Subtractive) Semantic Structures

One of the most remarkable achievements of mathematics education research of the past two decades has been the identification of a finite, well-defined set of semantic structures that arise in normal language and can be associated with word problems with structures which invite the use of additive and subtractive operations. It has been recognised that unless children learn to understand and be comfortable interpreting verbal and written texts which use any or all of these semantic structures they are likely to be experiencing difficulty in solving many mathematics problems.

Dawe and Mulligan's (1997) analysis of data from the NSW Basic Skills Testing Program (BSTP), for pupils in Grades 3 and 6, emphasised the importance of semantic structure on problem difficulty. Thus for example, 34% of Grade 3 pupils gave incorrect responses to the multiple-choice question: "Maria is 9. Her brother is 3. What is the difference between their ages?" Only 41% of Grade 3 pupils answered the following question correctly: "I am a well known number. Adding me to 12 gives the answer 12. What number am I?" (Possible answers: 0, 1, 10, and 12). The most popular wrong choice was 12, which was given by 29% of the respondents.

The fact that BSTP data were totally in the form of written responses to multiple-choice pencil-and-paper tests made it difficult to reach firm conclusions on why errors were made. Thus, for example, without interview data, one is left wondering whether the answer "12" was given because students guessed answers to the question randomly, or because the wording to the question contains the expression "the answer 12," or because of other reasons.

Dawe and Mulligan indicated that their analyses of BSTP data supported Lean, Clements, and Del Campo's (1990) contention that the semantic structures of questions made it difficult for pupils to comprehend the problems. Data from English's (1998b) study of the problem-solving and problem-posing performances of eight-year-old children, seemed to carry a similar message. English (1998b) reported that the linguistic pull of keywords in Change, Combine and Compare problems was the major source of error. Hyde, Power, and Zevenbergen (1999) reached precisely the same conclusion in their study of the performance on arithmetic word problems of 77 moderately to profoundly deaf students, ranging from Year 1 to Year 12. So too did Clements (1999a), in his study of responses to Change, Combine, Compare and Direct Comparison word problems by upper primary and lower secondary students in Brunei Darussalam. In other words, assuming those taking the tests had studied the topics being tested—which is a reasonable assumption for all the studies just cited—it appears to be the case that the semantic structures of questions, and the key words in the questions, were major factors influencing problem difficulty.

Maguire (1996) studied 282 word problems involving the four operations with whole numbers which were posed by a teacher of a composite Year 3/4 class in Victoria. He reported that the number of word problems in different categories differed quite noticeably. Some categories were well represented, but others (e.g., equalising addition and subtraction, and compare addition and subtraction), were not represented at all, or hardly represented. Maguire concluded that "far from providing 'a wide range of problems,' the problems were quite limited in range and often not relevant to the students' out-of-school activities" (p. 371).

We believe that one of the most important tasks facing mathematics educators and mathematics teachers at this time is to develop sound strategies for assisting learners to respond appropriately to problems which feature a wide range of semantic structures.

Word Problems With Multiplication (or Division) Semantic Structures

Although elegant classification schemes for word problems which require the use of additive or subtractive operations for their solutions have been developed and used extensively in mathematics education research over the past two decades, the same cannot be said of classification schemes which invite the use of multiplication or division. Clements (1999a) summarised the most widely used classification schemes in the latter category and concluded that, with the exception of the promising classification system developed by Mulligan (1992), most of the classification schemes that have been proposed are unnecessarily complex.

A different, but obviously related matter, is the range of methods used by young children when they tackle word problems which invite the use of multiplicative procedures for their solutions. Mulligan and Mitchelmore (1997) identified 12 calculation strategies used by lower primary school students, and from an analysis of these strategies developed three main "intuitive models": (a) direct counting, (b) repeated addition (or repeated subtraction), and (c) the multiplication (or division) operation (see also Mitchelmore & Mulligan, 1996). For Mulligan and Mitchelmore's (1997) study 60 Grade 2 or 3 children were interviewed, and the children's repertoire of intuitive models expanded following specific instruction on useful strategies.

Calculation strategies employed by the children when attempting to solve particular problems reflected the mathematical structure that they imposed on it. According to Mitchelmore and Mulligan (1996), "any of the observed strategies were readily explained by the size of numbers, the particular multiples involved, and the language used to describe them" (p. 413), especially superficial language cues. Of particular importance for this review was Mulligan and Mitchelmore's (1997) conclusion that although problem difficulty varied with semantic structure, the full range of intuitive models was employed across problems with different semantic structures. The structure of the preferred solution strategies did not necessarily correspond to the semantic structures of the problems.

Mitchelmore and Mulligan (1996) also reported that a major cause of error with comparison questions involving expressions such as "Sue has four times as many" was the linguistic cue "four times as many." Many respondents multiplied when division was required. This would appear to be another instance of the phenomenon documented by Lean et al. (1990)—respondents who cannot cope with the semantic complexity of a problem tend to replace the given problem with one which has a less complex structure. Furthermore, key-word approaches largely determine the semantic structure of the replacement problem.

According to Mulligan and Mitchelmore (1997), one explanation for the fact that the same intuitive models were used for problems with different semantic structures might be that every multiplicative situation had to involve "equal-sized groups." They emphasised that the intuitive model employed by a child reflected the mathematical structure that the child imposed on it.

Maguire (1996, previously mentioned) found that important multiplicative and division contexts such as comparison multiplication and division, and Cartesian product, were not given to the students. Also, the rules of thumb which were given were likely to reinforce common misconceptions (such as "when dividing, divide the larger number by the smaller," and "multiplication results in an increase, and division in an decrease").

Semantic Structure in Problems Involving Percentages

Dole (1999) and Dole et al. (1997) distinguished between the following three types of percentage application problems:

Type 1: Finding a part or percent of a number (e.g., 25% of 20 is *x*);

Type 2: Finding a number when a certain part or percent of that number is known (e.g., 20% of *x* is 6).

Type 3: Finding a part or percent one number is of another (e.g., *x*% of 15 is 5).

These three types of problems correspond to different semantic structures and, so far as students are concerned, these structures have different complexities. Dole et al. (1997) reported that interviews with Years 8 and 9 students revealed that only the most proficient students could recognise and cope with the different semantic and syntactic demands of Type 2 and Type 3 problems. Dole (1999) presented data indicating that a "proportional number line method" helped Year 8 students to solve percentage problems, even those with more complex language structures.

Ellerton and Clements (1998b) analysed data from a Year 8 class in Brunei Darussalam in which a teacher attempted to teach students to solve Type 1 and Type 2 problems. Most students learned to solve Type 1 problems like "Find 25% of $50." However, many found it extraordinarily difficult to comprehend, and solve Type 2 word problems such as "25% of a sum of money is $40. Find the sum." Ellerton and Clements (1998b) reported that the Year 8 teacher attempted to alert a class of 27 capable and enthusiastic students to language factors that can arise with such problems. However, on a written test given at the conclusion of these lessons, although 24 of the 27 students responded correctly to "Find 15% of $40," only 7 of the 27 coped with "25% of a sum of money is $40. Find the sum." No student in the class reasoned that because 25% is one-fourth, the required sum is therefore $40 x 4 = $160. Ellerton and Clements (1998b) concluded:

> The inability of the students to give correct responses in the test situation to anything other than very standard questions was clearly and overwhelmingly due to language factors. … The semantic structure of a question like, "25% of a sum of money is $40. Find the sum" is much more difficult for students whose first language is not English than might be imagined by those who have English as their mother tongue. (p. 179)

Ellerton and Clements (1998b) also pointed out that Newman research had repeatedly shown that most errors made by students in responding to written mathematical text are language-based. Upper-secondary school data analysed and reported in Noridah and Clements (1999) not only supports this claim but suggests that it holds at all levels of schooling and almost certainly in beyond-school contexts, too. It is also important to recognise that the language factor is probably as important for learners in classrooms where the language of instruction is their first language as it is for learners where this is not the case. Furthermore, student difficulties with mathematics word problems are more likely to arise from semantic structure than from an inability to cope with the "mathematical" concepts involved.

Wong and Rose Dalilah (1999) also studied the responses of Year 8 students in Brunei Darussalam to percentage word problems. They reported that although most students could answer questions which involved direct computations, many students had difficulty with questions which involved "calculating the base," because "they used multiplication rather than division" (p. 301). Wong and Rose

Dalilah concluded that "the semantic structure of this type of problem may be a hindering factor … but the use of the more difficult division operation may also play a part" (p. 304).

School mathematics problems requiring percentage calculations can introduce unusual contexts and often have difficult semantic structures. This became clear in research by Perry, Howard, and Miller (1999), who interviewed five Year 11 students who attended school in Western Sydney, to ascertain their levels of understanding of, and responses to, a question which appeared on the 1996 N.S.W. Mathematics in Society HSC. examination. The question was as follows:

> *Jo's Clothes* is having a sale. To find the percentage discount on an item, a customer rolls three dice and adds the uppermost numbers.
>
> What is the greatest percentage discount that could be received?
>
> For an item marked $100, how much would be paid if the smallest possible discount is rolled?
>
> In how many ways can three dice be rolled to obtain a total of 5?
>
> Calculate the probability of rolling a 5% discount.
>
> What is the probability of paying $95 or more for an item marked $100?

The interviewees had difficulty with specific terms like "uppermost." According to Perry et al. (1999) the subquestions were lexically dense and tended to employ the passive voice. The expressions "greatest possible discount" in part (i), and "smallest possible discount is rolled" in part (iii) are difficult semantic constructions and, according to Perry et al. (1999), the process of "nominalisation" made "the text much more abstract" (pp. 428–429).

Difficulties in Understanding Word Problems in Other Mathematical Contexts

The above discussions suggest that students often have great difficulty in comprehending mathematical text, or in linking a text with mathematics that they feel they know. As Stillman (1999) observed, "mathematical words and terms are often used in textual descriptions of tasks to clarify particular mathematical features, but these can impede rather than facilitate task access when the salience for the student is so low that they are ignored" (p. 456).

Furthermore, if learners are expected to generate text, many experience difficulty with the task of identifying and introducing appropriate mathematical symbols and structures which model the situation described in word problems. White and Mitchelmore (1996) reported that first-year university calculus students, all of whom had studied calculus in secondary school, had difficulty in expressing, in the symbols of differential calculus, word problems involving rates of change. Their analysis revealed that the students tended to be comfortable when manipulating the symbols, but had difficulty in writing equations to represent the quantities mentioned in the word problems. In other words, the students' major problem was one of mathematical modelling, or in Newman terms, transformation. White and Mitchelmore surmised that "the students had developed an 'abstract-apart' concept of variable rather than the 'abstract-general' concept that is needed for a successful study of calculus" (p. 79).

Noridah and Clements (1999), who studied upper-secondary A-level students' conceptions of inverse functions, reported findings similar to those of White and Mitchelmore (1996). Students were happy to manipulate symbols, but had virtually no understanding of the concept of inverse function. In particular, they did not understand the definitions or concepts of domain, range, and inverse. Students were content to adopt a "swap x and y routine" which would generate at least partially correct answers.

White and Mitchelmore (1996) attempted to improve their students' comprehension of calculus word problems involving rates-of-change situations. They reported, however, that "almost the only detectable result of 24 hours of instruction intended to make the rate of change more meaningful was an increase of manipulation-focus errors in symbolising a derivative" (p. 93). They concluded that the students had not achieved a mature concept of a variable, and that more attention should be given to assisting them to develop an understanding of the meanings of algebraic and calculus symbolisms.

Algebra and Language

It is obvious that the reading, writing, and interpretation of algebraic text are language-based activities; after all, algebraic text is replete with signs and symbols that communicate different messages to different readers. Furthermore, as Horne (1999) pointed out, major transformations within and between sign systems are often required. Thus, a student might be required to translate within a system by simplifying, or factorising, or expanding an algebraic expression. Or a student might be expected to link the solution(s) to an algebraic equation with the graph of a function.

Sometimes, however, the language aspects of tasks thought to facilitate algebraic learning are sufficiently hidden that they are not recognised by teachers or researchers. For example, the concept of equal, represented by the symbol "=," takes on different meanings in different algebraic contexts. Compare, for example, the meanings for (a) $x + 3 = 7$, (b) $y = 2\sin x$, (c) $\{(x, y): x^2 + y^2 = 1\}$ and $\sin^2 x + \cos^2 x = 1$. Sometimes the letter x is to be regarded as an "as yet unknown"; sometimes it is to be considered a general number; and on other occasions it is to be a variable which can be operated on as an object in its own right (Horne, 1999). Most teachers of mathematics have learned to recognise which meanings are appropriate in given contexts, but often students have not, and this can result in mismatches between teachers and students. Often, students—even among "mathematically capable" senior secondary pupils—have internalised important misconceptions with respect to elementary algebra

(Noridah & Clements, 1999; Pegg & Hadfield, 1999; Swedosh, 1999; Warren, 1999).

Some researchers believe that formal algebra is best introduced by asking students to identify functional relationships between variables defined by given tables of values. Those who support that view hold that such an approach is likely to facilitate students' development of the concept of a variable. However, others maintain that many lower secondary students cannot cope with such an approach. A group of researchers at the Queensland researchers (see, for example, Boulton-Lewis, 1999; Boulton-Lewis et al., 1997a, 1997b; Cooper et al., 1997; Pillay, Wilss, & Boulton-Lewis, 1998) have argued that this is only to be expected because the approach is quite complex. It assumes an understanding of algebraic and arithmetic syntax, the ability to formulate relationships involving two operations, and in some cases to relate these to concrete representations. According to Cooper, Baturo, and Williams (1999), "instruction to develop algebra knowledge should be seen as encompassing three stages, namely, arithmetic through pre-algebra (where arithmetic techniques are used with letters ...) to algebra (where operations act on variables)" (p. 178).

Cooper et al's (1999) assertion was supported by data from a three-year longitudinal study into the development of algebraic thinking by 33 students who moved from four primary schools to the same high school in Queensland (Boulton-Lewis, Cooper, Atweh, Pillay, & Wilss, 1998; Pillay et al., 1998). From the perspective of the main theme of this chapter, it should be noted that, since each of the three steps in thinking (arithmetic to pre-algebra to algebra) is associated with students being able to operate on and manipulate symbols, transformative language is required at each step.

In a series of studies, MacGregor and her co-workers investigated, among other things, the role of language factors in algebra learning (see, for example, MacGregor & Stacey, 1996a, 1996b, 1996c, 1997, 1998; MacGregor & Price, 1999; Stacey & MacGregor, 1997, 1999). After recognising that many secondary students do not easily learn how to express simple operations and relationships in algebraic notation, MacGregor and Stacey (1996a, 1997) found that lower and middle secondary students frequently misuse and misinterpret algebraic symbols, and are unable to cope with algebraic syntax. Even apparently simple tasks—such as "David is 10 cm taller than Con. Con is *h* cm tall. What can you write for David's height?"—were found to be more difficult than might reasonably have been expected. MacGregor and Stacey reported that on that task a common error is "D*h*" because students use this to represent "David's height." Another common error is "*h*10," because many students seem to be unaware of the convention that the sum of *h* and 10 is represented by "*h* + 10."

The fact that algebraic text seemed to be so poorly understood, from both receptive and expressive points of view, led MacGregor and Price (1999) to investigate the extent to which general language proficiency was related to algebraic understanding and competency. They experienced some difficulty in choosing/developing an appropriate research instrument for measuring language proficiency, and finally constructed their own test of *metalinguistic awareness*. This test contained six straight-language questions, six elementary algebra questions, and eight arithmetic questions. The straight-language items had no obvious association with any mathematics concepts (e.g., one item tested students' comprehension of the text "Trapped Dingo," by Judith Wright, and other items invited students to explain the meaning of proverbs, and to identify ambiguities in sentences). The researchers reported correlations between scores of metalinguistic awareness and algebra scores of 0.49 (for Grade 8), 0.34 (for Grade 9), and 0.62 (for Grade 10), and noted that "very few students with low metalinguistic scores achieved high algebra scores" (p. 461). It is interesting that these correlations between scores on pencil-and-paper tests of language and mathematics are of the same order as those between the word knowledge of students and mathematical performance in the TIMSS data (see Lokan & Ainley, 1998).

MacGregor and Price (1999) also pointed out that the algebraic sign system has its own grammatical rules and conventions that are not intuitively obvious, and have to be learned. Jourdan (1999) made the same point with respect to the sign systems of mathematics in general. MacGregor and Price (1999) also attempted to link their results with Cummins' studies, into the language skills of bilinguals, which suggested that maximum academic achievement would not be likely to be facilitated unless certain thresholds for linguistic proficiency were reached. Essentially, the main language variable (metalinguistic awareness) was not well defined, with the major instrument which operationalised it containing relatively few items, none of which had been validated. The authors noted that further research was needed to explore their hypothesis that "low metalinguistic awareness is an obstacle to learning algebraic notation" (p. 461).

Redden (1996) investigated the responses of 1435 students in Years 5—8 to a task on "how many matches are in the pattern". He explored the extent to which they used natural language and were able to state the algebraic rule for the number of matches in a general case. The task was such that a mathematically mature person would be expected to provide an algebraic response. Redden found that about one fifth of the students were able to give satisfactory natural language and symbolic notation responses, and that these forms of responses were dependent. Interestingly, this was true both for students in Years 5 and 6, who had not formally studied algebra, and for students in Years 7 and 8, who had studied algebra. Like MacGregor and Price's (1999) study, Redden's (1996) result suggests that students' natural language development is related to their algebraic performance. A problem-solving model proposed by Lowrie and Hill (1996) also recognised the importance of being able to link various modes of problem representation, including visual representation.

The Workplace and Curriculum Genres

One reason why many students have difficulty in linking algebra with anything is that algebra is often taught as if it is separate not only from real-world considerations, but also from other areas of mathematics (Kanes, 1996). Even the assessment criteria for algebra topics mandated in the Australian Qualifications Framework (AQF) courses, for industrial workers, tend to use words like "solve" and "sketch," which, according to FitzSimons (1997), are "skills which reproduce school mathematics techniques" and not "skills demanded by industry" (p. 165). FitzSimons (1997) maintained that textbooks, and teacher and student texts generated in such courses, do not reflect the interests, backgrounds, and workplace needs of the students.

FitzSimons (1997) provided the following example of a question associated with the learning outcome, "Solve vocational mathematics problems using linear and quadratic functions and their graphs":

> A researcher in physiology has decided that a good mathematical model for the number of impulses fired after a nerve has been stimulated is given by the quadratic function:
>
> $$y = -x^2 + 20x - 60,$$
>
> where y is the number of responses per millisecond, and x is the number of milliseconds since the nerve was stimulated:
>
> When will the maximum firing rate be reached?
>
> What is the maximum firing rate?

FitzSimons (1997) commented that the example "appears to be contrived, and makes no attempt to explain the physical model from which it was supposedly derived. Even students who are enrolled in the subject of Anatomy and Physiology can find no meaning in it" (p. 165).

Transcripts of actual workplace conversations reproduced in Kanes (1996) suggested that although mathematics concepts were used, the language was site-generated and did not conform to terminologies and language patterns taught in formal educational establishments (like schools). In particular, there was frequent, albeit subtle, use of linguistic devices such as metaphor and metonym. These were used "both to produce and regulate mathematical knowledge as respondents performed tasks required of them within a specific workplace setting" (p. 320). Kanes (1996) proceeded to reflect on what limits the range of meanings produced by the linguistic devices observed, and how (and whether) limitations are expressed within statements of task competencies.

According to Sträβer and Zevenbergen (1996):

> With the separation of the classroom situation from the actual problem situation vocational education shares a problem with general education: classroom teaching can always be characterised by the transformation of the knowledge to be taught into a linear text with a beginning and an end, separating the knowledge from its initial context, depersonalizing and compartmentalizing the knowledge to be taught. (p. 660)

The use of the word "text" in this passage reminds us that language factors pervade mathematics education contexts. Texts come in different forms, and are reflective of different discourse patterns, different genres. Wallace's (1999) analysis of the mathematics incorporated in three written trade texts (all developed with the intention of instructing readers on clutch maintenance) revealed very different genres. Only one of the texts—which was written in the active voice and used a talking/explaining approach—would be likely to communicate well with the average adult reader. According to Wallace, "the formal register of [the other two examples of] course instructional material contrasted appreciably with the register of the real-life working situation and therefore presented unnecessary problems for the student" (p. 394).

We suspect that Wallace's comments are as true for the various kinds of linguistic texts associated with mathematics classrooms as they are for written texts found in trade manuals. As this review has shown, many Australasian mathematics educators are now aware of the overriding importance of language factors as they affect mathematics learning. Although much has been achieved in our efforts to identify the main issues (see, for example, Anderson, 1996), we now need to find multiple solutions to the major problems that we now recognise. There is considerable support in the research literature for the view that school-based research teams involving teachers and other educators from a range of backgrounds, with different interests and areas of expertise, are most likely to achieve the breakthroughs which are needed (see, for example, Clarke, Clarke, & Sullivan, 1996; Groves & Doig, 1998; Howard, 1996).

Final Comments

The editors of MERGA's last four-yearly review (Atweh, Owens & Sullivan, 1996) noted that contributions to the broad area of language factors in mathematics teaching and learning represented "one of the major areas of strength in research in Australasia" (p. 7). There can be little doubt that during the period 1996–1999 not only has the volume and quality of research been maintained, but also it has been increasingly noticed by scholars in other nations (e.g., Adler, 1998; Becker, 1999). It is not surprising, therefore, that many of the works reviewed in this chapter have been published as articles in international refereed journals or as books or chapters in books released by international publishing houses.

Australasian mathematics educators have continued to urge the international mathematics education research community to ensure that language factors become a central concern of mathematics education research (Clements & Ellerton, 1996a; Dawe & Mulligan, 1997; Ellerton, 1997; Ellerton & Clements, 1998a; Southwell, 1997). At the classroom level, there is evidence that teachers can change their preferred classroom communication patterns if they are offered appropriate support (e.g., Clarke,1997).

In closing this review a fundamental question should be asked, "Is the now-traditional Australasian emphasis on research into language factors in mathematics education justified by the findings from the research?" One only has to reflect on some of the results obtained from Newman error analysis research, and some of the results from research into bilingualism and mathematics learning, over the past two decades, to realise that the answer is "Yes." Furthermore, within the period under review, analyses carried out of data from Australian students who participated in the Third International Mathematics and Science Study (TIMSS) indicated that language ability and mathematics performance are highly correlated. (Lokan & Ainley, 1998; Lokan, Ford, & Greenwood, 1996, 1997). Although composite student and parents' language background variables had relatively low correlations with TIMSS mathematics performance (at both the primary and secondary school levels), students' verbal - ability variable correlated at about 0.6 at the middle-primary school level, and around 0.5 at the lower-secondary level. These verbal ability correlations were higher than correlations for all other background variables in the TIMSS analyses, including variables covering parents' occupational status, home background, attitude (like/dislike mathematics), and attribution for success.

Of course, the sizes of correlation coefficients say little about the factors which are involved. In some contexts, language factors may not be as important for mathematics teaching and learning as others (e.g., Peard, 1999). It follows that an eclectic approach to researching links between language and mathematics is what is needed if we are to be able to tease out the details and educational implications of the complex relationships which previous research has identified. There is also a need to identify and explore relationships which have not been previously identified.

Unfortunately, such is the extent of the influence of language on mathematics teaching and learning that it is not possible in this review to cover all the relevant research publications that have been generated in Australia and New Zealand during the period 1996–1999. A decision was made, albeit reluctantly, not to discuss in detail research findings associated with language factors arising in papers in three main areas: assessment, technology, and group learning.

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Research in Problem Solving in Mathematics

Steven Nisbet
Griffith University
<s.nisbet@edn.gu.edu.au>

Ian Putt
James Cook University
<ian.putt@jcu.edu.au>

Introduction

Mathematical problem solving continued to be an area of research in Australasia during the second half of the 1990s. This research has occurred in the context of continued attention to the role of problem solving in teaching and learning school mathematics. Despite the recent highlighting of basic, often minimal, numeracy as a priority in the federal and state circles, problem solving continued to be described as an essential competency in school mathematics. During this period, the Commonwealth Department of Employment, Education and Training provided a grant to the Australian Association of Mathematics Teachers (AAMT) to support a project to "promote an informed view of the competency Using Mathematical Ideas and Techniques (UMIT) to educators in all relevant areas and provide strategies for the effective development of this competency in young people" (AAMT, 1996). UMIT is one of a set of work-related key competencies previously identified by a national vocational education committee (Mayer, 1992). The set includes collecting, analysing and organising information; communicating ideas and information; planning and organising activities; working with others and in teams; using mathematical ideas and techniques, solving problems; and using technology.

Because of the central role of solving problems in constructivist theories, considerable attention has been paid to the notion of problem solving as an essential activity for learning mathematics. The literature on learning and teaching in mathematics emphasises the significance of "constructivism" (a) as an approach to understanding the process of learning of mathematics and (b) as a guide to teachers as facilitators of learning. A scan of Australian professional mathematics teachers' journals reveals that teachers have reported frequently on their focus on problem solving and investigation in mathematics classes, and have encouraged their peers to do likewise (see, for example, Borg, 1998; Holton, 1997; Northcroft, 1999; Perso, 1997; Stephens, Ball, Montgomery & Waters, 1997; Stoyanova & Bana, 1997; Wilson & Brown, 1998). As a prominent facet of mathematics teaching at primary and secondary levels, problem solving has, not surprisingly, continued to be the subject of investigation for many researchers in Australasia.

This chapter covers research on problem solving conducted in Australasia or by Australasians in the period 1996-1999. The organisation of the chapter is similar to that used in the chapter of the previous review publication (Nisbet, Putt & Taplin, 1996) and is based on the following categories:

- contexts and applications of problem solving;
- techniques: (a) visual factors, (b) cognitive demand on problem solvers, (c) the use of algebraic and other special techniques in problem solving, and (d) the application of the SOLO taxonomy to cognition in mathematical problem solving;
- teaching strategies to facilitate problem solving, including metacognitive frameworks, collaborative learning, problem posing and use of open-ended tasks;
- catering for special groups;
- the role of affective factors in problem solving; and
- teachers and their professional development.

Contexts and Applications of Problem Solving

The first two studies reported in this section demonstrate the power of using real world contexts and applications of mathematical techniques in enhancing students' engagement with and performance in problem solving. The third study notes the difficulties primary-school children have in applying techniques in real contexts, and the fourth reports on the *problem solving in context* section of a large-scale testing program for primary-school children.

Stillman (1998) conducted a qualitative study to investigate the impact of prior knowledge of task context on students' approaches to application problems at the senior secondary level, and the effect of engagement with context on performance. Prior knowledge mainly enhanced students' understanding of the task, having most impact when students constructed a mental picture of the situation. Episodic prior knowledge seems to be more likely to have a positive effect than academic or encyclopaedic knowledge. Moderate to high engagement with a task context was not often associated with poor performance. Poor performance was more likely to be associated with no or little engagement. High engagement was not a necessary condition for success as the degree of engagement necessary for success appeared to be task specific.

Afamasaga-Fuata'I (1998a) investigated student's conceptualisations of quadratics by solving contextual quadratic problems. The results of her case study showed that using realistic situations as problem contexts invited a multiplicity of interpretations and methods for defining quadratic functional relationships. By reasoning empirically from the context of the problem, the students conceptualised quadratics iteratively and in terms of summation, in contrast to the most common view of quadratics as a product of two linear variations. Further, requiring students to verify and justify their strategies by cross-referencing between multiple representations of functional relationships and problem context led them to construct viable schemes to characterise quadratics in terms of rate of change, dimensionality and symmetry.

Two real-life word problems were given by Menon (1996) to 987 upper-primary students (462 boys, 525 girls) from 29 Singapore schools to determine whether these students could use number sense and reasoning, and whether there was a gap between the planned and implemented problem-solving curriculum in Singapore. One problem involved a simple division situation and overall 79% of the students solved the problem. However, from their responses Menon concluded that students did not always demonstrate number sense by relating their solutions to the real-life situation but were sometimes prone to indiscriminate and mechanical application of procedures. This was further evidenced on the second problem, which involved an inappropriate question. Menon argued that the curriculum needs to include more open-ended questions as well as questions with insufficient or extraneous information to allow students more flexibility in answering questions. This should lead to a narrowing of a perceived gap between the planned and implemented curriculum.

Stacey (1997) has commented on the Basic Skills Testing Program (BSTP) conducted in the state of New South Wales, in relation to problem solving in context. She notes that the BSTP provides useful information about some aspects of children's ability to use mathematics to solve real world problems. However the large-scale computer-assisted assessment technique does not provide a complete picture of a child's problem-solving ability. To cover the many dimensions of problem-solving activity, teachers need to develop a comprehensive plan using a variety of assessment techniques.

Cognitive Processes

Visual Factors

The studies involving visual factors demonstrate the effectiveness of using imagery and various visual representations such as drawings, diagrams, and mapping techniques to assist students across many grades in their mathematical conceptual development.

Lowrie (1996) examined the role of imagery among the problem-solving strategies used by Year 6 primary school students in a variety of mathematical contexts. The students attended a non-government school located in a large regional centre of New South Wales, and the data were gathered over a period of three years. The investigation focused on the ways in which students attempted new problems, and whether students tended to change their preference for problem-solving methods as they became familiar with specific topics. In particular, the role of visual imagery was studied in relation to the development of capability, confidence and the ability to transfer knowledge to other situations. The report describes how development of knowledge and skills interacts with preferred problem-solving processes, and particularly with the ways problems are framed and represented. Most of the participants in the inquiry were from the teacher/researcher's own class. It is argued that this naturalistic setting was an important factor in the generation of holistic interpretations of the data, and also in the use of a wider range of problem-solving processes than would otherwise have been the case. The study found that both visual and non-visual reasoning play an important role in problem solving; students will often use visual processing in the initial stages of problem solving, and then move toward more analytic, non-visual strategies when relevant patterns have been identified, or when a more informed understanding of the problem has been established. Importantly, visual methods allow students to negotiate the difficulties associated with a problem when conceptual limitations do not allow a student to complete the problem quickly and analytically. A dynamic problem-solving model emerged from the analyses of the quantitative and qualitative data. The model proposed that four distinctive features: (a) task difficulty, (b) problem representation, (c)metacognitive thought, and (d) the ability to apply knowledge to other situations, influence how students interpret, represent and go about solving problems and applying knowledge and learned techniques to other problem tasks.

Outhred and Sardelich (1998) studied the development of representations of numerical problems by children in their first year of school. Children's drawings of problem solutions were obtained from two classes during the period May to November in successive years. (Both classes were taught by the same teacher.) The children solved a range of numerical problems by modelling them with concrete materials, then drawing representations of the problem situations. Over the period of the study the representations became more structured, with groups delineated in various ways, including the use of letters and labels. By the end of the year, the children were writing their own problems and many children were representing problems symbolically. The authors suggest that the value of linking modelled and drawn representations may be that the drawn representation gives the children a means of recording their thinking and using this to explain their solution process. Moreover, asking children to explain how their drawings and their models are the same focuses their attention on similarities between the two structures from the beginning.

Shaw and Outhred (1999) investigated students' use of diagrams (i.e. frequency, success and reasons) in solving statistics problems. They looked at why students in an introductory statistics course at university do or do not use diagrams in problems which could be done with or without the aid of diagrams. The students were asked to solve three statistics problems and one non-statistics (linear algebra) problem for comparison. Most of the students did not draw diagrams, despite being encouraged to do so throughout the course. For all four problems, the students who drew diagrams as part of the solution process were far more successful than those who did not. The reason given by students for not drawing a diagram were that they were not sure it was worth the effort it took to draw it, rather than not knowing what to draw. Students seemed to want to calculate statistics without first obtaining a feel for the data. There was no confirmation for the hypothesis that students who had studied more mathematics at high school would be more likely to draw diagrams. The authors suggest that a greater emphasis be given to integrating the use of diagrams into the teaching of introductory statistics because of the higher success rates associated with their use.

Diezmann (1999) conducted a case study to evaluate the effectiveness of instruction in diagram generation with Year 5 students. She claims that, although the use of diagrams is advocated in mathematics, support for this instructional practice appears to be intuitive rather than evidentiary. In individual interview situations 12 students were presented sets of novel problems before and after instruction. If students did not spontaneously use diagrams, they were given general or specific prompts. The number of students who spontaneously generated diagrams increased from 4 to 11, and there were similar increases in the number of students who generated valid diagrams (with respect to representing movement, directionality and amount of movement). The results of the case study suggest that instruction can have a positive effect on students' diagram generation. However, the success of the program is dependent on the teacher's understanding of the role of diagram generation in problem solving and how diagram generation can be facilitated. The major implication for teachers and curriculum developers is that the mathematics curriculum should include specific instruction about the use of diagrams in problem solving.

Afamasaga-Fuata'I (1998b) investigated the use of concept maps and Vee maps in the teaching of mathematics problem solving to secondary (Year 11) students. It was found that the concept map was an excellent tool for illustrating the inter-relationships of concepts in a problem. The Vee map was an effective practical tool for exhibiting the interplay of the conceptual side and methodological side of a mathematics problem, and facilitated the generation of solutions to novel problems.

In a study of cognitive schemas used by high school students, Chinnappan (1998) examined the question of what the nature of prior mathematical knowledge is that facilitates the construction of useful problem representations in the domain of geometry. The quality of prior knowledge was analysed in terms of schemas that provide a measure of the degree of organisation of prior knowledge. Problem-solving performance and schema activation of a group of high- and low-achieving students were compared. As expected the high-achievers produced more correct answers than the low-achievers. More significantly, schema comparison indicated that the high-achievers accessed more problem-relevant schemas than the low-achievers. In a related task which focused on the problem diagram, both groups accessed almost equal numbers of geometry schemas. These results are interpreted as suggesting that high-achievers build schemas that are qualitatively more sophisticated than low-achievers which in turn helps them construct representations that are conducive to understanding the structure of geometry problems.

MacGregor and Stacey (1998) studied the cognitive models underlying algebraic and non-algebraic solutions to unequal partition problems. They claim that the structure of certain word-problems can be perceived in different ways,

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depending on the grammatical form of presentation of the problem and the student's expectation of how it will be solved. The results of their study involving 268 school students aged 14–16 showed that, for a certain class of problems, different problem presentations promote the construction of different cognitive models of the situation described. The data provide support for the hypothesis of Nathan, Kintsch, and Young (1992) that in the solution of algebra word-problems there are three components of interpretation and modelling: a propositional text base, a cognitive model of the situation, and a formal model of the mathematical relationships. However they show that, for certain problems, there are two equally valid cognitive models of the situation, only one of which can be linked to an algebraic representation of relationships. For problems of this type, the lack of correspondence between a cognitive model of the situation and an algebraic representation of relationships in the problem is a powerful obstacle to the use of algebraic methods.

The above studies show the effectiveness of physical representations of problems on cognitive models and solutions.

Cognitive Demand on Problem Solvers

The study reported in this section deals with how different problem-solving strategies involve different levels of cognitive demand. In a study with 121 female final-year secondary students, Stillman (1996) investigated the relationship between their written responses for a complex problem and the mathematical and cognitive processing used. A response map of the teacher-prepared solution was produced by the researcher and used (a) to classify the student scripts by global strategy and to identify major flaws in solutions in each category, and (b) to identify factors that contributed to failure. Data revealed four potentially successful and three flawed global strategies. The scripts for each of the potentially successful strategies were further categorised into clusters according to perceived patterns of mathematical structure revealed by the students' response maps. These maps allowed comparison of the cognitive demand of different strategies by reference to the visual summary of the analysis of the scripts. Furthermore, they revealed the difficulties faced by students in applying known facts and formulae in solving word problems as complex as the one investigated. The author discussed how teachers could use the response maps to determine the level of complexity of various mathematical problems used in the senior school and suggested further research arising out of this study.

Solving Problems Using Algebraic and Other Special Techniques

MacGregor and Stacey (1996) investigated lower secondary students' ability to solve some problems using algebra. They chose straightforward problems, expressed in words, where elementary algebra was useful for finding a solution. They gave students the instruction to write an equation for each problem and solve it. Some students did the test on three occasions, in August and October when they were in Year 9 or 10 and again in June in Year 10 or 11. The authors trace a progression of stages from naming quantities through describing relationships to writing equations and solving them. For the students, even when all relationships were recognised and correctly symbolised, integrating them into an equation was a common difficulty. Thus the failure to solve the problems did not lie in the students' failure to comprehend the written information, to understand the problem structure, or to see how the parts were related to each other, or to the whole. The main obstacles were the incorrect use of algebra syntax, and the failure to integrate the given information as an equation or set of equations.

After developing a method for teaching percent problem solving called *the proportional number line method*, Dole (1999) conducted a series of teaching experiments with Year 8 classes in an Australian inner city, single-sex secondary school. The intervention lasted for approximately 10 x 40-minute lessons. (The class on which Dole reported was part of a larger study.) Mean scores on the pretest (12%), and posttest (77%) showed a marked increase while the mean on the delayed posttest (76%) demonstrated maintenance of successful performance. Students' comments recorded in a journal after each lesson indicated their ready adoption of the proportional number line method. It was easily understood and led to success in solving percent problems and equations. Furthermore, there was some evidence of transfer to different proportional situations.

Yeap and Kaur (1996) conducted a teaching experiment with three groups of Year 9 students in Singapore. The experimental group consisted of students with low mathematical ability; one control group comprised students with similar ability to students from the experimental group while the other control group consisted of students with higher achievement test scores. The experimental group was taught the remainder theorem and binomial theorem using pattern observation, a problem-solving heuristic. Students in the control groups were taught the two theorems in a traditional manner where formal statements and algebraic forms of the theorems were given prominence. The high achievement control group ($n = 42$) performed significantly better than the experimental group ($p = 0.05$) but the experimental group ($n = 29$) performed significantly better ($p = 0.05$) than the other control group ($n = 39$) on the posttest which consisted of appropriate items from standard achievement tests. It was observed that the good students (high achieving control group), taught in an expository way, with a strong emphasis on algorithms, continued to excel in standard achievement test items. The experimental group, however, showed better performance in the achievement test than the class of similar ability taught in an expository way. The results of this teaching experiment lend some support to an instructional approach that placed emphasis on pattern observation. It was claimed that the experimental group students benefited in terms of mathematical processes through their mode of instruction although the authors were hesitant about claiming that these students gained a *deep* understanding of the topics from pattern observation.

Analyses Using the SOLO Taxonomy

Mulligan and Watson (1998) have presented an analysis of young students' development of concepts and problem solving in multiplication and division based on a multimodal SOLO model. Their analysis was drawn from two sources of data: (a) a two-year longitudinal study of 70 Grade 2 to 3 students' solutions to 24 multiplicative word problems, and (b) examples from a problem-centred teaching project with Grade 3 students. An increasingly complex range of counting, additive and multiplicative strategies based on an equal-grouping structure demonstrated conceptual growth through ikonic and concrete symbolic modes. The researchers claim that the solutions employed by students to solve any particular problem reflected the mathematical structure they imposed on it. They described a SOLO developmental model for multiplication and division in terms of developing structure and associated counting and calculation strategies.

Chick (1998) investigated the cognitive thinking employed by a research mathematician in solving a high-level problem. In the study she analysed the cognitive processes she engaged in herself while completing a PhD in pure mathematics. She raised the question of whether the cognitive behaviour adopted by mathematics researchers (while they put considerable cognitive effort into trying to expand the body of mathematical knowledge) is different from that of those who work on more standard mathematical problems. The study attempted to examine some aspects of mathematical cognition at the highest level of formal functioning. It illustrated how the structure of a mathematician's output—and, to a certain extent, its cognitive complexity—can be characterised by the SOLO taxonomy. Chick identified particular examples of a range of SOLO responses—unistructural, multistructural, and relational. Although there are similarities between the cognitive functioning of undergraduates mathematics students and post-graduate researchers (e.g. they both operate in the formal mode), she claims that there are enough differences to justify the claim that there are two modes of formal functioning—Formal-1 and Formal-2. Chick also discussed a number of philosophical issues concerning mathematical functioning at the research level.

Teaching Strategies to Facilitate Problem Solving

There are at least two broad categories of teaching strategies that teachers have at their disposal to enhance students' general problem-solving skill—the use of metacognitive frameworks and collaborative learning methods. These methods have been studied by researchers with students at many primary and secondary school levels, and such methods have been found to facilitate improved problem-solving performance and higher levels of thinking.

The second half of the 1990s has seen the emergence of research into students' problem posing, and its positive influence on students' problem-solving

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abilities. Studies reported here relate to the success of problem posing with students from different grades in the primary and junior high school, and teacher education students.

Metacognitive Frameworks

Lowrie (1998) conducted a case study of the development of metacognitive thinking in young children. A talented Year 1 child (aged 6) was challenged to "take another person's point of view" when responding to a series of questions based on a well-known folk tale. He argues that the use of effective questioning techniques allowed the child to develop a variety of metacognitive processes. The child was asked to consider a "what-if" scenario, to draw pictures, and to answer questions about the pictures. The drawing activity provided a mechanism that helped move the child to a higher level of thinking, to verbalise her thinking, and to take another person's (an illustrator's) point of view. The teaching process described in the case study allowed the child to reflect on her own thinking and see beyond the surface learning often accepted in the early years of schooling.

Two studies of teaching problem-solving processes were conducted by Durnin, Perrone, and Mackay (1997), one with a Year 4 class in Australia, and the other with a Year 5 class in the USA. Students in each class were randomly assigned to two groups. One group received teaching on problem-solving processes, while the other served as a control group. The students in the processes group did as well on the subject matter knowledge as the control group. However, on tasks involving processes both with content and transfer material, the processes group performed significantly better.

With Year 6 students, Adibnia (1996) investigated the effects on problem-solving performance of an instructional approach based on the cognitive-metacognitive framework of Garofalo and Lester (1985). A randomly assigned experimental class was taught by its regular teacher who implemented the instructional approach along with a chosen set of process problems. One control class was taught by its regular teacher and also solved the same problems but without the targeted teaching method, while a second control class was taught by its regular teacher but received neither the set of problems nor the targeted teaching method. Data were collected via the Collis-Romberg (1992) *Problem-Solving Profiles*, a Process Problems test designed by the researcher, and a questionnaire and a clinical interview to ascertain students' beliefs about and attitudes towards problem solving, and their cognitive and metacognitive activity during the phases of solving the problems. Data from the Process Problems test confirmed the effectiveness of the instructional intervention compared with the traditional methods of teaching. Students in the experimental class made significantly greater gains in performance than students in both the control classes. A differential effect of the experimental treatment on different ability levels was not confirmed as the interaction between classes, ability levels, and tests just failed to reach significance, thus indicating that the gain scores of the three ability groups in the experimental class were not significantly different when taken in comparison to the corresponding groups in the control classes. There was a significant difference in favour of the females in the experimental group on the Process Problems test and only the experimental group had significant positive gains in relation to beliefs and attitudes. Findings from this study should encourage further evaluative research into alternative models of instruction in problem solving in different curriculum areas for different age groups with varying mathematical ability.

Wilson (1998) studied the metacognitive thinking of upper primary school students in mathematics and strategies for assessing it. She found that most students always thought about what they already know and what the problem was asking them to do. They also made a plan to work it out and tried to remember if they had done a similar problem before. One key finding of the study was that existing definitions of metacognition are insufficiently precise and difficult to interpret in operational terms. Hence the author has provided a model of metacognitive behaviour which acknowledges three functions of metacognition—awareness, evaluation and regulation. A second key finding was that self-reporting appears to be a valid method for researching metacognition, however, corroborating evidence should be obtained through teacher observation. Wilson contends that if the assessment of students' metacognitive thinking is not integral to mathematics teaching and learning, then the improvement of student metacognition cannot be assumed.

In a study involving a group of Year 7 students in Singapore, Yeap and Menon (1996) sought to identify and describe metacognitive behaviours that occurred during mathematical problem solving. Ten students were asked to think aloud while solving three non-routine mathematics problems. Their verbal responses were audiotaped and transcribed verbatim. The resulting 30 protocols were divided into segments of behaviours and each segment was classified and coded according to a pre-determined taxonomy of behaviours (Foong, 1990). Detailed examination of the protocols of five students indicated that students did demonstrate metacognition during problem solving although their strategies were not always efficient and successful. However, the ability to monitor and regulate one's problem-solving process was central to success in non-routine mathematical problem solving. When students reflected upon their plans of action to ensure completeness and correctness, it led often to solutions. A systematic approach seemed to facilitate monitoring and regulating of the problem-solving process. The authors suggest therefore that it is important for teachers to incorporate more extensively the development of metacognitive processes and emphasise systematic thinking in their regular mathematics instruction.

As part of a study of children's knowledge and strategies in problem solving, Kaur (1996) asked 63 good novice problem solvers from Years 5, 6, 7, and 8 to complete in writing the statement "When my teacher gives me a mathematics problem to solve, this is what I do ...". Good novice problem solvers from each of the year levels were found to rely on individual problem-solving frameworks to guide them when solving problems. Of the 63 children, 52 had frameworks consisting of four phases (1. Understand/represent the problem, 2. Find a way to solve the problem, 3. Solve the problem, & 4. Check the solution). Eleven children (5 from Year 5, 3 from Year 6, and 3 from Year 7) had frameworks consisting of only the first three of these phases. The frameworks supplied by the Year 8 students were more detailed, comprehensive and sophisticated than those supplied by the students in the other grades.

In a study with junior high school students, Troyahn (1998) implemented an instructional program based on a framework for problem solving developed from the work of Schoenfeld (1985) and the Mathematics Curriculum and Teaching Program (Lovitt & Clarke, 1989). The framework took cognisance of Polya's (1957) model of problem solving together with metacognitive activity and affective factors. Two classes of Year 8 students served as an experimental class and a control class. Both classes were taught by the researcher who was assisted by the regular class teacher. Data were collected using (a) commercial and researcher produced tests of problem-solving performance, (b) a scale for measuring affective factors developed by Annesley and Clark (1990), and (c) stimulated recall interviews to ascertain students' metacognitive activity. Overall, there were significant differences in favour of the experimental class on measures of problem-solving performance which were attributed to the instructional intervention. The author was able to identify both the cognitive and metacognitive activity of the students and to show significant change in metacognitive activity in favour of the experimental group as a result of the intervention. There was only limited evidence that the intervention led to a change in affect in the experimental group. This study lacked generalisability because the focus was descriptive and used only two classes. However, it demonstrated that an instructional intervention can be successfully implemented in junior high classes in spite of all the obstacles that prevent continuity of instruction.

English (1999) studied the development of 7th-grade students' problem-solving abilities as they participated in an 11-week program of thought-revealing problem activities, comprising problem-posing and model-eliciting experiences. The study reported profiles of development of three students who displayed different levels of achievement in number sense and novel problem solving. The issues addressed include facility with problem structures, divergent thinking, and the processes involved in working on a model-eliciting task. The program appeared to be a positive and productive learning experience for the students. The author claims that, given that most of the activities were new to the students, such a program can be effective in fostering some of the understandings and processes required for their success in the coming years. Further details of English's research are presented in the chapter "Language Factors in Mathematics Education".

The studies noted in this section on metacognitive frameworks are illustrative not only of the effectiveness of using metacognitive frameworks in the improvement of problem-solving performance but also of the extent and variety of research in this area during a period amid calls for greater emphasis in the school curriculum on general problem solving skills and higher-order thinking (Burden & Williams, 1998).

Collaborative Learning

Malin (1996) investigated primary children's mathematical understandings and their ability to solve problems in groups. It aimed to encourage children to find the best or most efficient method of solving problems, by having them work in groups to discuss, question, analyse and finally come to a consensus on a method. She used a research method that draws loosely on the work of Del Campo and Clements (1987). It was through encouraging the children to discuss their work that they found that many individuals were more inclined to use a more efficient method when faced with a similar problem in the future. The research proved to be very successful, with not only the students displaying a great deal of interest, but also most teachers becoming enthusiastic and keen to use similar methods in their mathematics lessons again.

Geiger and Goos (1996) examined student interaction and discussion while the students were working on computer-based problem tasks in a senior secondary school classroom. Fifteen students studying an advanced mathematics course were encouraged to interact with the teacher and each other when working through the course materials in conjunction with computer spreadsheets. Analysis of verbal and observational data suggested that the problem task itself was an important variable influencing the degree of collaboration between students, and that the teacher's intervention could change students' engagement with the task. The findings have implications for specifying the teacher's role in relation to the use of computer technology in the mathematics classroom.

Barnes (1998) surveyed teachers on their opinions and uses of collaborative learning approaches in teaching secondary mathematics. Over 83% mentioned cognitive benefits, most expressing the idea that collaborative learning leads to enhanced understanding of mathematics. Others spoke of the benefits of being exposed to different approaches to a problem. However, it seems that teachers think that students do not necessarily learn from this exposure, because none of the teachers mentioned the metacognitive benefits from collaborative learning, such as, acquisition of problem-solving strategies. The use of collaborative learning techniques was not widespread in the secondary schools, especially at the senior secondary level. Barnes claims that this is because teachers believe that it takes more time to cover the syllabus using this approach. Even teachers who support collaborative learning, use it sparingly—about once a week.

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Problem Posing

A study was conducted by Lowrie (1999) in a Year 3/4 composite class over a 10 week period with two primary children, one (male) aged eight and the other (female) aged nine, both of whom were regarded as good at problem solving. The investigation used a case study format to collect data on the ways the two students engaged in problem posing when confronted with situations that required them to pose problems for a friend or for each other. It also sought to determine if children consider the needs and interests of their peers when posing problems for them to solve, and whether the student's own mathematical ability impacted on their ability to construct problems. Over the 10 weeks students' work samples and their reflections were gathered and a number of discussions/interviews with the children were audio-taped. The questions asked by the author during these discussions were designed to encourage the students to discuss their thinking processes prior to and after posing the problems. Data revealed that the student posing the problem did consider the needs and interests of their peers. There was evidence also which indicated that a student's mathematical knowledge impacted on the problems posed for an older peer. In fact after receiving feedback that the girl considered the first problem too easy, the boy could only increase the size of the numbers in the next problem to make it more challenging for her. Some key findings in the study related to the increase in the children's level of thinking about problem-solving processes and the amount of interaction and cooperation that was generated between the two students as they posed and solved problems. Lowrie used Stoyanova and Ellerton's (1996) problem-posing framework of free, semi-structured, and structured (see the chapter "Language Factors in Mathematics Education" for further details) but found little development occurred over the period of the study.

In a study with Year 3 students (aged 8 – 10 years), Maguire (1999) explored word problems generated by them from photographs of familiar objects and situations. In particular, he investigated: (a) whether there was a relationship between mathematical ability and the ability to generate word problems, (b) the type of word problem generated, and (c) their relationship to the students' mathematical understanding. From a class of 28 Maguire selected six students (two each of above average, average, and below average ability) using an experimenter-designed instrument based on arithmetic operations stipulated in the Victorian CSF mathematics statement. Students were asked to select a photograph from a set containing familiar objects and situations and to construct a difficult mathematics word problem for a friend to solve and then to provide a solution equation for the problem. This was done for four different photographs. Most of the students were able to generate a word problem that represented the situation in the photograph after an initial difficulty in understanding what was required of them. However they generally found difficulty in verbalising their word problems and all problems generated represented low-level mathematical activity. There was evidence for a relationship between achievement level and the word problems generated and all students had some difficulty with mathematical language with the lower ability having the greater difficulty. The type of word problem reflected those covered in class prior to the study and most were either separating or joining. Because problem posing was a novel experience for these students the author suggested that further research was needed to explore the effects of: (a) stimuli other than photographs, (b) more experience of problem posing, and (c) working in pairs on their performance of the task.

Southwell (1998) studied the development of teacher education students' problem solving through problem posing. Teacher education students in their first year of university were invited to take part in an eight-week program involving solving problems and posing problems on the basis of the given problem. Data were collected on problem-solving strategies used, progress in problem-solving skills, and the students' expressions of cognitive and affective factors. It was found that the requirement to pose a similar problem to a given problem helped clarify the students' thinking. Other important factors in the successful solving of the problems were the realism of the problem, and the students' capability for a visualisation strategy, such as drawing a diagram. Problems students perceived as having a practical use appeared to be more worthy of solution in the eyes of the students, and were more motivating for the students.

It has been claimed that research into the potential of problem posing as a means for developing students' understanding of mathematics has been hindered by the absence of a valid framework which links problem solving, problem posing and mathematics curricula. In response to this need, Stoyanova and Ellerton (1996) presented an overview of the frameworks previously used by researchers for investigating problem posing, and have proposed a new framework for research into students' problem posing in mathematics based on Krutetski's (1976) problem-solving categories. Central to the framework is the notion that every problem-posing situation can be classified as free, semi-structured, or structured. Examples of three categories of problem-posing situations that were used with mathematically-able lower-secondary students have been presented in an effort to assist the systematic implementation of problem-posing situations into mathematics classrooms.

Use of Open-Ended Tasks

Sullivan, Warren, and White (1999) studied Year 8 students' responses to closed and open-ended questions. One goal was to explore the effect of context. For the closed tasks, the context seemed to help for one set of tasks, but made no difference in tasks that included diagrams. For open-ended tasks, the context helped for the two sets of tasks, but seemed to make a third set (dealing with a complex concept) more difficult. Another goal was to compare responses to closed and open-ended tasks. In two sets of tasks, students found the open-ended tasks more difficult, suggesting that these required thinking above and beyond that required for the corresponding closed tasks. In the other case the open-ended task was easier. In explaining these differences the authors claim that the open-ended tasks which were more difficult required students to link two concepts and to use such links to conjecture and generalise. Such open-ended tasks may serve the role of stimulating students' thinking to higher levels. Both the context and open-endedness seemed to effect the focus and student response to the tasks. It seems that each type of task would contribute productively to classroom programs, and teachers could be encouraged to plan to use contexts and open-ended questions productively. In some cases the open-ended tasks may serve as a useful preliminary exploration of topics; in other cases, they may be better left until later.

It is encouraging to note the great interest in researching special teaching techniques and the consistency of the results in demonstrating their high levels of effectiveness in promoting performance in students' problem solving.

Catering for Special Groups

A number of researchers have investigated the teaching of problem solving in relation to special groups of students—young children, gifted students, students with hearing impairments, students at risk—and also in relation to gender differences.

Early Childhood

A study by Geoghegan (1998) investigated the emergence of relationships in a Grade 2 classroom constituted by the contemporaneous implementation of the three theoretical perspectives of (a) problem-centred learning, (b) constructivism, and (c) positive discipline. The study explored the organic interconnectedness, evolution, and creative processes of learning mathematics framed by these three theoretical perspectives from a systems theory viewpoint. The findings highlighted the reflexive complementarity of a mathematics program based in the theoretical perspectives of problem-centred learning, constructivism, and positive discipline. Furthermore, the study indicated that approaches to learning mathematics advocated by current reform agendas understate the significance of learning as being as much to do with control as it does with change. In light of the organically interconnected relationships of learning mathematics, a theoretical heuristic was developed to exemplify the implications of the findings of the study. Called SEARCH, the heuristic highlighted learning mathematics as a synergistic relationship constituted by an interconnectedness among social, emotional, physical, and cognitive development, each of which requires a balanced consideration in order to pursue the epistemological, ontological, and methodological paradigmatic frameworks embedded in current mathematics reform agendas.

Gifted Students

Hall (1997) investigated the teaching of talented primary school mathematics students using group work and non-routine mathematics problems. Results indicated that these students were very capable of solving this type of problem and that working in groups allowed students to communicate mathematically both orally and in writing.

Hearing Impaired Students

Hyde, Power, and Zevenbergen (1999) investigated the strategies used by a sample of deaf and hearing-impaired students in South-East Queensland when solving arithmetic word problems. The students (from Grades 1 through 12) came from both mainstream and Special Education settings and all had mastered basic number facts and had basic English competency as revealed by school records and teacher judgments. Data were gathered in two phases, firstly, through a survey of all students in the region and secondly, through an interview with the students in the sample. The survey which consisted of 24 questions involving the solving of arithmetic word problems (e.g., covering the notions of change, combine and compare), was done in class under the supervision of a teacher trained in mathematics and hearing-impairment and the class teacher. The interviews which were videotaped and transcribed were undertaken with a sign-language research assistant and students could choose to respond orally or by signing. The data revealed similar patterns of difficulty as those experienced by hearing students on the set of problems. There was a delay in the non-hearing students' performance in mathematics that had also been found by other studies. It was also found that the specificity of the language used in these word problems, particularly the complex semantics and lexical density, hindered the capacity of the deaf students to make sense of the tasks.

At-Risk Learners

Pearn, Doig, and Hunting (1996) reported on an investigation into the difficulties exposed by responses to a real-world division problem from a sample of Year 6 children who were interviewed as part of the development of an intervention study involving at-risk learners. The reasons why a division problem was used in the study were that children experience many difficulties at both the conceptual and procedural levels, and that division offers multiple solution strategies. Some of the children revealed faults with basic skills, but others had more deep-seated difficulties. One child lacked the metacognitive skills to keep track of his solution; another had developed a slow, cumbersome but meaningful method. The authors note the real classroom dilemmas facing teachers in the light of the difficulties experienced by the children in solving such problems.

Gender Differences

Many studies include a comparison of gender differences in performance as part of the investigation because it is generally easy to collect data on both genders (Miller & McInerney, 1995; Rice, 1995; see also the chapter in this book on Gender and Mathematics: Changing Perspectives and the work by Kota and Thomas referred to in the next section on Affective Factors). The study by Barnes (1997) reported here tapped into a large data set to examine gender differences in mathematical problem solving as evidenced in the data from the Basic Skills Testing Program (BSTP) in the state of New South Wales. At the Year 3 level, girls achieved better than boys overall and in the strands of Measurement and Space separately. In Year 6, girls achieved better than boys in the Number strand, while boys achieved better than girls in the Space and Measurement strands. No evidence was found for the claim that girls do better on easier problems, or that girls do better on tasks involving lower level skills, and boys better on higher level skills. There was no evidence for the hypothesis that girls are disadvantaged in multiple-choice items. There was some evidence that a small number of high-achieving boys may account for most of the observed differences on the assessments. However it was noticed that there was also a larger number of boys than girls among the lowest achievers.

The diversity of special groups represented in this section of the review illustrates the seriousness with which the mathematics education research community has taken heed of various calls for access to powerful mathematical ideas on the part of *all* students.

Affective Factors

Research into mathematical problem solving has identified the role of affective factors as well as cognitive factors in the development of students' problem-solving ability. The affective factors include self-concept, interest and enjoyment, along with beliefs about the nature of mathematical problem solving.

Thomas and Kota (1996) conducted a study of the role of arithmetic thinking and affective factors in algebraic problem solving. In the study, the authors recorded students' mode of thinking, problem-solving ability, mathematical anxiety and self-concept. Results showed a positive correlation for girls between anxiety and problem-solving performance, but for boys the positive correlation was with self-concept. They also found a reluctance on the part of students to use algebraic thinking in the problem-solving exercises. Kota and Thomas (1997) measured seven affective variables simultaneously with 345 Form 3 and 4 students from eight New Zealand secondary schools. They showed that there was a gender based gradient effect in the way the affective factors change in early adolescence which corresponds to a change in the problem-solving performance of boys and girls. The effect seems to result in a lowering of mathematical outcomes for girls but an improvement for boys.

Kota and Thomas (1998) tested the hypothesis that there is a significant relationship between problem-solving performance in algebra and affective factors such as self-concept, self-perception, interest, anxiety, and enjoyment. The authors claim that their study (using tests of algebraic skills and affective factors, along with student interviews) provides evidence linking the maintenance of high levels of affective factors with higher standards of algebraic problem solving. Although the process is cyclical, the authors claim that, with better performance increasing confidence, the way into the cycle appears to be via understanding. Affective factors drive the cycle, not performance. From the student interviews it is noted that self-confidence is an expression for self-concept and self-perception.

Anderson (1996) studied primary school teachers' beliefs about the role of problem solving in learning mathematics. A survey was used to gather teachers' favourite problems and comments about traditional versus constructivist perspectives on using problems in the classrooms. At least half of those surveyed chose problems that they believed promoted learning, but most teachers adopted a position that saw problem solving as an end in itself rather than a means in the learning process. The characteristics of problems that teachers saw as desirable were (a) being open-ended, (b) having a real-world context, (c) having a game context, and (d) being adaptable for use with many classes. Although half of the teachers surveyed mentioned a learning factor that indicated their awareness of the role of problem solving in learning mathematics, it is unclear whether the others were aware of the role.

It appears that there is a need for quite specific professional development and assistance for teachers who may wish to change their classroom practice to facilitate learning through problem solving and learning to solve problems. The role of affective factors in problem solving is just as important for teachers as it is for students.

Teachers and Their Professional Development

The role that teachers play in the development of problem solving in students has not been ignored by researchers in Australasia. Three studies reported here look at preservice teachers' and teachers' own problem-solving related abilities and their growth in handling a problem-centred curriculum.

Difficulties that preservice teachers encounter with problem solving have been investigated by Taplin (1998). First and second year primary student teachers were given 12 word problems and their solutions were analysed using a mapping procedure to assess the SOLO level of responses. The findings showed that the student teachers performed best on questions where they could use unistructural, ikonic thinking, where all necessary information was given in the problem, and where only one step was required to reach the solution. They performed least well on questions that required relational, concrete symbolic thinking, where it was necessary to employ several concepts from their previous experiences, where several interim results were required, or where more than one process was needed to calculate an interim result. Findings from the study contribute towards an understanding of the types of routine problems that should be included in a teacher education program and the processing skills that student teachers need to be guided to develop.

It has been claimed (Maguire, 1996) that in curriculum statements provided to schools, current teachers are encouraged to present to their pupils a rich variety of word problems. These word problems may be drawn from reference texts, real-life problem situations, or teachers generating their own word problems. How competent are teachers to do this? In addressing this question, Maguire investigated the ability of primary teachers to generate their own whole-number word problems. His analysis of a sample of such teacher-generated word problems revealed that these word problems failed to provide as diverse a range of problems as suggested in the resources made available to teachers, and were not encompassing enough to extend children's arithmetical experience. The generated problems also reinforced some misconceptions about the four arithmetic operations.

Clarke (1996) studied the professional growth of two Year 7 teachers through the highlights and challenges of a problem-solving classroom. The research examined the teachers' responses to critical incidents in the classroom which were triggered by events such as students' insightful responses (the most common), students' incorrect responses, students' challenges to the teachers' knowledge, or students' challenges to the teachers' beliefs. The teachers' responses to the incidents and the subsequent discussion showed a change in their practice and a growth in their understanding of the role of the teacher in a problem-solving classroom. Clarke maintains that the more a classroom resembles a constructivist, problem-solving environment, the greater the frequency of such critical incidents, and hence the greater the frequency of opportunities for the enhancement of student learning. She concludes that when students are encouraged by teachers to invent, conjecture, and solve realistic mathematical problems, there will be situations that are unexpected and challenging for the teacher. Avoiding these may lead to a limiting of students' thinking. Confronting them leads to a demanding role for the teacher, but also to displays of insightful student thinking.

It is clear from these studies that there are great professional-development needs on the part of teachers and student teachers with respect to their own problem-solving skills and their abilities to foster problem solving in the classroom.

Research Methodology

In relation to research method issues in the investigation of problem solving, Diezmann (1996) studied the development of data maps as a visual tool to analyse Year 5 students' strategies in problem solving. Data maps are defined as diagrams that serve to organise and display data, and orient the decoder. Specifically, the data maps use codes for interview components, student actions, and level of success. She proposed that the study has shown that visual data analysis using these data maps is plausible, and has enormous potential in qualitative research studies.

Some studies reported in this chapter have also used tools such as the SOLO Taxonomy in the analysis of data (e.g. Stillman, 1996; Chick, 1998; Taplin, 1998). Other studies have sought to examine students' thought processes while solving problems and have used techniques such as stimulated recall interviews (e.g. Troyahn, 1998), clinical interviews (e.g. Adibnia, 1996), and think aloud techniques (e.g. Yeap & Menon, 1996). It would appear that there is still a fruitful area of research into students' cognitions while solving problems.

Conclusion

Problem solving in school mathematics has been a continuing area for research in the four years since the 1992-1995 review. It is encouraging to note that a substantial number of studies have been concerned with what happens in the classroom either in terms of effective strategies for facilitating the learning of problem solving or in examining the thought processes of students in the process of solving problems. Of particular interest is the work being done with problem posing and it is hoped that the overview presented by Stoyanova and Ellerton (1996) will stimulate further studies in this area with students across the whole spectrum of education.

In the previous review there were a small number of studies that examined students' affective responses to mathematical problem solving. Again in this review this area has been covered with a number of studies that focused on beliefs and attitudes while others included these areas within a larger study. There still appears to be a pressing need to research the affective aspects of mathematics learning if anecdotal reports from teachers and mathematics educators are any indication.

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Technology-Assisted Mathematics Education

Gary Asp
University of Melbourne
<g.asp@edfac.unimelb.edu.au>

Barry McCrae
University of Melbourne
<b.mccrae@edfac.unimelb.edu.au>

Introduction

All of the studies reported in this chapter are concerned in some way with the impact of computer or calculator technology on mathematics education. They have been categorised mainly according to the type of technology or software used, with an additional category for studies primarily concerned with assessment issues. The categories in order of discussion are:

- Computers
- Internet and multimedia
- Non-graphics calculators
- Graphics calculators
- Computer algebra systems
- Logo and dynamic geometry software
- Assessment

Computers

What happens when teachers are asked to make use of computers in their classrooms as an aid to the learning of mathematics? Thomas, Tyrrell, and Bullock (1996) described results and discussed the implications arising from a year-long New Zealand study examining, from the practicing mathematics teachers' perspective, issues arising from the introduction of computers. The study began in the second term of 1993 and involved 12 volunteer teachers from six schools and 3 support staff. All teachers had some experience using computers, but only 4 had used a computer in their mathematics teaching. Two computers were provided for each school. The teachers engaged in cycles of classroom-based action research involving the use of computers, and kept daily logs recording what they did and their personal impressions of success or otherwise. A questionnaire was used to track any changes in the teachers' attitudes for four categories: style of assessment, style of teaching, view of mathematics assessment, view of mathematics teaching. Results showed a significant change in teachers' attitudes for each category towards a more favourable view of the role of the computer in their teaching and assessment. The resulting change in perspective, which for some teachers took as long as a year to achieve, was characterised by:

- a lessening of control and greater use of guided discovery learning that made use of discussion and group work;

- a willingness to learn along with the students;
- a desire to plan lessons involving the computer where its role is a tool for learning; and
- an ability to make mathematics and its implications, and not the computer, the focus of concern.

Two important influences in bringing about the shift were the constant availability of the computer in the classroom, and the encouragement, motivation and assistance provided by the support staff.

To explore the classroom use of computers in the teaching of mathematics throughout New Zealand, a nationwide survey of primary and secondary schools was commenced early in 1995. Thomas (1996) used a representative sample of 1500 replies from all levels of schooling from primary to secondary to report findings from the study. Nearly two thirds of the teachers at both primary and secondary level indicated they used computers in their teaching, but less than half of the primary teachers and only about a quarter of the secondary teachers used computers at least once a month. The major obstacles to greater use were seen to be lack of confidence, arising in part from lack of satisfactory training, and lack of resources including both hardware and software, with the former emphasised more by secondary teachers and the latter more by primary teachers. Teachers indicated they wanted improved confidence, better classroom management strategies, clearer understandings of good software, and good ideas for classroom learning activities from training in the use of computers. The predominant areas of computer use at the primary level were number and geometry, and at the secondary level were graphical work and statistics. Teachers at all levels reported using computers to promote skill development as well as for problem solving and investigations. It would appear that the teachers have good intentions concerning the level and nature of computer use, but further training and resources are needed to bring these to fruition.

The two studies discussed above explored the use of computers largely from the perspective of teachers, but it is also important to consider the students. Vale (1997) investigated how boys and girls in a Year 10 class behaved when they used computers for the learning of mathematics. Video of lessons was transcribed and coded to indicate the participants, the cognitive nature, the content, the attitudes and feelings about people and the computer, and the degree of collaboration displayed in the recorded interactions. Analysis indicated that boys were more interested in the hardware and the networking system, whereas the girls were more likely to view the computer as a tool for accomplishing tasks and tended to personalise their interactions with the computer. Related findings were reported in another study by Vale (1998) that used a questionnaire to explore the attitudes of 49 students from a lower/middle class secondary school in Melbourne towards the use of computers. All students in this study were able to make regular use of computers. Most indicated they wanted to use computers as part of their study of mathematics because they "viewed computers as a source of success in mathematics, a source of pleasure, a source of relevance, a source of power or a combination of these factors" (p. 634). An analysis of explanations for answers indicated gender differences in the views expressed. While recognising the benefits of using computers, girls were more likely than boys to qualify their support for the use of computers. Boys tended to associate computers with pleasure and the importance of developing skill in their use, while girls tended to associate the use of computers with success in mathematics, giving prominence to the potential of computers for improving understanding or eliminating difficult tasks.

As teachers work towards becoming exemplary users of computer technology in the teaching and learning of mathematics it is important to have an idea of what knowledge and skills they require. Rollo and Bobis (1996) examined existing literature to develop a list of criteria indicating expertise in instructional computing, and carried out a case study of one expert teacher to validate and refine the list. As one may expect, many of the listed items are important to any form of teaching, with or without computers. Items solely relevant to computing were: (a) strategies for linking on- and off-screen computer work, (b) strategies for linking on-screen mathematics to the curriculum, (c) knowledge and understanding of the content of computer programs, and the mathematical implications of this knowledge for students, (d) an understanding of how and what students will learn from their use and (e) interaction with a computer program. Chinnappan's (1997) case study of one student teacher's efforts to provide learning and pedagogical implications arising from the use of the program *ANUGraph* suggests that the capabilities identified by Rollo and Bobis cannot be developed in isolated computer training sessions that focus only on how to use a program. As found by Thomas, Tyrell, and Bullock (1996), actual experiences using a computer program in classroom situations are also needed.

A number of classroom-based studies using a variety of computer programs or applications have been reported. Using a case study approach with 14 seven-to-eight-year old children working in a computer environment based on *Geo-Logo*, Yelland and Masters (1997) found that the dynamic and interactive nature of the environment made it possible for the children to coordinate spatial and numeric concepts to solve novel problems. The computer provided immediate graphical feedback on the reflective modifications children made to their thinking about aspects of the problem tasks and how to solve them. This enabled the children to build and test ideas for themselves and, in so doing, make sense of mathematical ideas and problem-solving strategies. However, Masters and Yelland (1996) reported that the children working to complete tasks in the *Geo-Logo* environment did not spontaneously use executive skills related to problem-solving unless the teacher was there to support and encourage them to use these higher order thinking skills. Lowrie (1998) reported similar findings in a case study of two young children, aged 7 and 8 years, using the problem solving software package *Madeline Thinking Games*. He concluded that the flexibility of the computer environment enhanced the metacognitive interactions and the activities directed to the negotiation of meaning between the teacher and the student. As a consequence, the children overcame initial difficulties in linking screen images with real three-dimensional space to develop understandings of perspective, proportion and angle.

Geiger and Goos (1996), in a study with Year 11 students using spreadsheets, examined the effect of the computer as a tool for data collection and manipulation and as a catalyst for the exploration and examination of patterns emerging from the data. By examining the function of students' talk and the structure of student interaction they found differences in the social organisation of students' work. The nature of the task students were given to perform was identified as the major determinant for the differences, indicating that if computer tasks are to elicit high level verbal reasoning these tasks need to be carefully structured. Furthermore, teacher intervention is needed, particularly for those tasks that might be seen by students as solely producing data for exploration. The teacher's role should be to place the task in context, to ensure its problematic nature is not concealed and to direct discussion towards conjecturing and argumentation rather than simply information exchange.

The versatility and effectiveness of using the computer and appropriate software to engage students in meaningful learning of important mathematical concepts has been explored in a number of other studies. Price (1997a) developed and used a computer model of the place-value material Multibase Arithmetic Blocks (MAB) to provide different ways of representing a number and ease of movement from one representation to another. Price reported that using the computer with the software enabled the interviewer to persist in presenting counter-examples until the student's misconceptions could not be maintained in the face of the evidence provided. However Lewis (1996), from an investigation involving the use of a commercial computer software package as a pictorial transition device in linking modes of representation of knowledge, found that use of the software was not a necessary link between concrete and symbolic modes of representation. When designing software, as in the case of Price, or choosing computer software to use with students, it is useful to know design features that will ensure student engagement with the software. Abercrombie and King (1996) investigated the effects on Grade 8 students of different motivational elements in versions of a public-domain fraction drill-and-practice program and found that graphical representation together with aural feedback provided the best motivating and interest-stimulating mathematical environment for individualised learning.

Arnold (1996b) documented and examined ways in which six secondary students and two groups of six preservice teachers used software tools for learning algebra. He found that strategic use of appropriate software, characterised as "goal-directed, flexible, insightful", did support the development of algebraic skills and understandings. These included improved facility in algebraic manipulation and in use of different representational forms. Graphical representation was the most favoured form of representation, but its use was often automatic and superficial. A table of values was more difficult for students to interpret, and less frequently chosen. Computer algebra tools, enabling symbolic manipulation, were most effective for investigations and development of extended algebraic processes such as equation solving. Two impediments to the effective use of the software tools were identified: (a) held beliefs of mathematics learning that devalued exploration and external support in preference to repetitive individual skill development by teacher dominated instruction; and (b) the failure of some algebraic forms (including simple expressions and tables of values) to provide clear signals provoking meaning and action strategies (see also, Arnold, 1996a).

Lamont (1996) conducted an aptitude (sequential or simultaneous information processing style)-treatment (computer function-graphing or traditional teaching) interaction study with two co-educational Year 11 calculus classes. Each student was categorised as high or low on the basis of the ratio of simultaneous to sequential information processing indices. A comparison of means favoured the function-graphing approach with the highs from the function-graphing approach deriving greater benefit than the lows, but the differences were not statistically significant. In a quasi-experimental study also involving two Year 11 classes, Nevill (1998) compared a computer-based graphical approach to the learning of derivative concepts and the elementary rules of differentiation with a traditional approach based on the formal definition of a derivative involving limits. The computer function plotter (*ANUGraph*) made it possible for students to explore concepts in context but without the complications of the limiting process and its notations. On the posttest, which required the use of differentiation rules and the interpretation of results from standard applications of these rules, the graphical-approach class performed significantly better. It is suggested that this may have been due to the computer mediating enhanced links between concept image and concepts definition (see Tall & Vinner, 1981). Another study on the use of computer graphics for establishing integration concepts was carried out by Ye and Thomas (1997), and Wong (1998) emphasised the value of computer-graphic activities to promote visual thinking.

In a preliminary study involving five students in Year 7 and six in Year 8, Thomas and Hall (1998) sought to determine if the students could improve their *proceptual* understanding of equations (see Gray & Tall, 1994) by using a computer program that provided an environment for investigating the algebraic concepts of expression and equation. The students spent six weeks working through the program's various levels. A test comprising 40 linear equations to be solved, increasing in difficulty from equations of the type $x + a = b$ to $ax + b = cx + d$, was used as the pretest and the posttest. A test on the difference in means from pre- to post-test revealed significant improvement for both year levels. More than half the students used a method of solution in which the same operation was applied to both sides of the equation to produce a simpler equivalent equation on route to the desired and eventual $x = a$ form. Eight of the

11 students overcame the cognitive gap described by Hercovics and Linchevski (1994) to solve $5x + 12 = 3x + 24$.

A number of other papers report on potential classroom uses or benefits to be derived from the use of computer software, particularly spreadsheets, but are supported only by the author's beliefs or student affirmation regarding their use. See for example Jun (1997), Lockwood and Boland (1998), Looi and Tan (1996), Lowrie, Hill, and Hemmings (1996), Richards (1997), and Yoong (1997).

Non-Graphics Calculators

Considerable evidence from research studies and projects undertaken prior to 1996 shows that favourable benefits can be derived from the use of calculators for the learning and teaching of mathematics (see Hembree & Dessart, 1986; Shuard, 1992; Stacey & Groves (1996). It is not surprising then that most studies involving basic calculators reported after this time focused on gaining a picture of the level and nature of existing classroom use, and of factors affecting this use. Given the rapid rise in use of graphics calculators in the secondary school, particularly in senior levels, it is also no surprise that all but one of the studies reviewed here related to the use of calculators in primary schools.

Jones and Tanner (1998) found that Welsh secondary students who claimed free and frequent use of calculators performed significantly better than other students on a test of basic arithmetic skills taken without the use of calculators. However, only a moderate percentage of the students reported free (23%) or frequent use (41%). Sparrow and Swan (1997a, 1997b), and Swan and Sparrow (1997), provided evidence that simple four function calculators were not used as frequently as one might expect in the teaching of primary school mathematics, and when they were, not for purposes that tap their full potential as aids to children's learning. They conducted a survey of primary school teachers in Western Australia to determine the extent and nature of calculator use in the primary classroom, impediments to calculator use, and attitudes of teachers towards the use of calculators in the primary classroom. Responses to 16 survey questions, and clarifying comments, were received from 1297 teachers teaching at either Year 1, Year 3, Year 5 or Year 7 levels. Results showed that most teachers (71.6%) were in favour of all students using a calculator, but only a little more than a third (36.8%) reported that they permitted students to use calculators often or to have complete freedom to use a calculator at any time. When calculator use did occur it was restricted by age or grade level, generally done under the direct guidance of the teacher, and carried out for trivial purposes. Frequently reported uses were for doing and checking work, for games and puzzles, and for formal calculator lessons. Teachers made almost no reference to using calculators in problem solving contexts. Anderson (1997) also found that one of the least frequently used teaching strategies for problem solving was to have calculators available for student use. Teachers in Sparrow and Swan's study who reported using calculators as tools for learning and understanding mathematics were more likely to be aware of research-based ideas on calculator use, or to have observed the benefits of calculator use arising from their own use of calculator activities in classroom teaching situations. Sparrow and Swan suggested that an emphasis on standard written algorithms appeared to obstruct the use of calculators in realistic applications, and that a direct impediment to full and appropriate calculator use was the generally reserved and negative attitudes and beliefs of teachers.

In an attempt to better understand the computational process, and in particular the role played by calculators in this process, Swan and Bana (1998) developed a model describing the interplay between mental, written, and calculator computation. They tested this model with a sample of 12 students from Year 5 to Year 7 and found that, when given free choice of computational methods, students often chose to use calculators for questions involving big numbers and decimals. They also reported that some students used calculators when unsuccessful with mental or written methods and some used the calculator only as a checking device. In another study involving similar aged students that also explored choice of computational procedures, Price (1997b) found that students used calculators to perform calculations more often when the teacher was not present. It may be that demonstrated teacher beliefs, or students' perception of teacher beliefs, regarding the role calculators should play in computation influenced student choice of computational methods.

White (1998) examined teacher beliefs regarding the use of calculators in upper primary mathematics classrooms. Responses to a questionnaire were obtained from 115 teachers located in primary schools with an enrolment of more than 100 students. The typical teacher respondent was a female between the age of 36 and 45 with 11 to 15 years teaching experience. Teachers felt that the positive contributions of calculators (e.g. speed of computational facility and the assistance gained in problem solving) offset possible disadvantages of their use (such as causing laziness, dependency, blind acceptance of answers, lack of encouragement of thinking skills and cheating). The teachers perceived strong support for their beliefs from other teachers and students, but only moderate support from their principal/supervisor, and direct opposition from parents. Overall, the teachers in White's study showed a positive intention towards student use of calculators, a perception of approval from the social environment for this intention, and a perception of personal control over the implementation of the intention. This holds hope that the teachers will sooner or later translate their intentions into action.

Since teacher beliefs and attitudes appear to be vital for the successful implementation of appropriate calculator use in the mathematics classroom, it is essential that this issue be addressed in preservice courses. Biddulph (1996) included a four-week calculator component in an optional subject on technology in mathematics education taken by 24 third year Bachelor of Education (Primary) students. Reflective statements by the students showed a major shift in their thinking concerning calculators, from narrow and limited use to wider application as learning and teaching tools. Two major factors mentioned by students as reasons for their changed views were reading research literature on calculators (9 of the 24 students) and the experience of working with children in planned classroom activities making use of calculators (22 of the 24 students). It would appear useful to alert student teachers, and teachers, to research proven benefits that can be derived from appropriate calculator usage as discussed, for example, in Stacey and Groves (1996), Groves (1996), Groves (1997) and Groves and Stacey (1998). It would appear even more important for teachers to trial classroom calculator based learning activities and to reflect on the nature and degree of success of the learning experiences these activities provide.

Graphics Calculators

In the 1990s the graphics calculator became a major form of technology used in many Australia secondary schools to teach and learn mathematics. A major stimulus for the take-up and use by students and teachers of these calculators in some states was systemic decisions by accreditation bodies that permitted students to use graphics calculators in external examinations for Year 12 mathematics subjects.

In 1995 the Victorian Board of Studies decided to permit, from 1997 onwards, the use of graphics calculators in its external examinations for Year 12 mathematics subjects (Victorian Board of Studies, 1995). A survey of all secondary schools and TAFE colleges within Victoria was conducted in September 1997 to provide information on a number of issues associated with the Board's decision. Data were collected from the mathematics coordinator and the VCE mathematics teachers. Reports by Routitsky and Tobin (1998), Routitsky, Tobin and Stephens (1998), and Tobin, Routitsky and Jones (1999) provide the following findings:

- Independent non-Catholic schools prescribed the calculators more often than did Catholic schools, which prescribed them more often than did government schools.
- About 80% of the full cohort of schools had one or more class sets of calculators.
- Personal ownership by Year 12 students varied according to mathematics subject enrolment, with students in the more mathematically demanding subjects reporting higher levels of ownership (41% of classes in the most demanding subject had greater than 95% of students owning a graphics calculator).
- Teachers from government schools had a significantly lower level of expectation that all students would have access to calculators in 1997 or in 1998 than did teachers from non-government schools.
- There was "strong teacher support for the Board's policy to introduce graphics calculators despite a prevailing view that there is inevitably

some social disadvantage in the transition period" (Routitsky & Tobin, 1998, p. 490).

- When the level of graphics calculator access in the classroom increased, teachers' perceptions of the graphics calculator's usefulness in the classroom, its potential to reduce the time for explanation and to make learning easier, and its capacity to improve understanding, all tended to increase,

In 1992 Queensland's Board of Senior Secondary Studies initiated new mathematics subject syllabuses that were to be completely implemented in 1995. Included in the syllabuses was a request that students become familiar with the use of computers and/or graphics calculators for learning and doing mathematics. Carss and Drumment (1997) reported that several Queensland senior mathematics teachers who were evaluating the implementation of the new syllabuses indicated a degree of teacher reluctance to comply with this advice. For graphics calculators, issues of screen size, cost, and the fact that student time to learn how to use the calculators was not time spent learning mathematics were raised. After visiting, observing and talking with staff in four schools that were making use of graphics calculators, Carss and Drumment suggest that a possible reason for the slow uptake of graphics calculators may be the time needed for teachers to rethink, redo, and relearn the mathematics they already know and have comfortably taught before without calculators. However, as shown by Tynan and Dowsey (1997) in their reflections on the Victorian Certificate of Education Graphics Calculator Project, teachers can be assisted and accelerated in doing these things by a program of appropriate professional development directed at raising awareness of the impact and role of graphics calculators in the teaching and learning of mathematics, to increasing skill in the use of the calculators, and to promoting thoughtful integration of the calculators into the learning and assessment of mathematics. Tynan and Dowsey found that, when provided with extensive support materials and related instruction on the use of calculators, teachers felt empowered rather than intimidated by the technology. However, they also found that teachers, at this initial stage of learning to use graphics calculators, gave highest priority to fitting calculator usage to existing practices, rather than examining how graphics calculators might bring about changes to the curriculum.

Penglase and Arnold (1996) provide a comprehensive and critical review of graphics calculator research from 1985 to 1995. Anyone wishing to gain useful insights into the nature of and outcomes from graphics calculator research conducted during this period should consult this review. Most of the studies examined are non-Australasian. MERGA's earlier reviews of research, covering the periods 1988–1991 and 1992–1995, make reference to most of the Australasian studies that are found in the Penglase and Arnold (1996) review. Penglase and Arnold suggested that determining the ways a graphics calculator can be used to provide for maximum learning and determining the teaching practices and learning environments to bring about this maximum learning are two key outcomes that graphics calculator research should provide. Having reviewed the research to 1995, they suggested (in 1996) that there still remains much to be done before these outcomes can be fully realised. Hence, it might have been anticipated that there would be a number of Australasian studies in the period 1996-1999 that focused on these important classroom related issues or, for that matter, on any aspect of learning or teaching involving graphics calculators. This was not the case, and those few studies appearing after 1996 are now discussed.

Geiger (1998) used a questionnaire and interviews to determine the views and attitudes of twelve students towards the use of mathematically enabled technologies in a collaborative classroom setting. The students had classroom access to two models of graphics calculators, including one which provided symbol manipulation capabilities, as well as access to computers, with function graphing and spreadsheet software, both at school and at home. In order to gauge possible changes in perceptions over time, the questionnaire was administered twice—when students had lesser and when they had greater experience using the technology. The Likert style items of the questionnaire, and follow-up interviews, explored attitudes towards the use of technology in learning mathematics, preferences for working together or alone and for use of computer or graphing calculator technology, and ways the technology was used both at home and at school. Results indicated strong overall support from the students for the use of the technologies; particularly in the way they enabled and supported mathematical learning and exploration in unfamiliar contexts. Greater experience with the technologies lead to greater awareness of its strengths and weakness, and may have been a factor in producing some movement towards a preference for working alone rather than together. Students indicated that they were more likely to feel the need for interaction with others when faced with a situation where a genuine problem had to be confronted. This relates to an earlier finding from Geiger and Goos (1996) highlighting the crucial role played by the nature of the learning task in fostering meaningful collaborative interaction in technology-based learning environments.

Faragher (1999) argued that there are some students who feel that graphics calculators are not helpful to them in their study of mathematics. From her research in two Queensland secondary schools she identified common characteristics of students who expressed a negative reaction to the use of the calculators. Most of these could well apply to any student who experiences difficulty in learning mathematics. This suggests that having to learn to use a graphics calculator, as well as having to learn the mathematics, is seen by some students who are already struggling with the mathematics as an additional and unwanted burden, something also noted by Tynan and Asp (1998). From interviews of students who disliked using graphics calculators, Faragher determined that they lacked a deep understanding of underlying concepts, which, she felt, made it impossible for them to consider alternative methods provided by the calculator. Yet Kissane (1997, 1998) and Tobin (1997, 1998), and others, might argue that a deeper understanding of the concepts can be directly fostered by learning these concepts with the aid of a graphics calculator. Faragher (1999, pp. 241–242) suggested some strategies to assist students who might initially be adverse to using a graphics calculator for such purposes.

Graham and Thomas (1997) conducted a classroom-based study with five schools in the United Kingdom to determine if a graphics calculator (TI-80) algebra module would provide a useful environment for students to build an understanding of the notion of variable. An experimental-control design was used with one experimental (graphics calculator) class and one control class in each school. The classes were top and middle ability groups of students aged 12–14 years. A volunteer teacher, who also chose the control class of similar background and ability, taught the graphics calculator class in each school. (It is not clear who taught the control class.) Each treatment completed a three-week algebra module covering the same work that was largely based on the substitution of number values for letters in algebraic expressions and rules. The graphics calculator classes made use of the multiple line display on the *home* (work) screen and the calculator's *store* capability to input different values for the letters in expressions. Once an expression was entered onto the home screen, it was easier for the graphics calculator group to change input numbers for the expression in order to explore how the output number was effected. Graham and Thomas argue that the graphics calculator provided a better environment to gain correct feedback on predictions, thus enabling a better development of "an understanding of letters in algebra as stores with labels and changeable contents" (Graham & Thomas, 1997; pp. 3–13). This is supported by posttest results favouring the graphics calculator treatment over the control treatment, and also by results on particular questions examining conceptual understanding of the meaning of letters. By contrast, Mueller, Pedler, Anderson, and Bloom (1997) found university students were not comfortable with using the graphics calculator in examinations.

Galbraith, Renshaw, Goos, and Geiger (1999) proposed four roles for technology in relation to student learning. Excerpts from a graphics calculator-based lesson, along with comments, illustrate how these roles can assist in mapping and analysing the characteristics of a classroom community of practice. Whether or not one subscribes to the authors' sociocultural perspective on learning, the proposed roles for technology should be of interest to researchers. The lowest level is that of technology as master. Here the complexity of usage limits student activity to a few operations over which they have competence and the student, without sufficient mathematical understanding, blindly accepts the output produced. In the next level, technology as servant, the user is in control, applying the technology as a reliable fast mechanical aid to complete mathematical tasks whose output is largely regarded as authoritative, and not questioned. In the third level, technology as partner, the technology is seen as a companion with which to explore, and not just a tool for doing something. There is also an awareness that an outcome needs to be judged against criteria other than the technology-produced response, with a consequent recognition of the need to balance the *authorities* of mathematics and technology. At the highest level, technology as an extension of self, the technology provides an extension of the user's mathematical abilities, becoming an essential part of the students' mathematical repertoire, something that shares and supports their mathematical argumentation.

Several articles concerned with graphics calculators, but not reporting on research studies, appeared in professional journals and conference proceedings during this period. These articles suggest ways to use the calculators, or describe potential benefits for student learning arising from their use, or raise and discuss implications to curricula and assessment arising from their use. See, for example, Dunne and Brown (1996), Howlett (1998), Kissane (1997, 1998), and Tobin (1997, 1998).

Internet and Multimedia

Michener (1997) explored the potential and the reality of the use of the internet in the teaching and learning of mathematics. He found that mathematics teachers in Australia, like their USA counterparts (Becker, 1999), currently made little use of internet-based activities and that, world-wide, there had been very little research on the use of the internet in the teaching of mathematics. The small number of such studies reported here, none of which actually concern the teaching of mathematics in a school, is a reflection of these findings.

Callingham and Griffin (1997) reported that responses via the internet were disappointingly low to an extended investigative task, available in hard copy form or on the internet, attempted by over 1200 students as part of the Year 9 Numeracy Assessment and Monitoring Program undertaken in Tasmanian government schools in 1997. The major issue appeared to be lack of convenient access during mathematics lessons, though there was evidence that many students were "not disposed to use computers for a maths project" (p. 11) even under favourable conditions. In addition, the quality and quantity of the internet responses was far less than had been expected. This is partly attributed to the relatively low skills of the students and partly to the drawbacks of the internet interface which did not support graphic input and did not allow work to be saved for another occasion. The provision of an email link for mailing attachments largely overcame the first limitation, but the authors concluded that the interface did not seem to stimulate students' thinking and called for further research into students' cognitive processes when faced with a computer interactive task.

A case study in mathematics education in which data were gathered from the internet was discussed by Forgasz, Leder and Lynch (1996) after they first discussed the general research potential of the internet and the sampling and ethical issues that arise when it is used to gather data. The aims of the study were to determine how mathematics and related areas are portrayed in the print media, and whether individual sex-role orientations appear to affect the *reading* and interpretation of media reports. A questionnaire to determine gender role orientations, three newspaper articles (dealing with gender-related educational and career issues) together with a series of questions about their content, and a small number of items requesting biographical information were emailed to five electronic mailing lists concerned with women, gender, feminism, or education. Recipients were asked to respond by email or, if they wished to remain anonymous, by conventional post. The results of the survey were not presented. The authors highlighted two unanticipated difficulties that arose through the use of the internet: the need, because of copyright concerns, to get written permission from the writers and publishers of the newspaper articles; and the contamination of the sample by much wider distribution of the survey, beyond the targeted mailing lists, than they had expected. On the other hand, the use of email facilitated the respondents making additional comments, allowed incomplete responses to be immediately followed-up and meant that the data did not have to be transcribed.

In an apparently related study, Lynch and Leder (1996) described a project in which a similar (three-part) questionnaire was distributed via email to students in a particular teacher's class at a Californian college of business administration. Completion and return of the questionnaire was voluntary. Lessons learned included the need for flexibility in finding a partner for internet projects, the need to pay attention to the technological set-up and support available to the students, and their competence in using it. However, Lynch and Leder stated that the most important lesson learned was that the involvement of technology alone is not enough to ensure that students will be engaged with a task. Internet projects, like all classroom projects, need to be integrated fully into the classroom program and not be just an add-on activity.

Foley and Schuck (Foley & Schuck, 1998a & 1998b; Schuck & Foley, 1998) conducted an intervention study in which a web-based conferencing tool (*TopClass*) was used to promote dialogue between students training to be primary teachers and respected mathematics educators from a number of countries. In their first lecture, the students were asked to complete, anonymously but supplying their own code name, a 41-item questionnaire. The questions probed a student's beliefs on the nature of mathematics and mathematics learning, attitudes to computer conferencing, and views on the usefulness of the internet and computers in learning. Each item had to be scored on a five point Likert scale ranging from *strongly disagree* to *strongly agree*. The students were then given two statements, one on the nature of mathematics and the other on the cultural context of mathematics, and asked to write down their initial responses to the statements.

Subsequently, the students were given reading material offering a wide range of views about the nature of mathematics, including the responses of the mathematics educators to the focus statements which had been posted on the discussion site. As part of the subject assessment, the students had to work in self-selected groups of 5 or 6 to produce a group response to one of the focus statements. About midway through the semester, after all responses had been posted, each group had to formulate a second response that took into account the views of the other groups and any further postings from the mathematics educators. Near the end of the subject, the students had to complete the pre-intervention survey again, supplemented by 10 open-ended questions about the project. Each student kept a reflective journal throughout the semester. From the 145 students (140 female, 25 male) who participated in the study, the results of the 51 students who used the same code name on both surveys were statistically analysed. Two subsets of survey items, those concerned with beliefs and those concerned with attitudes to computer conferencing, were tested for significant changes in means, both item-by-item and overall. In addition, responses to the open-ended questions (n=80 to 90 for each question) and extracts from a selection of reflective journals were analysed for common themes.

Although only 3 of the 18 items about beliefs showed a statistically significant ($p<0.05$) gain in mean values, the entire group taken together showed a clear increase ($p<0.01$) towards a more liberating view of mathematics (Schuck & Foley, 1998). This positive finding was supported by journal entries that showed many students originally held a very narrow view of mathematics, but that their view broadened throughout the semester. However, for those items concerned with the value of computer conferencing, all but one had significant decreases in their means ($p<0.05$) and the overall mean decreased significantly ($p<0.01$) (Foley & Schuck, 1998b). The responses to the open-ended questions showed that many students did value the opportunity for group collaboration afforded by the web conferencing, and the access it gave to the views of others, but that they had difficulty with accessing and using the technology and working in groups.

Technical problems also plagued Kidman and Nason's (1997) pilot study to establish a (mathematical subject-matter) knowledge-building community using a shared database for computer conferencing. The participants in the study were 100 students in the fourth year of a Bachelor of Education (Primary) course (BEd). Students, lecturers and researchers could post ideas and comments on the database, but technical difficulties and access problems prevented some students from making any postings. Technical problems aside, the researchers concluded that, to maintain a computer-mediated knowledge-building community with such a cohort of students, the following two problems needed to be resolved: (a) how to develop commenting skills in the students, and (b) how to overcome many students' fears that they have little to offer a discourse on mathematics education.

Nason, Brett, and Woodruff (1996) used a similar, but more developed computer system (CSILE—*Computer Supported Intentional Learning Environment*), to mediate the construction of a knowledge-building community within two elementary school classrooms in Toronto, Canada. The participants were three groups of students (Grades 3 and 5/6) undertaking a series of mathematical investigations, their two teachers and researcher (Nason). The investigations involved two components: (a) face-to-face inquiry and (b) computer-mediated justification. Data sources were the CSILE log, student and teacher interviews, ethnographic observations of the classrooms, and samples of the students' work. It was found that the high level of knowledge-building exhibited by the students during the inquiry component was not maintained in the computer environment. The authors attribute this to the loss of community, which had been the impetus for much of the enthusiasm shown in the face-to-face work, and to the difficulty of synthesising a primarily oral/iconic process into text format, exacerbated by CSILE's crude graphic tools. Like Callingham and Griffin (1997), they call for human-computer interfaces that overcome these problems.

A number of papers published during the review period report the development of multimedia resources, generally packaged as CD-ROMS with accompanying text material, for use in the preservice training of teachers. Typically, the theoretical basis and perceived benefits of the resource are discussed and there is a description of how it is used or is to be used, but little or no research evidence concerning its actual use is presented. Zevenbergen (1996), for example, reported on the development of a CD-ROM that was soon to be used by students to self-pace their learning of key conceptual and process knowledge in the first mathematics education unit of a BEd. course. Chambers and Stacey (1999) describe three multimedia resources used with undergraduate education students: *Computers and the K–6 Classroom: Kids Can Do!*, *Teaching and Learning about Decimals*, and *Teaching and Learning about Whole Numbers*. *Teaching and Learning about Decimals* consists of background information, case studies (including QuickTime movies) of children with common misconceptions about decimals, and teaching strategies and ideas (including games that challenge misconceptions). In *Teaching and Learning about Whole Numbers*, users view video-clips of children performing mathematical operations and verbalising their steps. It is claimed that students using each of these resources show deeper thinking and enhanced reflective practises about how children think.

Sullivan and Mousley (1996) describe a small scale investigation of an interactive multimedia resource presented on CD-ROM that included videotapes of mathematics lessons, pre- and post-lesson interviews with the teachers, documents and readings relevant to the lessons, and graphic representations of classroom interaction data, *Learning About Teaching* (see Mousley & Sullivan, 1996). The subjects in the study interacted with the resource in groups for about 10 hours before participating in debriefing seminars and individual interviews. Each user maintained a journal, each group session was observed and notes taken, and a computer log was kept of the pathways used by the groups through the resource. Sullivan and Mousley concluded that case material presented in this interactive way can enhance learning about teaching, but that further research is needed into the amount and form of intervention that is needed to support its optimum use.

Herrington, Herrington, Sparrow, and Oliver (1998) described a resource, consisting of two CD-ROMs and written materials, designed to develop preservice teachers' knowledge and practices for teaching and assessing mathematics in K–12 classrooms. Users can view video clips of real classes, hear the views of the teachers and students, and read advice from novice teachers and expert mathematics educators. The paper includes an evaluation of six student teachers' use of the different assessment techniques featured in the resource during a teaching round that took place five weeks after the use of the resource. Each student and supervising teacher was interviewed within three weeks of the completion of the round, and their comments were analysed. It was found that all six students used a variety of techniques to assess children's understanding, with five of the students attributing this to the interactive multimedia resource.

With the rapid increase that has taken place in recent years in the availability, accessibility and reliability of the internet and multimedia resources, it is expected that there will be considerable research activity in this area in the next four years. There is certainly a real need for research that addresses their current and developing use within school classrooms.

Computer Algebra Systems

The use of computer algebra systems for the learning and teaching of mathematics is an important, developing area of research. A computer algebra system (CAS) provides tables, graphs, and symbolic manipulation, thereby creating an environment with potential for the linked use of numerical, graphical, and symbolic results in the learning and doing of mathematics. In the past 10 years most practicing mathematicians, some tertiary students, but fewer school mathematics students, have used a computer-based CAS such as MAPLE, MATHEMATICA or DERIVE. The emergence of affordable versions of these software applications, and the more recent emergence of calculators incorporating them, will make CAS accessible to, and used by, many more mathematics students. This "widespread availability of symbolic manipulation by computer algebra requires a reassessment of the role of the central concepts and skills of secondary mathematics" and will result in changes to existing curricula, assessment and teaching styles because it "challenges the algorithmic algebra and graphing that forms the central thread of secondary mathematics" (Stacey, 1997b, p. 388; Stacey, 1996, p. 15).

Some ideas on what CAS can do and how it can be used in learning and doing mathematics are reported by Blyth, May, and Connell (1997), Leigh-Lancaster (1996), and Leigh-Lancaster and Stephens (1996). Leigh-Lancaster and Stephens use illustrative examples from senior secondary mathematics to document the scope and nature of CAS, and raise some teaching, learning and assessment issues arising from CAS use. They too suggest that CAS will "challenge some aspects of the culture of school mathematics, especially in the amount of time spent on teaching and practicing routine algebraic manipulations, and our current constrained contexts to assess algebraic understanding" (p. 31).

One such challenge to school culture was in evidence when a teacher refused to authenticate a Year 12 mathematics student's work in a major assessment task because the student used a CAS to do algebra manipulations that he could not do by hand (Roberts, 1997). Stacey (1997a) believed that the student's work could have been authenticated, but the results of a survey of Roberts' colleagues later in the year (McCrae, Dowsey & Stephens, 1998) left no doubt that he had their overwhelming support. Only 9% of the 160 VCE mathematics teachers who responded to the survey (of 126 randomly selected schools) would have authenticated the work in the circumstances.

In her response, Stacey (1997a) argued that student access to and use of CAS will require teachers to employ different criteria from that used in the past for determining if a student understands and can use an algebraic procedure. She suggested (p. 17) that a student might be said to understand a procedure if they:

- know what the procedure is for;
- know what sort of input is required;
- know under what conditions the procedure can be used;
- can make sense of the answer that the software produces; and
- have some capacity to detect errors in answers provided by the software

Can CAS help students to learn the procedures and concepts of algebra? Tynan and Asp (1998) conducted a teaching experiment with two Year 9 classes to explore the effects of CAS use in the learning of early algebra. The two classes studied a common linear functions unit, one using a calculator (TI-92) that had a CAS and the other using a graphics calculator (TI-82) that did not. Both classes took the same pretest and posttest in which calculators were not allowed, and the CAS class also took an additional posttest in which their CAS calculators were allowed. Teacher and student journals provided additional data. Results indicated that for equation solving tasks CAS students were more inclined to use the algebraic method of performing the same operation on both sides of an equation than did the non-CAS students, who tended to make more use of the *backtracking* method. CAS appeared to help students focus on choosing an appropriate sequence of algebraic manipulations leading to the solution of an equation and contributed to the formalisation of the approach. (This may have implications for issues concerning backtracking that were raised by Stacey and Macgregor, 1999.) Also, when required to solve problems presented in word statements the CAS students were more likely to construct and solve an equation than attempt to use arithmetic-based backtracking or trial-and-error procedures, yet they were no more successful in completing the tasks than the non-CAS students. In addition, "most students were enthusiastic about using CAS as a dynamic checking device, and post-test results indicate CAS use did not reduce student's by-hand algebra skills" (Tynan & Asp, 1998; p. 627).

Can CAS help students to learn the procedures and concepts of calculus? Calculus was the content focus for five reports from studies conducted in the period of this review. McCrae, Asp, and Kendal (1999) explored the use of a

CAS-active introductory calculus course with three classes of Year 11 students. All students were loaned a TI-92 for use in class and at home. The class teachers taught differentiation to their class in 20 CAS-active lessons designed by the researchers in collaboration with the teachers. The approach taken focused on concept development and applications, and teachers were requested not to give the usual emphasis to the development of by-hand skills. Three tests, the first two of which were CAS-active, were given at the completion of the 20 lessons. The first test consisted of multiple-choice and short-answer questions, the second of extended responses, and the third of a selection of items from the first two tests. The multiple-choice items were also given to a comparable Year 11 class and a standard Year 12 class. Test results showed that, when they were allowed to use CAS, the students in the CAS classes could solve test items with high procedural and low conceptual demand (symbolic items) with the same degree of success as the Year 12 class. Furthermore, test results showed that when compared to the non-CAS Year 11 class they had not suffered unduly from less emphasis on by-hand skills. Results on test items with high conceptual and low procedural demand (core items) also indicated that the conceptual understanding of the CAS students was better than that of the non-CAS Year 11 students, but not as good as the Year 12 students. On the two CAS-active tests students indicated if they did or did not use the CAS to answer each question. Considerable differences in the level of CAS usage on the symbolic and core items was found between the three classes, and greater success in correctly answering questions did not necessarily follow from higher CAS usage.

In two reports arising from the above study, Kendal and Stacey (1999a, 1999b) included a third category in their discussion of the test items—those offering a choice between graphical and algebraic approach (options items). With this complete set of test item data, they gave a more detailed analysis of the relative performance of each class on each item type and of the relationship between CAS usage and success. Differences in learning outcomes for the classes were described and documented from test item performance and from related data collected by means of class observations, interviews of students, and student responses to questionnaires, challenging questions, and log sheets. Kendal and Stacey attributed these differences to the "different teaching styles, personal philosophies and cognitive preferences of the teachers", with one teacher privileging technological and algebraic approaches, one conceptual understanding and by-hand approaches, and one graphical and conceptual understanding built from illustrating algebraic ideas graphically (1999a, pp. 134-135). They suggested that the multiple representation capability of CAS will lead to a diversity of teaching methods, and they asked if there is a privileging or teaching style that is most advantageous for students?

Two other studies involving the use of a CAS in the learning of calculus were reported by Hong (1997) and Hong and Thomas (1997). Hong used 36 high school students (age 16–19 years) in her study and Hong and Thomas used an initial cohort of 161 first year university students in their study. All students were pre- and post-tested with a questionnaire that measured conceptual understanding of integration. All the high school students and seven of the tertiary students (who volunteered) used spreadsheets and a CAS (MAPLE) to investigate the processes of integration. The seven tertiary students were matched with seven others who did not use the spreadsheets and CAS. Interview data was collected from 17 high school students and the seven matched pairs of tertiary students. Results from the data sources suggested that students using the spreadsheet and CAS in their study of integration developed a deeper conceptual understanding of integration, and were not limited to viewing integral calculus as a series of procedures and associated algorithms.

Most of the worldwide research done on CAS to date has involved tertiary level students. The three remaining Australasian CAS studies that were reported during the period of this review are no exception. A study by Chinnappan and White (1997) set out to explore the effect of the computer algebra system MAPLE on student ability to access and use mathematical knowledge. Two students from a standard calculus course who had about six months experience in using the CAS were given a problem task involving concepts in calculus, analytical geometry and linear algebra. After a period of ten minutes in which students read and discussed the problem, they talked their way through their attempt to solve it, and this was video recorded and transcribed. In a brief examination of this data, some links were made between prior knowledge and particular MAPLE commands along with the suggestion that these commands and their execution may have prompted student retrieval and use of this prior knowledge.

Raj and Malone (1997) investigated the effects of a computer algebra system (DERIVE) on the learning of mathematics and on attitudes towards mathematics of Papua New Guinea students enrolled in a Foundation Year Engineering Mathematics course. The course had been largely structured around procedural knowledge that emphasised routine methods. It was felt that using a CAS in a computer laboratory to complete specified tasks might help students to recognise patterns and make generalisations, and generally promote a more interactive form of learning. In addition, it was hoped these sessions would reinforce learning encountered earlier in lectures, extend this learning to real-world applications, and promote an investigative attitude and approach to mathematical problems. A quasi-experimental design that divided the students into experimental (CAS) and control (non-CAS) groups, with one or more groups of each type assigned to the same tutor, was used. Analysis of achievement scores on a common review exercise established the equivalence of the CAS and non-CAS cohorts. Analysis of student performance on test data, presumably done without access to CAS, revealed that neither the treatment nor the tutor had produced any significant difference in achievement after two semesters, with no significant interaction between treatment and tutor. Furthermore, analysis of attitudes questionnaire data showed that the treatment did not produce any significant difference in attitudes. It was noted however that student free response comments did show a positive attitude towards the use of the CAS. The nature of the performance assessment tasks, which might be assumed gave emphasis to procedural knowledge, may have been a contributing factor in the lack of observed differences in achievement and in student attitudes towards mathematics.

Pierce (1999) examined whether the use of a computer algebra system (DERIVE) by undergraduate students for their learning and assessment in an introductory calculus course encouraged them to: (a) use multiple representations; (b) negotiate meaning through discussions with other students; and (c) negotiate meaning through their own interactions with the CAS. Pierce's initial class observation that, in a CAS environment, students did move freely between algebraic and graphical representations of functions was confirmed by questionnaire results. It was also evident from these results that students showed a reluctance to use tables to solve problems. Pierce suggested that one reason for this outcome may be the use of tables primarily in the early development of concepts and rarely in problem-solving situations. Questionnaire data also clearly indicated that when students used CAS and shared a computer they did discuss what was on the screen. They also deliberately interacted with the CAS to extend and clarify ideas. Overall questionnaire results indicated a positive acceptance and "use of CAS in a cycle of student input ⇒ computer output ⇒ student reflection ⇒ leading to new student input" (p. 435). In one questionnaire, students were presented with problem types and asked to give the tool they would use to solve it for speed, confidence, and learning. CAS was clearly chosen for speed and sometimes for confidence, but paper-and-pencil was chosen more often for confidence and for learning, indicating that students still felt they learnt most when they worked by hand with paper and pencil.

Logo and Dynamic Geometry Software

In the early to mid-nineties, Yelland and colleagues carried out a number of studies in early childhood and primary school settings with two versions of Logo (*Geo-Logo1* and *Turtle Math2*) that were specifically designed to support children learning mathematics (Yelland, 1997). The early studies concerned gender differences and are summarised in the previous volume (Barnes & Horne, 1996). Later studies were more concerned with the development of spatial and measurement concepts and are reviewed in the chapter on space and measurement in the present volume. The overall focus of Yelland's research was the importance of the teacher in mediating learning and how children collaborate to solve problems within the Logo environment. A clear result of the research was to support "the use of scaffolded learning in the technological environment, so that learning experiences are not only more meaningful and engaging for young children, but also so that children may use metacognitive strategies more effectively and consistently" (Yelland, 1997, p. 6). Another noteworthy outcome was children's application of mathematical concepts well beyond that expected of them in traditional curricula.

Turner (1999) investigated the use of Logo to support constructivist learning in a post-primary mathematics program. The school concerned was an independent girls' school in Melbourne, with a strong academic focus and traditional structure. Twelve teachers were involved, with the researcher (also a teacher at the school—but not of mathematics) providing expert Logo support, including the provision of a series of Logo microworld activities. The study took place over a three year period and centred on Years 7 through 9. Once again, it was found that the involvement of teachers as learning facilitators was crucial to advance students' understanding of new concepts. However, teacher acceptance that Logo could be used to support constructivist learning of mathematics had to be fostered throughout the study.

The main finding of Turner's study was that the formal requirements of the mathematics curriculum eventually overwhelmed individual teacher interest in the constructivist potential of *Logo*: "Teacher preference was for computer use as an 'efficiency tool', directly applicable to mathematical concepts as predetermined within the subject's curriculum topic list, over supporting students learning to program as a basis for student-directed mathematical exploration or problem solving" (p. vii). This supports Yelland's (1997) claim that teachers will not use technology as a learning tool for their students until its use becomes embedded in curricula.

Dynamic geometry software, such as *Cabri Geometry* and *The Geometer's Sketchpad*, has the potential to return geometry to greater prominence in school mathematics curricula. Green (1997), for example, explained advantages he has found when students use dynamic geometry packages, especially when a reversal approach to theorems is employed. For example, in starting with the statement that equal angles are subtended by an interval, students can explore where the angles lie. He poses the expectation that what is taught, how it is taught and how students are assessed will change as technology is further used in geometry. Engebretsen (1997) used quotes from students' reports of their projects about triangles and vectors to show how much they enjoyed using Cabri on the TI-92 and how it advanced their investigations. Particularly interesting is one group's description of the importance of formulating "in depth conjectures" that were "more than just guess and check".

Not surprisingly then, the effectiveness of such software in enhancing the teaching and learning of geometry emerged as a significant area of research during the period of the present review. A crucial factor when working in a dynamic geometry environment is that geometric constructions based on visual appearance alone—so called *by-eye* methods—are not *drag-resistant*, that is they do not retain the required geometric properties when individual components are dragged. Hence, dynamic geometry software forces students to think very carefully about the properties of the figure they wish to draw and manipulate.

The study by Vincent (1998a, 1998b) using *Cabri Geometry* was undertaken in an independent girls' school in Melbourne, with 12 members of an accelerated Year 7/8 class (ages 11–12), over a four-week period. The girls had their own notebook computer, but had not used Cabri prior to the study. They were divided into two groups on the basis of van Hiele pretest scores: Group I were those at Levels 0 or 1, and Group II were the remaining students. Both groups completed six lessons with structured Cabri worksheets and then the Group II students completed several construction tasks with Cabri, documented by means of Cabri files and recorded conversations. All of the students increased their van Hiele levels for some or all of the four concepts involved (squares, right-angled triangles, isosceles triangles and parallel lines).

The students in Vincent's study frequently commenced their Cabri constructions with by-eye drawings, even when previous experience had shown them that such constructions would not retain all the required properties when dragged (Vincent & McCrae, 1999b). This observation supports the belief of Healy, Hoelzl, Hoyles and Noss (1994) that, for many students, visual constructions provide an essential scaffolding for Cabri constructions. Vincent's study also found that use of Cabri enhanced the use and understanding of appropriate geometric language, particularly when students actively discussed their constructions with each other (Vincent & McCrae, 1999a).

A study by Cashman (1997) also demonstrated the effectiveness of Cabri in enhancing the geometrical language, and reasoning, of students. Action research was carried out on a Year 8 (ages 12–13) class of 28 girls of average ability. The students worked in pairs for 11 lessons completing Cabri investigations following worksheets prepared by the researcher. The development of geometrical understanding within the students involved, as determined by a van Hiele pre- and post-test, was not significantly different from other classes which received conventional instruction. This may have been due to flaws that were exposed in the teaching methodology employed and which pointed to the need for a better balance between exploration, instruction, and feedback. Vincent (1998b, p. 651) similarly observes that it is important that Cabri tasks are structured carefully and that teacher intervention, though it should be kept to a minimum, is essential at times.

Stewart (1998) also concluded that teacher intervention was necessary with student-centred materials in a study that investigated a constructivist approach to the topic of congruent triangles in Year 9. A total of 15 students of low to high ability used worksheets, based on constructivist principles, to complete Cabri activities involving the main congruence conditions. The students were spread across four classes, taught by four different teachers, and the school concerned was the same independent girls' school in which Turner's (1999) Logo study was conducted. Two of the teachers let their students work through the worksheets at their own pace, providing individual assistance as required, whilst the other two teachers provided more direction to their classes. Pen-and-pencil testing both before and after completion of the worksheets indicated that only the most able students had learnt much about congruence concepts even though most tasks were completed satisfactorily. However, the teachers involved indicated that the participating students were better prepared for the formal teaching of congruence later and suggested that teacher intervention providing consolidation at the end of each task was necessary to assist students develop the relevant concepts.

Stewart (1999) reports a very positive feeling about an alternative investigative approach to congruence trialed with all Year 9 classes at the same school the next year. This only took a single lesson and consisted of teacher-managed discussion based around the question "What information do you require in order to draw exactly the same triangle as [the] one I have already drawn?" (Lopez-Real, 1996). Students used *Cabri Geometry* to draw their triangles, with one laptop shared between two students. According to Stewart (1999), the students "learned in a cooperative way as they engaged in discussion about concepts and strategies. It provided a sound conceptual framework for further learning" (p. 356).

A study by Redden and Clark (1997) investigated students' attempts at inferring generalisations about angle measurements from data obtained by manipulating diagrams drawn with *The Geometer's Sketchpad*. The study was conducted with two Year 7 (age about 13) mathematics classes, with a total of 41 students and taught by their usual teachers. Both classes were undertaking their first unit in geometry, and results were reported for two activities: the first one was concerned with complementary angles and the other with vertically opposite angles. Working in pairs at a computer, the students were provided with a Sketchpad diagram (having appropriate invariant properties) which they could manipulate and so observe what effect this had on the relevant angle measures (and, in the case of the pair of complementary angles, their sum). At the conclusion of each activity, the students were asked to "Express your findings in the form of a rule". Redden and Clark concluded, on the basis of the students' responses to this question, that the Sketchpad environment facilitates expressions of generality, but that these expressions of generality exhibit variability in structure in both the language used and their geometric insight. The researchers used the SOLO taxonomy to help categorise the responses.

There was an improvement in response quality for the vertically opposite angles activity (actually the third investigation undertaken) over the complementary angles activity (the first investigation). This development was conjectured to be at least partly attributable to follow up discussions after the earlier activities—an explanation in accord with the observations of Vincent and McCrae (1999a) and Stewart (1999) referred to above.

Dix (1999) compared the performance of Year 8 students on geometry assignments presented in two formats: (a) technology-rich, requiring the use of *Geometer's Sketchpad*, and (b) traditional. This well-designed study was also used to investigate two more general questions: whether the use of computers influences student motivation and whether there is a gender difference in students' attitudes towards computers. Two Year 8 classes of similar age and mix of ability, in a public school in Adelaide, were involved in the study. Class A comprised 28 students, 11 female and 17 male, and Class B comprised 19 students, 8 female and 11 male. The same teacher taught the topic Construction and Design to both classes in exactly the same way. Two equivalent versions of two assignments, Tessellations and Angle Sum in a Polygon, were created and both classes received both assignments but in alternating format. Each assignment had to be completed in a maximum of three periods and could only be worked on in class time. Results from the assignments and a common end-of-unit test, containing two questions specifically pertinent to the assignments, were used to assess student achievement.

No significant difference was found between the achievement of students on the assignment answered using *Geometer's Sketchpad* and their achievement on the equivalent assignment answered using the traditional methods of pen and paper. Dix considered this a positive outcome considering the students' limited previous experience with Sketchpad (three orientation lessons conducted for each class as part of the study), and argued that Sketchpad is thus "an appropriate and effective mathematics tool for teaching and learning geometric principles" (p. 196). Her finding with regard to the effect on student motivation was that it did appear to be positively influenced, but that a corresponding shift in achievement was not apparent. Pre- and post-administration of a computer attitude survey showed that the male students had a significantly more positive attitude to computers than the female students, both before ($p = 0.02$) and after ($p = 0.01$) the study. There was a significant positive change in attitude of the male students during the study ($p = 0.007$) and some positive shift in attitude of the female students ($p = 0.14$). Clear gender differences were observed in the students' use of Sketchpad, with male students more willing to experiment with it while female students were much more task focused.

Each of the dynamic geometry studies referred to so far has concerned junior secondary school age children. Brown's (1997) report, however, is concerned with primary school age children. A Grade 6 and two Grade 5 classes at an elementary school in Alaska were provided with a Texas Instruments TI-92 calculator for one week. During this time the students completed a number of investigative activities using the calculator's inbuilt Cabri Geometry capability. Investigations of the sum of the angles on a straight line, in a triangle and in a regular polygon were some of the activities involved. The students were able to complete the investigations on their own and "had developed a better understanding of geometry at an earlier time in the year than had usually been achieved in the past" (p. 108).

The study of O'Connor (1999b) is unique amongst those reviewed here in that its focus is the role of dynamic geometry software in the development of dynamic visualisation strategies, rather than in the learning of geometry. The study was conducted in a Melbourne boys' senior secondary college where more than 80% of the students come from working or middle class, non-English speaking families. Forty-nine Year 11 (ages 16–17.5) students volunteered to take part in the study and a test of spatial ability and a computer proficiency questionnaire were used to select the nine who were involved in the entire study. Three of the nine had high spatial and computer proficiency (PC) scores, two had relatively low spatial and PC scores, two had high spatial and low PC scores and two had low spatial and high PC scores.

The learning phase of O'Connor's study consisted of four 50-minute sessions, each one involving four or five related activities. The focus of two of the sets of activities (Comparing Areas and Tangrams) was the orientation of objects to one another, while the focus of the other two sets (Rolling Objects and Rod Loci) was the production of a locus by a point on an object. For each activity, the students first predicted the outcome, explored the situation using a Sketchpad representation prepared by the researcher, and then concluded with a short analysis of the accuracy of their prediction. It was found that the students' visualisation strategies did improve over the course of the study, but that their general spatial ability did not improve significantly more than the 40 students not involved in the entire study. The rate of development of visualisation strategies appeared to be linked to general spatial aptitude; nevertheless, a *predict-test-evaluate* approach to instruction is recommended for all ability levels (O'Connor, 1999a, 1999b).

O'Connor's (1999b, pp. 74–78) discussion of the implications of his study for teaching and learning, includes most of the findings common to most of the dynamic geometry studies that have been reviewed here. Most notable of these common findings is that dynamic geometry software, in particular *Cabri Geometry* and *The Geometer's Sketchpad*, can be used with little prior instruction as an effective aid in the learning of geometry from upper primary school through to senior secondary school. Learning seems to be enhanced if tasks are carefully structured, there is appropriate teacher intervention and students are encouraged to discuss their work—however, these should be regarded more as common observations than findings as none of the study designs really controlled for any of these factors.

It appears that *Logo*'s place in the mathematics classroom has been taken over by dynamic geometry software. However, its current incarnation, MicroWorlds, is known to be used in many Australian schools and hopefully some studies involving its use in mathematics learning will be published in time for the next review. Perhaps the most exciting area for future geometry research, however, is the use of dynamic geometry in modelling problems of a dynamic nature such as linkage problems (Schools Council, 1970).

Assessment

At the time of writing of the previous volume, the need to address issues surrounding the use of graphics calculators in assessment in advanced mathematics had become apparent (Barone-Nugent, 1997; White & Pegg, 1996, pp. 156–7). In early 1995, the Victorian Board of Studies (1995, March) announced that graphics calculators could be used in Victorian Certificate of Education (VCE) examinations commencing in 1997. At much the same time it was announced that graphics calculators could be used in Western Australian Tertiary Entrance Examinations (TEE) commencing in 1998. In Victoria, only

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graphics calculators without CAS (computer algebra systems) were approved, but Western Australia allowed the use of a calculator (HP 38G) with limited CAS capability. From the outset, Western Australian examiners could set questions that *required* the use of a graphics calculator, but such *graphics calculator active* questions were not allowed in VCE examinations until 1999.

With the initial requirement that VCE examinations be *graphics calculator neutral*—that is students were not to be advantaged by having a graphics calculator, Jones (1995) investigated the potential impact of graphics calculator availability on the 1994 examinations for VCE Mathematical Methods 3/4. He used the categories: (a) *no impact* beyond that of an ordinary scientific calculator, (b) *impacts* by providing alternative, but mathematically valid, methods of solution and (c) *trivialises* by providing alternative methods of solution that require little or no mathematical input from the user. McCrae (1996) extended this analysis to the 1994 VCE Specialist Mathematics examinations. Jones and McCrae (1996) found that the main impact of graphics calculators would be to broaden the methods available to answer many questions, since they allow the user to employ graphical and numerical approaches instead of conventional algebraic solutions. A small number of question types, in particular multiple-choice items involving the selection of an appropriate graph, would be trivialised and so would be no longer useable in their existing form. Tobin (1997) found that the availability of graphics calculators would have had a similar impact on the 1995 and 1996 examinations.

To discover whether VCE mathematics teachers agreed with their assessment as VCE examiners of the potential impact of graphics calculators, Jones and McCrae (1996) asked a volunteer sample of 27 VCE mathematics teachers to categorise the impact of graphics calculator availability on 13 multiple-choice, calculus questions taken from the 1995 VCE Mathematical Methods 3/4 examination paper. The categories were: (a) has no impact and so the question could remain unchanged; (b) impacts but the question could remain unchanged; and (c) impacts and the question would need to be modified or replaced. It was found that the teachers and the examiners generally rated questions in a similar manner, but that where clear differences existed, the examiners tended to be more conservative regarding fewer of the questions as being suitable, in their existing form, for a graphics calculator neutral examination.

To assist with the design of mathematics examinations in which students have access to graphics calculators, Kemp, Kissane, and Bradley (1996) describe and exemplified a typology of how graphics calculators are expected to be used in answering examination questions. They identified, and illustrated with examples, 10 types of usage grouped into three major categories: (a) graphics calculators are expected to be used, (b) graphics calculators are expected to be used by some students but not by others (i.e. graphics calculators could be used), and (c) graphics calculators are not expected to be used. Kemp, Kissane, and Bradley also discuss the awarding of partial credit for incorrect answers when students have access to graphics calculators and highlight the need "to be more cautious and explicit to students in giving either advice or instructions on showing their work in an examination" (p. 47).

Bradley (1999) discusses students' uses of graphics calculators in the Western Australian 1998 Applicable Mathematics TEE as evidenced by their written papers and from talking to a small sample of students. In accord with Kemp, Kissane, and Bradley's (1996) typology, the examination paper contained a mixture of questions for which graphics calculators were either required, could be used at the students' discretion, or were not expected to be used. Bradley found that generally students used graphics calculators where they were expected to use them, but that many students lost time and accuracy through inefficient use of calculators or data entry error. She concludes that more attention needs to be given to students learning how to use their calculators efficiently, including whether it is appropriate to use a calculator at all for a particular task.

The use of graphics calculators in the Western Australian 1998 Calculus TEE was also investigated (Anderson, Bloom, Mueller, & Pedler, 1999; Forster & Mueller, 1999; Mueller & Forster, 1999). For each of six questions in which graphics calculators were either expected to be used or could be used to advantage, examination markers recorded the methods students used and the marks awarded for a systematic sample of 404 (21%) of the total 1882 scripts. In addition, students' working was further perused on 172 (9%) of the scripts, six candidates (3 female and 3 male) of differing abilities were interviewed in the week following the examination, and two markers were interviewed after marking had finished.

Mueller and Forster's (1999) analysis concentrates on errors that can be regarded as misuses of graphics calculators or misconceptions about their outputs. They found that interpretation and transcription of graphs were major areas of difficulty for many students, and suggested that more instructional time needs to be spent on exploring the limitations of calculator graph displays (and other limitations of particular graphics calculators). Mueller and Forster also found that a significant number of students did not use their calculator as a first option for graphing even when there was a time advantage for doing so, though they may have used it for checking their answer. This uncertainty about when to use graphics calculators was accompanied by their under-utilisation due to lack of familiarity with all of their appropriate capabilities.

Forster and Mueller (1999) explored the ways in which students used their graphics calculators effectively. In addition to using the standard graphical or numerical features, many students made sophisticated use of other capabilities (in particular, parametric graphing). Students' interview responses highlighted the usefulness of their graphics calculators to check traditional calculus methods. Forster and Mueller also discussed the need for adequate rigour in written answers based on calculator output, and drew attention to two concerns of teachers: whether more is being expected of students because of the availability of technology and whether it is resulting in calculus giving way to numerical methods.

With the increasing availability of affordable, hand-held calculators having CAS capabilities, an emerging priority is to research the impact this will have on mathematics assessment. McCrae (1996) found that over 60% of the 1994 VCE Specialist Mathematics paper would have been either trivialised or severely impacted by the availability of a CAS calculator. Leigh-Lancaster and Stephens (1996) analyse the impact of CAS availability on three questions, relating to functions and algebra, taken from end of Year 12 examinations sat in 1995 in New South Wales and South Australia. This analysis was conducted as part of a comprehensive analysis of Year 12 mathematics examinations in most Australian States to show how access to CAS is likely to change the ways in which such questions are approached. They conclude that there will be less need for students to practise routine algebraic manipulations, but more need to appreciate and move between multiple representations of functions and expressions.

Stephens and Leigh-Lancaster (1997) presented a sample assessment task, probing students' understanding of cubic functions and their graphs, to illustrate what they believe are key issues in the design of assessment tasks when students have access to CAS. Although the availability of CAS lessens the symbolic manipulation demands placed on students, it is clear that the task cannot be done successfully without a sound understanding of cubics and the algebraic techniques needed. The design principles identified by Stephens and Leigh-Lancaster as being necessary for such tasks included (a) the need for a greater emphasis on concepts and understanding, and (b) the task is driven by mathematical considerations rather than to reward technological expertise. Anderson, Bloom, Mueller and Pedler (1997) similarly emphasised that assessment should be driven by the objectives of the mathematics curriculum, not by the available technology, and that CAS availability means that there needs to be a reduction in routine drill and practice examples and a greater emphasis on examples requiring students to interpret information or to give evidence of their reasoning.

An impact of mathematically able software (Leigh-Lancaster & Stephens, 1996), such as CAS and dynamic geometry software, on coursework assessment in particular were noted by Dix (1999). She found that much more exploration and investigation occurred in a technology-rich environment, resulting in the completion of several versions of each assignment and the extension of the task beyond its original scope. This can result in an excessive workload on students, as occurred with the school-based Common Assessment Task for VCE Mathematical Methods, causing its redesign (McCrae, Dowsey, & Stephens, 1998, p. 354).

In summary, research reviewed in this section both showed the need to modify assessment methods in anticipation of students having access to increasingly sophisticated technology, and how assessment was responding to this need. A critical question that looms is the extent (if at all) to which it is going to be necessary to conduct technology-free mathematics assessment to ensure that we can validly and reliably assess whether students possess the skills and understanding that are necessary to use the technology effectively. But what are these essential, prerequisite skills and understanding?

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Research in Teaching and Learning Algebra

Elizabeth Warren
Australian Catholic University
<e.warren@mcauley.acu.edu.au>

Introduction

"Algebra can be thought of in many ways, as patterns, functions, and relations, as language, representations, and structures based on generalised arithmetic, or as a tool for modeling mathematical ideas and problems" (National Council of Teachers of Mathematics, 1998, p. 221). The algebra curriculum debate has focused on the shift from an over-emphasis on symbolism, manipulative skills and rote memorization of facts to a balance between procedural and conceptual knowledge, the development of problem solving abilities, and the role of technology in assisting the development of algebraic understanding. All of these issues are reflected in the recent research conducted by Australasian researchers.

This chapter is divided into three main sections. The first section deals with research relating to students' understanding of algebraic concepts and the cognitive and non-cognitive factors that are believed to influence particular misunderstandings. The second section focuses the contexts and applications for learning algebra, and the third section covers studies on the teaching of algebra including research relating to the teacher and different approaches used for teaching algebra.

Students' Understanding of Algebraic Concepts

Bridging the Didactic Gap between Arithmetic and Algebra

Recently in the literature the distinction has been made between algebra, algebraic thinking, and pre-algebraic thinking. Algebra consists of two components—representing numbers and quantities by literal symbols and calculating with these symbols. The focus is on a letter-symbolic form. By contrast, algebraic thinking refers to a broader range of representations. Students are engaged in algebraic thinking when they represent the relationships between quantitative situations. This representation could include the letter-symbolic form but could also include graphs, spreadsheets, geometric patterns and natural language. Beginning algebra students are required to move from arithmetic thinking to algebraic thinking. This transition is believed to involve a move from knowledge required to solve arithmetic equations (operating on or with numbers) to knowledge required to solve algebraic equations (operating on or with the unknown or variable). This transition is referred to as pre-algebraic thinking, the cognitive gap or didactic cut between arithmetic and algebra.

Two approaches for bridging this gap are recommended in the literature. The first involves generalising the patterns in arithmetic and the second involves generalising the patterns found in functional situations such as number patterns, visual patterns and tables of values. Students seem to have varying success with these approaches. In Australasia, studies have explored cognitive factors that influence success when using both of these approaches. The next section reports the findings of this research. The first group of studies investigated the transition from arithmetic to algebra (pre-algebraic thinking) purely in terms of generalised arithmetic.

Algebra as Generalised Arithmetic

Students' transition from arithmetic to algebra was investigated in a three year longitudinal study by Boulton-Lewis, Cooper, Atweh, Pillay, Wilss, and Mutch (1997; Boulton-Lewis, Cooper, Atweh, Pillay, and Wilss, 1998; Boulton-Lewis, Cooper, Pillay, and Wilss, 1998; Cooper, Boulton-Lewis, Atweh, Pillay, Wilss, and Mutch, 1997; and Pillay, Wills and Boulton-Lewis, 1998). This study examined students' growing ability with the commutative, associative, and distributive laws; inverse operations; order of operations; meaning of equals; and, meaning of the variable. The study followed 33 students from the beginning of early algebra instruction (Grade 7) to the completion of algebra instruction (Grade 9). Once a year, a 20 minute interview was conducted with each of the students. No intervention by the researchers occurred during the period of the study. Students simply attended their normal mathematics classes. The results indicated that by the end of Grade 9 most students had moved from not being able to explain the commutative law for addition and multiplication to satisfactory explanations. The distributive law continually caused some students problems. Most did not exhibit difficulties with inverse operations or order of operations throughout the three year period. The results also showed that the notion of equals (in the incomplete solution) denoting the answer did not change. For equals in the complete solution, understanding moved from inappropriate in Grade 7 to inappropriate or algebraic in Grade 8, to algebraic (denoting an equal or balanced relationship) in Grade 9. One outcome of this longitudinal study was the proposal of a model that is believed to describe algebraic development. The model is based on the premise that students' difficulties stem from dealing with inappropriate cognitive loads in their early algebraic experiences.

The model is intended to aid the transition, and suggests a sequence for learning complex equations as given in Figure 1. The model reflects increasing levels of processing loads. As concepts become well known they are subsumed into concepts at a higher level. Thus learning complex equations is the end product of a learning sequence of mathematical concepts including binary arithmetic ($2 + 3 = 5$), complex arithmetic ($35 \div 7 + 8 = 13$), and binary algebraic operations ($x + 3 = 11$).

ARITHMETIC

$2+3 = 5$; $35\div7+8=13$;

Operations, laws, numerical answers, and equals as meaning each side of equals is the same value

PRE-ALGEBRA

$x+7=16$; $3(x+7)=24$; $3x$

Recognition of unknown, then variable in equations, then expressions, concatenation, equals as meaning each side of the equation is the same

ALGEBRA

$x+3=2x-1$; $x+3y+4x-2y=15$

More than one unknown or variable, operating on or with the unknown/variable, equals as equivalence.

Figure 1. Developmental model for the transition from arithmetic to algebra.

Cooper and Williams (1997) performed a teaching experiment using the model as the framework for developing a sequence of teaching episodes. Two ten lesson programs were presented to two Grade 8 classes. At the completion of the teaching experiment, 14 students were interviewed. While it was claimed that the results indicated that the model appeared to provide a framework for effective teaching, some concerns were expressed. It was suggested that the model should consist of separate steps performed over time but rather as steps that should be followed for each new notion and principle introduced to students. Cooper, Baturo, and Williams (1999) further reported that the correct meaning of equals appears to need ongoing reinforcement, and that students' confusion between equations and expressions is very resilient.

Algebra as Representing Functional Relationships

The need to contextualise the initial instruction in algebra has been the focus of considerable discussion and research in recent years. In this case, the variable is introduced through a functional approach, with the focus on the language of generality. Research findings indicated that language and specific thinking skills play an important role in success with this approach.

Redden (1996a, 1996b) investigated the role that natural language plays in successfully reaching symbolic solutions. The sample comprised 1435 children aged 10 to 13 years. Half of the children had no formal exposure to school algebra. The children were presented with four number pattern items and were asked to describe them in their own language and in symbols. Using SOLO's taxonomy (Biggs & Collis, 1982), categories for both types of responses were delineated. The results indicated that there was significant association between the natural language of pattern description and the symbolic notation used to represent the language. However, the research did not indicate whether one was a precursor of the other. A follow-up study (Redden, 1996a) with 26 of the original students was conducted two years later. This study investigated whether the symbolic language of algebra followed children's natural language development. In order to ascertain children's ability with natural language to describe generality in patterns, they were presented with patterns involving uncountable examples. The results indicated that natural language description of number patterns seems to be a necessary prerequisite for representing the patterns in algebraic notation.

Other processes also seem important for success with the functional approach (Warren, 1996, 1997). In this study a written test was administered to 379 students (13–15 years) and interviews were conducted with 36 of these students. The results indicated that success calls on an array of thinking processes not generally believed to be needed for early algebraic understanding. In particular, spatial visualisation, that is the skill of mentally manipulating, rotating, twisting and inverting pictorially represented stimuli, seemed important for generalising from visual patterns. Successful students also seemed to be able to perceive the visual pattern from differing perspectives. They also quickly changed their approach to reaching a generalisation once they realised the generalisation was not applicable to all the presented cases; that is, they exhibited an ability to think flexibly.

Cognitive Theories and Developing an Understanding of Algebraic Concepts

Cognitive theories with regard to developing an understanding of algebraic concepts are prevalent in the literature. Many Australasian researchers have investigated and used three theories of cognition in their recent research, namely, Biggs and Collis (1982), Küchemann (1978) and Sfard (1991, 1992).

Briefly, Sfard claimed that in reaching algebraic abstraction, students pass through three phases: interiorization ➔ condensation ➔ reification. The interiorization phase occurs when some process is performed on familiar mathematical objects. The condensation phase occurs when the process is condensed into manageable sizes. Reification is the transition from an operational mode to a structural mode where the process is an object in its own right. This transition is often referred to as encapsulation. Küchemann (1978) described the range of students' views of algebraic symbols in terms of hierarchical levels, namely, letter is ignored, letter is evaluated, letter as object, letter as specific unknown, letter as generalised number and letter as variable.

The development of children's understanding of the variable concept are reflected in these levels. Stacey and MacGregor (1997) confirmed the existence of a further two categories, these being, *x* as multiple referents and *x* as an alphabetical label.

The Development of Understanding of Algebraic Concepts

Many capable Australian students also experience difficulties with school algebra (Pegg & Hadfield, 1999). For this study, 1 000 scripts were randomly selected from capable Year 12 mathematics students' final public examination in NSW. The cohort in this sample was drawn from the top 50% of papers for 1995, 1996, and 1997. Significant problems identified in student responses included an inability to recognise the difference between two squares, a tendency to interpret expressions as equations to be solved, an inability to understand the place of numerators and denominators in fractions, and difficulties in monitoring basic algebraic and arithmetic operations while undertaking the solution process.

Quinlan (1996) investigated the difficulties that students experience when developing an understanding of the meaning and use of algebraic symbols. Quinlan's research synthesised and built on previous research projects (e.g., Harper, 1979; Küchemann; 1978; Collis, 1975). In this study, the sample of 208 Year 7 and 309 Years 7 to Year 12 students answered a number of written tests and were interviewed with regard to their understanding of the variable. Two major findings were reported. First, at least some minimal understanding of the meaning of symbols (e.g., letter as object, letter evaluated) is related to success in writing symbols in algebraic form. Second, students' success in using symbols is related to developing an understanding of symbols, that is, understanding symbols as representing generalised numbers and variables. The findings supported the notion that students with better understanding of the variable experienced greater success on algebraic tasks and vice versa.

Many of the difficulties experienced by school age students persist at a tertiary level (Boustead, 1997; Coady & Pegg, 1996; Galbraith, Pemberton & Hains, 1996). Coady and Pegg investigated the extent to which 150 first year university students had developed a formal level of reasoning in terms of the SOLO model (Biggs & Collis, 1982). The elements are abstract concepts and the operations are concerned with determining actual and deduced relationships between the elements. The results indicated that some students were operating at a formal level and some were not. It was conjectured that formal reasoning requires high levels of cognitive growth in both procedural and structural aspects of algebra. These findings were supported by Boustead (1997) and Galbraith et al. (1996). Boustead's research examined algebraic understanding of final year high school students (n = 37), first year university students (n=540) and second year university students (n = 340). Their algebraic understanding was examined according to their understanding of (a) the order of operations, (b) properties of numbers, (c) structure of obtaining solutions, (d) possible range of solutions, and

(e) false generalisation. Although most of the secondary students exhibited difficulties in all areas, these difficulties were more widespread with the first year students and slightly less widespread with the second year students. Galbraith et al. examined the algebraic knowledge of students entering university mathematics courses. Their results also revealed a number of misconceptions students held with regard to some fundamental algebraic concepts. Both Boustead and Galbraith et al. concluded that students do not seem to markedly improve their algebraic skills through learning more advanced mathematical concepts. One conjecture was that most of the success that students experience at school is based on limited understanding of the variable with most of the algebraic processes rote learned. This conjecture is supported by White and Mitchelmore (1996).

White and Mitchelmore (1996) researched a group of 40 university students' performance on calculus application problems. The type of problems used in this study could be solved using algebra and symbolic calculators. Student responses strongly suggest the major source of difficulty was an underdeveloped concept of the variable. Students tended to treat variables as symbols to be manipulated rather than quantities to be related. In their previous mathematical experiences they seemed to have learned to operate with symbols without regard to their contextual meaning. White and Mitchelmore suggest that modeling a contextualised rate of change problem using algebra represented a higher level of abstraction than abstraction of algebraic concepts with no context, indicating additional complexities to Sfard's theory. It was hypothesized that success in university calculus courses requires a concept of the variable at or near the point of reification.

The importance of conceptual understanding, not only in the algebraic domain, but also in other areas of mathematics was supported by Thomas and Hong (1996). This research investigated student thinking and misconceptions when dealing with the Riemann integral. The sample comprised 47 senior high school students studying calculus. The students responded to a questionnaire designed to probe conceptual understanding of the Riemann integral. It was conjectured that the high number of students who were unable to make any attempt at solving the problems where there was no obvious process, indicated a lack of conceptual understanding. They seemed to view the integral sign as a process rather than a concept, that is, they have failed to encapsulate the integration process.

Reasoning processes. The role of reasoning processes in reaching the point of reification has also been investigated in recent research. English and Sharry (1996) visited Sfard's theory and suggested that the transition from condensation to reification requires students to divide their attention between calculations involved in dealing with the mathematical problems and the objects themselves. They hypothesized that analogical reasoning assists this transition. Their research investigated 10 students' (aged 16–18 years) ability to classify algebraic expressions and suggested that the development of the correct categories entails the ability to reason analogically.

The importance of the role of higher-order thinking in the algebraic domain was also supported by Tularam (1997) and Warren (1996, 1997). Tularam investigated sixteen Year 11 students' ability to solve novel algebraic word problems using talking aloud and clinical methods. The results of the study showed that algebraic knowledge, higher-order thinking and affective factors were associated with success in novel algebraic word problem solving, with higher order thinking being the most important factor in successful performance.

Language proficiency and learning algebra. MacGregor and Price (1999) investigated whether awareness of symbols (knowing the numerals, letters and other mathematical signs can be treated as symbols detached from real world contexts), awareness of syntax (a recognition of well formed algebraic expressions), and ambiguity are associated with success in learning the notation of algebra. The paper explores the interaction between the ability to reflect on and analyse spoken or written language and the ability to form algebraic expressions. Three tests were administered to 1236 students aged 11 to 15. The tests consisted of language items requiring symbol awareness, language items requiring syntax awareness and items devised to measure students' ability with algebra. The results indicated that the language items were too easy. New items were selected for the language component and the tests were administered to another 400 students. The results of 340 students' responses were analysed. All of these students had English as their first language. Some students with good metalinguistic skills had many of the algebra items incorrect, while for others, poor metalinguistic skills did not affect their ability to learn algebra.

Most of the above research has focused on the difficulties students have in developing algebraic concepts; that is, knowledge generation. The next section summarises the results of the research pertaining to knowledge utilisation, especially with regard to solving word problems. The ability to formulate solutions to word problems has received a great deal of attention in recent research.

The Context and Application of Algebraic Concepts

Formulating Equations from Word Problems

MacGregor and Stacey (1996a, 1996b) tested a sample of students (n = 90) on their ability to solve word problems. The problems could all be solved by non-algebraic means. Two crucial steps that are believed necessary for reaching a solution are representing the relationships in the problem and integrating these relationships in one variable. The results indicated that the major difficulties experienced by these students was the misuse of algebraic notation rather than the ability to comprehend the problem and formulate a solution path. The preferred solution method was trial and error. The same sample was re-tested ten months later (MacGregor & Stacey, 1996c). During the intervening period the students had been trained to use a systematic trial and error approach to problem solving. Although the number of students who could correctly answer the problems increased, their use of algebra did not substantially increase. The results indicated that although they understood the underlying structure, many were not using algebra to represent the structure. One conclusion from this study is that representing a problem algebraically is not an intuitive process. It needs to be explicitly taught. It was also suggested that selecting word problems that were trivial, that is those which could simply be solved by arithmetic means, could contribute to the solution paths chosen from the students.

Students' level of understanding of the variable concept also seems to impact on their ability to solve word problems (Küchemann, 1978). Differing interpretations of letters in school algebra seem to produce differing patterns of reasoning. Stacey and MacGregor (1997, 1999a) from their research of 900 13–15 year old students found that one quarter of the sample did not work out correct answers to some very simple problems. The difficulties seemed to revolve around the cognitive discontinuities involved in the shift from using arithmetic methods to algebraic ones to reach a solution, that is, bridging the didactic gap. Follow-up interviews revealed how different uses of a letter to symbolise the *unknown* affected their attempts to use an algebraic method. For example, x referring to different quantities in the one equation and x referring to different quantities at different stages of the solution affected the algebraic method chosen.

Other factors beyond student's level of cognition seem to influence the way algebraic notation is interpreted. Some of these are interference of new learning, analogies with other systems (in other parts of mathematics or school), problem presentation, and affective factors.

Interference of New Learning and Other Symbol Systems

MacGregor and Stacey (1997) presented two thousand students aged 11–15 years with a series of pen-and-paper tests. Some schools tested the same cohorts of students over a 13 month period, and some used the test with different year levels. Students' errors fell into the Küchemann categories (letter ignored, numerical value, abbreviated word, and unknown quantity). As students progress through the school system they seem to exhibit a new array of misinterpretations. The results indicated that some errors were due to interference of new learning. The older children showed interpretations not seen in the younger children's responses (e.g., the letter as a label associated with the name of an object, the letter is always 1). Students' responses to interviews supported this conjecture. One commented that "x without a coefficient means one x", supporting the error that x equals 1. The concentration of this belief in a particular school indicated that this was new learning at that particular school. The interference of new learning was also supported by the widespread misuse of exponential notation in the older grades. Some students made incorrect analogies with this symbol system. It seemed that older children have more opportunities to make mistakes because they believed they were expected to use more advanced notation or they were affected by partly-learned new schemas interfering with problem representation.

Problem Presentation

MacGregor and Stacey (1998) investigated whether the problem presentation influenced the solution path chosen. They presented 268 students (ages 14–16) with two types of unequal share problems. The first type consisted of sum of unequal parts, for example, "Mark has $5 more than Jan. Altogether they have $47. How much does each have?" The second type consisted of division of the whole into unequal parts, for example, "$47 shared between 2 people. One gets $5 more than the other. How much does each get?" For the first type, the situation model (the representation of events described in the problem) and the problem model (its formal mathematical structure) are believed to be compatible. In the second problem, the two models are not compatible. Transformation of the description of the problem is required in order to arrive at its formal mathematical structure. It was hypothesised that students would experience more success with the first type of problem as the problem presentation supports the problem model. The results indicated that success was not significantly influenced by the problem presentation. Only students using non-algebraic models were significantly influenced in their choice of method by the problem presentation. The division-into-parts type problem prompted the use of the share-and-adjust method. While the problem presentation seems to influence the cognitive model constructed at the first stages of comprehension, it seems successful students form a cognitive model of the situation that is dynamic and continuously re-assembled and transformed. It was hypothesised that there are three stages to interpretations of problems, namely, the prepositional text base, the cognitive model of the events described, and the formal mathematical relationship model.

The influence of the problem presentation on the cognitive model constructed was also investigated by Lopez-Real (1997). Lopez-Real's study revisited the reversal error in algebra. The purpose of this study was to investigate whether different syntactic structures of problems affected students' facility level with the problems. The sample comprised 221 secondary students whose first language was Chinese. For problems where the syntactic structure suggested a sum model (e.g. p is 6 more than q) many students used a difference model to represent the problem. It was conjectured that the students mentally translated the problem into Chinese before solving it. The translation of the problem suggests a difference model. The results from 84 English students presented with the same problems supported this conjecture. They tended to use a sum model to represent such problems. It seems that the syntax of the problem plays a part in determining the mental model constructed.

Affective Factors

The relationship between the affective domain and problem solving domain was reported in Thomas and Kota (1996), Kota and Thomas (1997, 1998), and Kota (1997). Two tests were administered to 345 students (13–15 years). The first test measured seven affective factors (self-concept, interest, anxiety, self-perceptions, usefulness of mathematics, mathematics intrinsic motivation, and enjoyment of mathematics). The second test measured student's ability to solve algebraic word problems. The tests were administered twice within a period of one year. Initial results, reported in Thomas and Kota (1996) indicated that anxiety and self-concept had a positive effect on the problem-solving ability of the girls, especially younger ones, whereas for the boys the positive effect on their problem solving ability was self-concept, especially for the older ones. When examining the girls' results over the one-year period, it seemed that the relationship between the affective domain and algebraic problem solving changed as they increased in academic level and chronological age. The positive correlations persisted for the low achieving group of girls but reversed for the high achieving group of girls. The results for the boys were opposite. The effects of the affective factors were maintained for the high achieving group but decreased for the low achieving group. That is, for all boys the correlations between affective factors matched their performance but for girls, as they matured, they moved in the opposite direction. The conjectured hypothesis was that, for girls cognition ceases to be a reward by itself when they face a cognitive challenge in mathematics. It was suggested that in order to sustain achievement levels for girls, extra cognitive input, such as affective factors, needed to occur. While these studies showed that achievement and affective factors are linked, they do not prove that one is causal of the other. Thus, in order to test the hypothesis that affective factors rather than performance drives success, further research is needed. Interviews with a sample of fifteen of these students revealed two other factors that were causal to students' performance, namely, student's level of effort and the teacher (Kota & Thomas, 1998). It appeared that the teachers played an important role in improving students' beliefs, attitudes and feelings towards mathematics. This influenced students' ability to work and understand mathematics. It was proposed that understanding is the key to the affective domain, and students look outside themselves for this understanding.

Teaching Algebra

The Teacher

Menzel and Clarke's (1998, 1999) research focused on teachers. They sought to examine the terms teachers employed to describe the content, structure and nature of algebra and to describe teachers' pedagogical knowledge and teachers' personal beliefs about algebra. They studied five teachers over a two year period.

This research documented the diversity of influences and beliefs contributing to the way teachers interpret the nature and purpose of algebra to their students. For these teachers, algebra was important because it (a) could assist students in their VCE studies, (b) is useful as a tool for problem solving, (c) is an important component of cultural education, and (d) develops thinking skills. Their conceptions of algebra differed from each other and some teachers presented algebra as something to do rather than emphasising the central ideas and concepts of algebra. Despite teachers' individual and differing interpretations of algebra content, their beliefs about content specific instruction practices were similar. However, teachers also often lacked the detailed knowledge needed to both identify specific student difficulties and construct situations that might assist students to overcome their difficulties. The consequences that this has for students attending these classes need further exploration.

Teaching with Technology

A *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) recommends that the teaching of algebra should centre on the study of functions and their graphical representation and written representation. This recommendation was based on limited research. Over the last five years a vast amount of research has been conducted on the teaching of algebra. Most of this research has dealt with the use of computer algebra systems and graphics calculators. A full summary of this research is given in the previous chapter on "Technology-Assisted Mathematics Education". A few comments are given here on some of these studies.

A number of studies explored the use of technology in the learning of algebra. Graham and Thomas (1997) researched the use of graphical calculators to provide an environment in which students (aged 12–14 years) began to build an understanding of the variable. Pre- and post-tests were administered with questions on the tests based on Küchemann's (1981) research. The results indicated that, compared to the control groups, four of the five calculator groups significantly improved their understanding of the letter as a specific unknown and a generalised number.

By contrast, Mueller, Pedler, Anderson, and Bloom (1997) found that older students tended to resist the use of graphics calculators when solving linear algebra problems. Students, when allowed to use the graphics calculator in their final exam in a linear algebra university course, appeared not to feel sufficiently comfortable with the graphics calculator in the examination situation. This could simply reflect a limited experience with the graphics calculator during prior mathematical learning at a school level. These findings were supported by Pierce (1999) in a study of 30 undergraduate students. Most students felt that the use of computer algebra systems contributed to their learning but that pen-and-paper experiences were more important.

Tynan and Asp (1998) investigated the impact of a hand-held computer algebra system (CAS) on Year 9 students' performance and patterns of algebraic thinking. The findings indicated that students in the CAS group persisted more with algebraic methods, particularly when the level of difficulty of the manipulation increased. However, they were no more successful in completing such tasks than students from the control group. With regard to the commonly held belief that frequent use of such packages reduces students' by-hand algebra skills, some of the better students believed that using CAS was more tedious than solving the problems by hand and some less able students saw CAS as an additional burden.

CAS was also used with three lowest ability Year 11 mathematics classes in a study by McCrae, Asp, and Kendall (1999). While the CAS students performed at a higher level than the non-CAS calculator classes in Year 11, they did not perform as well as the Year 12 classes. The results also delineated how the teacher affects the types of solution approach favoured by students. One teacher privileged technological and algebraic approaches and the students' preferred approach was to use algebra by CAS when solving problems. Another teacher privileged conceptual understanding and by hand approaches, and this was reflected in the preferred solution processes exhibited by this class on the tests.

Computer aided learning has also been investigated. Hong and Thomas' (1997) research involved seven first year university students who had already completed an introductory calculus course. The aim of the research was to investigate the use of computer based modules to develop conceptual understanding of integration, that is, see the integral as both a process and a concept. The computer is believed to relieve the learner of the need to encapsulate the process before obtaining a sense of the properties of the object. Two tests were administered to 161 students. The tests consisted of both routine and non-routine integration questions. Using the results of a pretest, the seven students were matched with seven other students from the control group. Students in the experimental group were involved in supplementary computer tutorials. The results of the posttest indicated that the experimental group performed significantly better on both the procedural and conceptual questions. Raj and Malone's (1997) research with students in Papua New Guinea using DERIVE does not support these findings (see the chapter on "Technology-Assisted Mathematics Education".

In summary, there seems to be conflicting research with regard to the use of technology in supporting the development of conceptual and procedural knowledge in the algebraic domain. This seems to be influenced by both the teacher involved and prior experience with technology in a mathematical environment.

Student-Teacher Interaction

Nicolls (1998) in a small case study investigated the role that language and social interaction has to play in the construction of students' knowledge about variables. In this limited study, he concluded that a high level of shared meaning between teacher and student resulted in the student achieving a considerable amount of understanding. Further research in this area needs to be carried out before any conclusive statements can be made.

Teaching Materials

The materials used when teaching algebra also need to be critically examined. It seems that misleading materials can cause difficulties for developing understanding of the variable. MacGregor & Stacey (1996b, 1997a) in their studies found that students who were exposed to materials explicitly stating that the letter represented an abbreviated word exhibited the misconception that the letter stood for a label. This error appeared to be fairly resilient. Stacey and MacGregor (1999b) also expressed concern with regard to curriculum increasingly based on arithmetic thinking. They suggested that in many texts there is a predominance of problems that do not really need algebraic thinking for their solution and that equation methods that are based on arithmetic thinking are being retained and promoted in some instances up to Grade 10.

. Curriculum materials based on the use of concrete materials (containers and objects) are being implemented with beginning algebra students across Australia. Atweh, Cooper and Boulton-Lewis (1996), Boulton-Lewis et al (1997), and Cooper and Williams (1997) present findings from their research which is that students do not appear to connect the teacher's material representation to their own mental representation. A gap seemed to exist between the concrete and symbolic representations. This is claimed to be due to the additional cognitive demand the materials bring to the learning. By contrast, Wright (1999) concluded from the results of his case studies involving a variety of teaching approaches, that concrete materials appeared to have considerable merit in assisting students to focus on the arithmetic structure behind the rules which they had developed.

Teaching Approaches

Generalising from patterns and generalising from arithmetic. The difficulties associated with both of these approaches are cited at the beginning of the chapter in the sections Algebra as Generalised Arithmetic and Algebra as Representing Functional Relationships.

Other approaches. While some teaching materials cause misinterpretations, some methods are conjectured to support students' cognitive growth. Mathews (1997) reports on a teaching experiment in which students learned to translate and solve word problems with two unknowns using two variables. The traditional approach involves representing part of the problem with a single variable and then representing all other variables in terms of that variable. The sample comprised 210 students in their first year of solving algebraic word problems. The control group experienced the traditional approach for solving word problems in two unknowns. At the conclusion of the teaching experiment, two written tests were administered to both groups (one with word problems with one unknown and the other with word problems with two unknowns). The data supported the hypothesis that students are more successful in solving word problems if they are taught from the beginning to use two variables when there are two unknowns in a problem.

Issues Related to Research Methodologies

Methodologies for Collecting Data

Generally students' thinking about key concepts in algebra has been initially explored by using written tests (e.g., Quinlan, 1996; Stacey & MacGregor, 1997; and Redden 1996a, 1996b). Warren (1998, 1999) explored the use of written tests for gauging students' understanding of the concept of a variable. The questions used in the written test were based on questions commonly found in past research (e.g., Küchemann, 1982; Harper, 1979; Quinlan, 1992). However, the original questions were reworded and prompts were added. The results indicated that the framing of the question influenced the types of responses given. The use of appropriate prompts helped elicit students' full understanding. Semi-structured interviews were carried out with a sample of students who completed the written tests. These interviews gave greater depth to students' responses and some of the conclusions conjectured from the results of the written test in fact were found to be incorrect. The results of this research indicated that care must be taken when conjecturing about students' understanding from written responses. The context of the question and how it is posed need to be considered when reaching conclusions.

Arnold (1996) presented an alternative methodology based upon Kelly's (1955) personal construct theory. The methodology involved three processes. In the first stage students were presented with a series of ten cards and were simply asked to sort the cards into groups. The cards displayed a range of common algebraic visual prompts, such as, symbols, expressions, equations, graphs, and tables of values. In the second stage students were presented with the same cards sorted into deliberately chosen groups of 3, such as $(x - 1)(x + 1)$, $2x - 1 = x + 7$ and $4 - 3x$. Students were asked to pick the odd one out. The purpose of this stage was to illicit descriptions of the categories according to the student's own language. For example, one student described the categories in terms of numerals, pictures, graphs, equal coordinates and symbols. The third stage involved pairing the student's descriptors on a continuum. Students were then asked to place the ten cards on the paired continuum. From the results, concept network maps were drawn for each student. Computer technology was used to assist the presentation of the groups of cards and the recording of the results. It was believed that the networks represented both the nature and relative strengths of the relationship between the various constructs.

Harris (1997) reported on the beginnings of a two year case study of children developing algebraic understanding. The case study will explore how beginning algebra students construct their initial algebraic understanding of the variable and function. An initial framework for analysing and reporting the results was presented. The framework suggested is the model of a dynamic theory of the growth of mathematical understanding.

Learning Frameworks

Many students experience difficulties in algebra and much of the recent research has investigated these difficulties. Horne (1999) called for the need to have the findings of research used to develop a framework that acknowledges students' thinking rather than outcomes (as most curriculum documents seem to be based on), that is, the integration of research with curriculum, where growth points in development are delineated and acknowledgement is made of separate developments in different areas. This is one of the challenges that we need to face in the next five years of algebra research.

Conclusion

Researchers in Australasia are concerned with many questions of importance in the teaching and learning of algebra, for example, (a) the transition from arithmetic thinking to algebraic thinking; (b) the cognitive development of algebraic understanding; (c) the application of algebraic concepts; and (d) the teaching of algebra (approaches and the role of the teacher). All of the findings have implications for the algebraic learning environment. Curriculum developers, textbook writers and classroom teachers need to take cognisance of many of these findings, particularly with regard to the teaching approaches which support cognitive development and the transition between arithmetic and algebra. While the teacher still assists the development of algebraic understanding, the centrality of this role is changing. The use of technology is becoming more prevalent. Research is beginning to delineate how to use such technology and the benefits it has for developing algebraic understanding. There are however, many questions needing further investigation, particularly in the area of the use of technology, teaching materials, and teaching approaches in assisting the effective transition from arithmetic to algebra and the development of conceptual understanding of algebraic concepts.

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Making Connections with Space and Measurement

Tom Lowrie
Charles Sturt University Wagga
<t.lowrie@csu.edu.au>

Kay Owens
University of Western Sydney
<k.owens@uws.edu.au>

Introduction

In the corresponding chapter of the last four-yearly review Owens, Mitchelmore, Outhred and Pegg (1996) identified a large body of research associated with space, geometry, measurement and visualisation conducted by researchers throughout Australasia. One of the most outstanding features of the chapter, apart from the quality of research undertaken during this period of time, was the diverse range of topics covered. These topics were drawn together diagrammatically as a starting point for discussion of the field by the Special Interest Group at the annual MERGA conference in 1999. The final diagram (Figure 1) presents: (a) a range of content-based mathematics topics under the headings of spatial concepts and measurement concepts; (b) a range of mathematics processes associated with spatial and measurement thinking, and (c) a range of contexts for learning. This diagram can be viewed as an outline (or framework) for the topics to be covered in this chapter. Implicit in the diagram are the relationships between these topics. For example, mathematical processes and contexts for learning impact on conceptual knowledge.

In some studies the mathematical processes have been examined within the *content areas* of space and/or measurement concepts whereas on other occasions these processes have been studied within other content areas, including number concepts.

Other studies have focused on the development of conceptual understandings, presenting three theoretical perspectives. One perspective is a variation of the van Hiele theory (e.g. Pegg, 1998); another is that of abstraction from physical contexts that represent a complex concept such as angle (e.g. Mitchelmore & White, 1998a, 1998b), and the third is a framework for early space based on imagery strategies (e.g., Owens, 1999b).

Assessment has been used as a tool for research for measuring change (e.g., Vincent, 1998), and as part of a learning experience (e.g., Dix, 1998). Assessment tasks have also been used to develop a greater understanding of how students have been thinking through summarising students' results (e.g., Shaw & Outhred, 1999), by examining work samples (e.g., Owens, 1999 c; Owens & Outhred, 1998; Whitland & Pegg, 1999) and by observing processes used by students during the tasks (e.g., Owens, 1996). In addition, theoretical frameworks have also been translated into specific tasks and questions either to examine the framework or to assist teachers in their assessment of students.

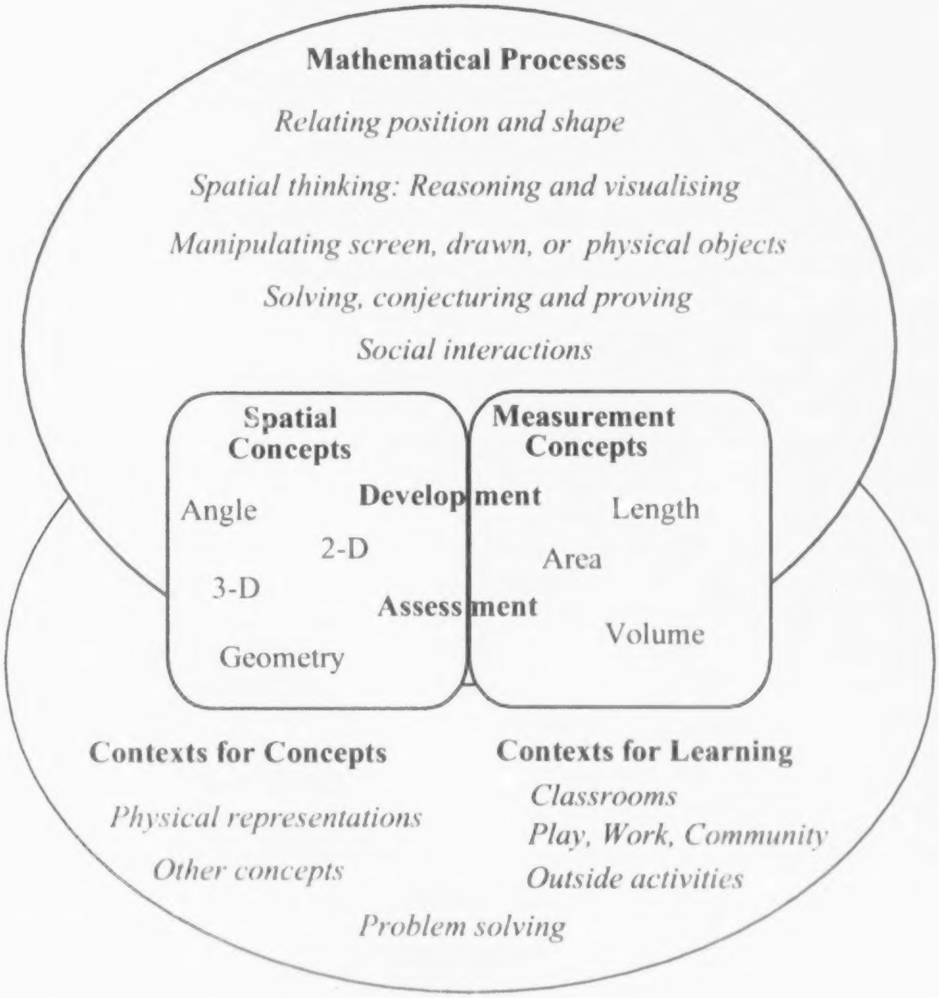
Mathematical Processes
Relating position and shape
Spatial thinking: Reasoning and visualising
Manipulating screen, drawn, or physical objects
Solving, conjecturing and proving
Social interactions
Spatial Concepts
Measurement Concepts
Angle
Development
Length
2-D
Area
3-D
Assessment
Volume
Geometry
Contexts for Concepts
Contexts for Learning
Physical representations
Classrooms
Play, Work, Community
Other concepts
Outside activities
Problem solving

Figure 1. A representation of the topics covered in the chapter.

When discussing spatial concepts the terms *space* and *geometry* are used. While space is often used in connection with primary-school mathematics and geometry with high-school mathematics, these two terms are inextricably linked, with the latter representing the more formal development and application of spatial concepts. As such, mathematical content relating to (a) understandings of two-dimensional (2D) and three-dimensional (3D) shapes and space (including angles); (b) spatial concept development; and (c) understandings of formal geometry including proofs, are considered within the space strands of most national and/or state syllabus documents. The term *measurement* is linked to concepts associated with perimeter, area, and volume. These concepts are considered within the measurement strands of most national and/or state syllabus documents. However, developing both types of concepts depends on spatial thinking.

Mathematical processes associated with visualisation, imagery and problem solving form strong relationships with the content areas of space and measurement in the present review. Some of the reviewed studies have considered the role of imagery in the early stages of the problem-solving process while others have investigated the extent to which diagrams and drawings assist in problem representation and measurement concepts.

Spatial Concepts

Representations of Three-dimensional Shapes

Leeson, Stewart, and Wright (1997) investigated the way in which four young children, of varying abilities, constructed three-dimensional shapes from 2D representations. One of the tasks, for example, required the students to build a replica of a structure after viewing a picture of the object made of five cubes. Another task assessed whether students could match 3D shapes from nets. A capable kindergarten child was more likely to complete tasks effectively when concrete materials were provided. Tasks that required the student to visualise shapes were not completed successfully. As expected, the more advanced Year 1 student used standard geometry terms to describe the tasks. Great variations in young students' spatial and measurement knowledge were also found in a study conducted by Stewart, Leeson, and Wright (1997). Over a four-month period most students' spatial knowledge improved more than their measurement knowledge. Interestingly, there was an apparent link between high-scoring students' numerical ability and their space and measurement ability.

Lowrie (1998b) also looked at the relationship between 2D and 3D environments in a study conducted with the use of computer simulation software. The study investigated the way in which two young children (aged 7 and 8) engage in a computer-based problem-solving activity that required a degree of visual-spatial reasoning. The activity encouraged the children to construct spatial understandings associated with perspective and representation when using computer software that aimed to develop problem-solving skills. The activity required the children to construct a room on screen with the use of a series of furniture icons that could be moved into position in the virtual room. The children found it difficult to link three-dimensional space with images presented in simulation programs. As a result, it was argued that computer-based problems that appear to target visual and spatial understandings can be completed by children without really encouraging them to engage in mathematical thinking. Not withstanding the limitations of a study based on two children, the study found that simulation programs associated with perspective and representation are most effectively used in situations where a teacher is able to interact with the user so that important understandings can be developed.

Everett (1999) described a pilot study involving 30 students in Grades 1, 3 and 5, in which she investigated the students' ability to visualise 3D shapes. In particular, the study examined whether, in their visualisations, students focus on critical or non-critical aspects of 3D shapes and whether any differences exist between students' visual images, verbal descriptions and drawn representations. An interview-based assessment was devised to address the research questions. Results showed that although the quality of some students' visualisations may improve with grade level, non-mathematical or non-critical qualities feature strongly in most children's visualisations. Everett suggested that further research was needed to investigate whether a teaching program, either whole class or individual, could improve children's visualisation of 3D shapes and what specific aspects of these shapes should be the focus of such a program.

Owens (1998 c) also reported on responses by 30 students (aged 9 and 10 years) to a pilot test of 3D spatial thinking. The test involved 2D drawn representations of manipulated 3D visual objects and seeing parts of 3D objects. The results indicated lack of experience in visualising the manipulation (e.g. folding) of 2D shapes to form 3D shapes, in interpreting pictorial representations of 3D shapes, and in imaging patterns such as rows of blocks in 3D objects.

Two Dimensional Spatial Concept Development

Van Hiele theory. A large body of research related to, or associated with, the van Hiele theory has used levels to assess students' mathematical understandings (e.g., Currie & Pegg, 1997; Currie & Pegg, 1998a, 1998b; Pegg, 1997; Vincent, 1998). Researchers have also outlined difficulties in interpreting responses in terms of the van Hiele levels and proposed future research directions for the consideration of the levels (Lawrie, 1998; Pegg, 1998).

Pegg provided the most significant summaries of this research in which the second level of van Hiele's theory has been separated into two parts (Pegg, 1997, 1998; Pegg & Davey, 1998). Pegg (1997) also expanded upon the descriptions offered in Levels 2 and 3 of the theory. Pegg maintained that ideas associated with the SOLO Taxonomy (Biggs & Collis, 1982) allowed for more inclusive criteria to be used to broaden the level descriptions. In the case of Level 2, Pegg suggested that concepts need to have a visual basis and individual answers should have a strong real-word referent for students. With respect to Level 3, students should have an ability to develop an overview of relevant elements so that appropriate generalisations may be formed. Pegg also showed that students have an ability to work with algebraic symbols, that stand for real numbers, at this level. In summary, the suggested levels are:

- *Level 1* Figures are identified according to their overall appearance. Properties play no explicit role in this identification process.

- *Level 2A* Figures are identified in terms of one single property. This property is usually some aspect associated with the length of sides but it need not be this.
- *Level 2B* At this level several properties are known. The important feature of this list is the independent (isolated) nature of the properties to one another.
- *Level 3* Relationship between previously identified properties of a figure are now established. There is an ordering of the properties, i.e., one or more properties are seen to imply other properties. Also at this level relationships between figures are established.
- *Level 4* Deduction is understood and students can develop proofs. In terms of properties at this level, multiple definitions of shapes using necessary and sufficient conditions can be established. (Pegg, 1997, pp. 241–242)

Whitland (Whitland & Pegg, 1999) used this model to categorise the responses of Year 2 students' descriptions of basic 2D shapes. The researchers felt that these students were likely to give responses indicative of Levels 1 and 2. Six students completed a questionnaire and an interview that was designed to explore the students' understanding of squares and oblongs. The case studies revealed an extensive range of responses. All students were able to recognise that a square had four sides but only one student was able to explain or describe any features of the figures presented. No student gave the same name to two squares presented in different orientations with a non-standard orientation being describes as a "diamond" by four of the students. It was argued that differences between global attributes that may build upon Level 1 understandings and features, that are building blocks toward the notion of property at Level 2A, need to be considered if the van Hiele model is to be relevant in describing the development of young children's geometric understandings.

Currie and Pegg (1997) investigated 24 secondary-school students' understanding of class inclusion as it related to different triangle groups. The students were asked to identify and justify relationships among the seven different types of triangles in the class of triangles. The notion of class inclusion was identified as an important Level 3 characteristic reflecting cognitive development in the van Hiele Theory. In the Currie and Pegg (1997) study student's responses were classified into seven types after examining: (a) the nature of the links formed when grouping different triangles; and (b) developmental patterns that existed in the different student responses. The first type of response indicated the use of a single feature or property to relate the properties together, such as acute angles or equal sides. Increasingly sophisticated responses were classified as students considered the extent to which the equilateral triangle was a subset of the isosceles class of triangles. The student responses revealed that there appears to be a developmental path that enables students to gradually acquire the skill of class inclusion. Currie and Pegg

(1998b) provided further details from a small selection of students.

In another study associated with class-inclusion, Currie and Pegg (1998a) presented the same students with activities involving quadrilaterals. The more complex structure of relationships associated with quadrilaterals required additional categories to those that emerged in the Currie and Pegg (1997) study. Both studies showed that there were developmental patterns in the acquisition of class inclusion concepts. The later study also highlighted the difficulty that students' face when attempting to understand relationships within the quadrilateral family. In addition, the study provided a structured framework for teachers to follow when attempting to move students toward more sophisticated understandings of class inclusion with respect to quadrilaterals. Nevertheless, as Ellerton, Clements, and Clarkson in the chapter on "Language Factors in Mathematics Education" clearly point out there is a real danger that the notion of levels has been imposed on the data leading to a loss of valuable knowledge because of a blinkered interrogation and interpretation of the data.

Lawrie (1997; Lawrie & Pegg, 1997) evaluated the extent to which two different coding systems (Gutiérrez, Jaime, & Fortuny, 1991; Mayberry, 1981) provided insights into the geometric understanding of 61 first-year primary-teacher trainees within the van Hiele classification levels. Lawrie (1997) found that the Mayberry coding system provided a more direct and quicker method of determining a student's van Hiele level of understanding of geometry whereas the method developed by Gutiérrez *et al* was a more comprehensive method. The study found that refinements in both methods were needed.

The Mayberry (1981) system was designed specifically to assess the van Hiele levels of preservice students. Lawrie (1996b) used the instrument with sixty first-year primary-teacher trainees revealing that most students had not progressed further than Level 2 for most concepts. In other words, the students were able to recognise concepts and list their properties but did not understand the relationships between the properties. This was despite the fact that almost two thirds of the students had completed a senior secondary mathematics course that included a geometric segment. Lawrie (1996a) commented that the distribution of the Australian results was similar to those presented by Mayberry in the USA (1981). Lawrie noted that the results of this study indicated that students attempting senior geometry courses may not have been at the level towards which instruction in those courses was aimed. As such instruction was not meaningful for the students, their progress through the levels was not facilitated.

Geometry. Lawrie (1998, 1999) developed a written test based on the Mayberry interview items for determining the van Hiele levels. The Rasch partial credit model was used to check whether the data fitted the model and to determine item difficulties (Adams & Khoo, 1993). Lawrie explored the inconsistencies using interviews with little success but found from the Rasch analysis that certain parts of the Mayberry test encouraged different levels of thinking than those intended. For example, item difficulty scores for parts of items written for Level 3 about the circle had item difficulties similar to items written for Level 2. It was for items on the circle that most students performed well. A question associated with recognition of an isosceles triangle was more difficult than other items in Level 1 because it was in fact an equilateral triangle and therefore required, in Lawrie's view, a Level 3 skill.

Lawrie (1999; Lawrie & Pegg, 1999) used van Hiele levels to determine the appropriateness of a set of questions in assessing Year 10 students' understanding in geometry. In selecting the questions, the researchers ensured that each question was phrased so as to probe for a statement indicating the depth of a student's reasoning in geometry. In almost all cases, questions were to have the potential to be answered at more than one van Hiele level. Although the questions were carefully designed, over two thirds of the questions did not provide the researcher with sufficient information about the students' van Hiele levels of understanding. It was concluded that written questions do not necessarily result in the intended levels.

Imagery development. A different approach has been taken by Owens (1999b; Owens, Leberne, & Harrison, 1998) to the early development of Space. Based on Owens' earlier research (e.g., 1996b, 1998) and the body of literature on visual imagery (e.g., Lowrie, 1997; Owens & Cements, 1998; Presmeg, 1986) and spatial abilities (from both a factor analytic approach and an information processing approach), she has developed, with a group of school mathematics consultants. a framework for early Space. The framework has three broad strands: (a) Orientation and Motion; (b) Part-Whole Relationships; and (c) Language and Classification. However, the key to development across these strands is the development of imagery. Initially students begin with Emergent Strategies as they begin to recognise shapes, orientation, and language. Then Perceptual Strategies develop in which physical representations are used to think spatially. Pictorial Imagery Strategies are then used by students who can work with an image when physical representations are not needed for decision-making. Pattern and Dynamic Imagery Strategies are expressed when students associate specific relationships between parts of shapes such as a grid and when students think dynamically by moving images around or by internally changing shapes through extending, pulling, or folding parts. Efficient Strategies tend to develop as language assists recognition of properties of shapes, position relationships, and imagery associated with turning and folding experiences. Efficient Strategies involve reasoning and the beginnings of proof that develops through imagery as well as language. Owens argued that significant developments seem to occur as students move through the different kinds of strategies. Investigative tactics such as turning, flipping and superimposing physical manipulatives, folding, and drawing can assist with learning. Language also intertwines to specify aspects of shape and position that are noticed and used by the learner in a problem-solving situation (1996a, 1996b). Students' responses to the assessment tasks (1999b) have been categorised according to concept and strategy. This framework extends notions of Space mathematics by considering conceptual development and imagery development in analysis of content development.

Contexts for Concepts: The Case of Angles

An abstraction model of development. After initial investigations using six pieces of apparatus which reflected contexts in which students are likely to become aware of angles, Mitchelmore and White (Mitchelmore, 1996; Mitchelmore & White, 1996) began to develop a theoretical perspective in which angle contexts were the beginnings for students to abstract the concept of angles. Several tasks were given to Year 2 students from which it became clear that young children did have an initial contextual understanding of angles and that they were beginning to associate the different contexts into a general angle concept. Mitchelmore (1997) presented students in Grades 2 and 4 with models of physical angle situations and angle models including a bent straw. Children as young as Grade 2 were able to say that a rotation of a small doll from forward to left was equal to that from right to forward but they did not interpret turning as an angle. At best, 76% of Grade 4 and 54% of Grade 2 represented the turning with one line in the final direction, fewer drew a trace of the turn, and only 33% of Grade 4 and 8% of Grade 2 students represented the turn by two radii. The students could not match it with another angle representation, e.g. a tile corner. Interestingly, despite lessons on angles as turning, the Grade 4 students could not give a definition of an angle and in fact 58% said that the doll turning was not related to angles. Mitchelmore extended the study and theory (1998a, Mitchelmore & White, 1996) to ask 36 students from Grade 2, 4, and 6 (both genders from two schools) about the similarities of different turning situations, knob, door, doll and car on road map. Not all students could describe extreme cases of the effect of turns of the stove knob or door models. They gave reasonable alternative examples of each kind of turning but the selecting of abstract models for the knob for making a 60° turn was extremely difficult. By Grade 6, most students had developed a broad turning concept that included at least rotations and hinges like doors as subconcepts. Mitchelmore concluded that teaching about angles with clear lines, drawing these turns, and using static angles with two distinct lines will assist the early development of angles.

Mitchelmore and White (1998b) built on this study and hypothesised a fairly complex array of physical angle situations involving turn, meeting, slope, corner, bend direction, and opening (this link to a region was given more prominence in this study). Each situation was expected to have two distinct areas of manifestation. For example, a corner is a part of a rigid object. It can be represented by planes such as walls or by lines such as the edges of a table top. A bend could be an object or a path such as a deflection of a ball. Mitchelmore and White noted that students frequently experienced each of the angle situations prior to school. For example, students understood it was harder to walk up a steep hill. The researchers argued that these *situated angle concepts* developed into *contextual angle concepts* such as hills and cranes both providing examples of slopes. In the next stage these angle contexts were linked to form an abstract angle concept. In the 1998 study, 36 Year 4 students were presented with pairs of the six physical contexts, and six angle models. Students were asked to select the abstract angle models (e.g. lines on plastic interlocking circles), or straws with pipecleaner joins) that could represent each physical situation. It is interesting that the majority of children in the study claimed not to have done anything like this in school; 4 mentioned angles, 4 mentioned turning, 2 mentioned corners and tessellations, and 28 mentioned right-angles (4 claiming to know about left angles). Although the children came from only two schools (from two classes in each), greater familiarity with angles in other schools is unlikely as the state Syllabus and teachers do not encourage it.

It remains to be seen whether young primary students can learn more about angles and whether teachers can build on their intuitive experiences. Although Mitchelmore and White have provided a complex diversity of contexts and theorised on how development might occur, Owens' (1996a, 1998b) study illustrates how students were learning in such contexts. The teaching focused on the context of angles on shapes and the forefinger and thumb were turned apart to mark the arms of the angle. The larger angle was marked by a larger turn or the overlaying of the angles exposing a larger region. Sticks were also used to represent angles as part of the outline of a shape. Not all students in the classroom were able to notice or disembed the angles but a number of Year 2 and 4 students were able to make these simple links between turning and fixed arm positions on a shape. In fact, all 12 students from a range of abilities in both years who were able to talk with the teacher in groups of three achieved this understanding. Owens pointed out that the learning was part of problem-centred activities in which students needed to be responsive as a result of their cognitive processing. The focus of students' attention was a key to learning about the angles and to distinguishing size of angles from the size of sides or the overall shape. Owens suggested that there was little value in emphasising general levels of development to explain students' learning.

Measurement Conceptual Development

Length

Yelland and Masters (1997) explored young children's understanding of the concept of length in a curriculum unit that incorporated both computer and non-computer based tasks. The Year 3 unit from the *Investigations in Number, Data and Space* Queensland curriculum involved the children in geometric, numeric and problem-solving activities over a period of eight weeks. Basic to the unit was the notion that young children learn most effectively when they actively construct ideas and when they are given the opportunity to engage in tasks that are both meaningful and appropriate to their level of development.

The research, involving 14 children (mean age 7 years 9 months) who worked collaboratively in pairs, revealed that tasks that engage children in actively constructing paths of various lengths were conducive to the development of the concept of length. Children participated in pre- and post-tests that sought to elicit information about their understanding of length. The most salient observations pertained to the children's ability to calculate specific lengths after the computer and non-computer experiences. Before the research only three of the 14 children could calculate the perimeter of three shapes (square, rectangle and triangle) accurately. However in the posttest they were all able to complete this correctly. Moreover, in the pretest the children were asked to complete the missing measures on partially drawn items. Only six children were able to do this based on a visual approximation strategy, that is, they said it was 40 because "it looked like it!" In the posttest the number who were successful rose to 9 but what was more interesting was that they gave the reasons for their answer in terms of accurate calculations of length based on the information provided in the diagram. The same pattern was evident in another test item where the children were required to determine the length of the side of a house in a diagram. The study revealed that young children's understanding of length was enhanced after their participation in the unit and that the choice of paths seemed to be a useful springboard for the understanding of 2D shapes and their properties.

Preservice graduate primary teachers in Singapore were asked to devise a problem for students that would illustrate whether the students understood the concept of perimeter and then they were asked whether a problem had sufficient information to find the perimeter of a drawn rectilinear figure. Results suggested about 80% of the problems suggested by the preservice teachers required procedural rather than conceptual understanding and about a quarter of the preservice teachers seemed to expect all lengths were needed to obtain a perimeter indicating that they themselves lacked an adequate concept of perimeter that could be applied to a problem (Menon, 1997).

Area

A number of studies have looked at the vexed problem of how students develop concepts associated with area. Kidman and Cooper (1996a) provide a succinct summary of theoretical perspectives on area. They divide the studies into those that followed a logical-operational approach and those using information-processing techniques. The former group of students include Piagetian and other approaches which emphasise that students first use a one dimensional approach. However, the conclusions did not hold up under research scrutiny as they were limited by ignoring perceptual judgements. The later studies, post 1975, were influenced by teaching experiments and by a research methodology that assesses students' quantitative judgements and allowed a blend of stimuli to attribute to an observed resultant. For each individual, regression equations provide a diagnosis in simple algebraic terms. This methodology was consolidated into the Information Integration Theory (Anderson & Cuneo, 1978) and was used by Kidman (e.g., 1999) to determine whether students were using an additive or multiplicative rule to estimate the size of areas. Kidman's 1996 pilot study (Kidman & Cooper, 1996b) with three children indicated that certain tasks were suitable for the students to be used in the main study. She also checked the suitability of two methods of analysis. Regression equations were established but insufficient details are given to explain how these were obtained.

Kidman (1997, 1999; Kidman & Cooper, 1997) asked students to estimate on a linear scale the area of rectangles (wooden blocks painted to look like chocolate bars) and parts of rectangles which varied in area and perimeter. She found that a sample of 36 students in Years 4, 6, and 8 used additive judgement rules. Students in Year 8, however, were in transition and beginning to use the normative multiplicative rules. There were intra-subject differences dependent on the particular "chocolate" block, its alignment, and the type of removed ("eaten") section. In the 1999 report, Kidman found that the students used a range of strategies when using additive and multiplicative judgement rules. Those using the additive rule tended to rely on rulers or fingers for measuring length while those using the multiplicative rule predominantly used overlay and positioning strategies.

Sixteen preservice primary teachers were interviewed for their understanding of subject matter knowledge about area, about the nature and discourse of mathematics, and about mathematics in society, and for their disposition towards mathematics. They were interviewed while engaging in eight area tasks (Baturo & Nason, 1996). The tasks were intended to elicit their concrete, computational, and principled conceptual knowledge although the results suggested that they had an intuitive understanding of the fact that fewer larger area units were required to cover an area than smaller units. The reason for this was the lack of principled conceptual understanding or even concrete knowledge of area or the relationship between it and multiplication. It seemed these students had not established the links between arrays and multiplication. Students were also confused about area related to 3D objects, and were unable to link school mathematics to a problem about whether a round pool would fit into a rectangular backyard. Students' computational knowledge was rule dominated, many were not sure about the area of a square with side 2.8 metres, some were unable to tell a reasonable answer, some forgot "which end to use to move the decimal point," and some forgot conventional symbols for area units.

Outhred and Mitchelmore (1996) showed that students in middle and late primary school who were able to draw grids of tiles on rectangles were more likely to be able to work out areas. Outhred identified a number of difficulties for students (e.g. establishing the idea of covering without gaps and overlaps) and suggested a range of developments.

Interestingly, Outhred's results reflected the responses made by students on items requiring students to visualise the tiling of shapes with differently shaped tiles (Owens & Outhred, 1996). Students' spontaneous drawings indicated certain approaches such as drawing tiles in rows, recognising grids and patterns, recognising the lack of adequacy of their own drawing, or less helpful strategies like drawing individual tiles or ignoring gaps or tile size. The 200 students were in Years 2 and 4 in four schools, and attempted the items on three occasions, and three-quarters of them completed ten classroom spatial activities after the first attempt. The shape of the area and the tile affected performance. In general, half the students had difficulty visualising the tiling of shapes, but students who participated in spatial activities were generally more successful in determining the number of tiles that would cover a shape. Students who drew the tilings were more likely to be successful on the trapezia items that involved triangular tiles (Owens & Outhred, 1997, 1998). The tiling items were part of a test of spatial thinking, Thinking About 2D Shapes (1992), and scores on the overall test were very highly correlated with results for the tiling items except that students had difficulty with the number of rectangular tiles needed for a square (Owens & Outhred, 1997, 1998).

McPhail (1998) reported that a Year 1 class were able to establish the idea of a region, to use a pattern to cover a shape, and to use efficient counting strategies to decide how many tiles were needed to cover a shape (both equilateral triangles and square tiles were used). At the end of each lesson, three children were interviewed to establish what they had learnt from the activity. The observing teacher also gave her comments on the value of the lesson and what she had noticed during the lesson. Furthermore, the simple use of a boundary on the edge of the tiles provided opportunities for students to draw grids of the tiles. McPhail noted that early primary school lessons did not need to just focus on establishing the concept of surface or area and that the tiling tasks which focused on pattern overcame the activities degenerating into counting-only tasks (cf. Outhred, 1993) but rather focused on covering areas.

Another study associated with the concept of area was carried out by Ferrer and Vistro-Yu (1999) during a typical fifth grade lesson in a school attached to a university in the Philippines. In groups, the students were given the task of estimating and finding the area of a kidney-shaped figure. They were given grids on acetate, and small tiles—both of two sizes—rulers, scissors, and string. Students tried to use prior knowledge including that of using the circumference like they had for a circle and drawing the polygon that just fitted the kidney-shape. This unusual polygon was then divided up and measured with a ruler to calculate areas of rectangles. All groups tried counting the squares and estimating additional whole squares when using the grids. Interestingly, some students were unaware of how best to use the two different-sized square units. This small study reflected students' approaches in two other quoted studies. The researchers noted that students were able to apply prior knowledge to a novel problem, they knew what units to use, they appreciated the importance of estimation, and they worked well in cooperative groups.

A comprehensive commentary on the performance of NSW Years 3 and 6 primary school students on Basic Skills test items about length, area and volume highlighted the difficulties that students were having across a range of important concepts in measurement (Outhred, 1997). Comments on the items indicated how students might have thought in answering the problems, indicating the difficulties of pen-and-paper tests despite the use of phrases such as *distance around its edge* rather than the use of the term *perimeter*. The importance of comparing one length with another in another orientation was the most difficult item. The importance of estimating, visualising, using appropriate repeatable units, the relevance of real life examples, and the difficulty of scale were noted. Where overseas studies could be related, similar results were attained. In general, student performance was not as high for area and volume questions as for length.

Volume

Outhred's (1997) examination of Basic Skills test results showed that for volume, visualising was more difficult than for length and area. For example, students were more likely to be wrong in counting an irregular stack of blocks than in calculating the number in a rectangular box. The items included consideration of occupied space, interior space, and displaced volume. There is a need to link spatial, numeric, and measurement concepts in such items.

Spatial Thinking

Diversity of Spatial Thinking

In a similar analysis of results from NSW Years 3 and 6 students on Basic Skills items, Owens (1997a) highlighted the diversity of visual imagery skills that students needed in answering the questions. Among these were the skills of transforming images, recognising shapes, analysing plans and recognising parts of shapes, and appreciating alternative perspectives. Some of the plan items were most difficult for students. Imagery seems to be affected by experiences, quality of images, interpreting of diagrams, remembering images, and assisting the understanding of images with verbal representations.

Comparing Outcomes from Different Space and Shape Tasks

Owens (1999c; Owens & Geoghegan, 1998) continued to explore how spatial thinking was correlated with a range of different tasks. Three of the tasks were (a) making a map from home to school, (b) a tangram task, and (c) making a described simple shape in a barrier game. Grades were given for responses to these tasks by considering outcome statements in *Mathematics—A curriculum profile for Australian schools* (Curriculum Corporation, 1994). There were two grades given for each national profile level. Grades correlated with scores on a selection of items from the test Thinking About 2D Shapes (Owens, 1992), while the grades for the tasks themselves did not correlate either due to the small number of students (between 21 and 26) or from the distinct outcomes expected in each task. Only 17 out of 34 students who had completed two of the tasks were classified at the same Australian national profile level (Curriculum Corporation, 1994) for all tasks. These results bring into question the validity of using individual tasks for grading students' spatial development.

Mathematical Processes Used in Space and Measurement

Visualising and Spatial Reasoning

A number of studies have examined the importance of visual representations in mathematical reasoning. Mulligan and Thomas (1998) found that the processing of images plays an important role in the development of young children's understandings of numeration and number operations. Although the children's external representations do not always correspond to conventional mathematics they argued that teaching and learning situations that allow students to describe, draw and discuss their personal *mental images* should be encouraged. The images children used to describe number concepts were often idiosyncratic but helped generate powerful understandings through both static and dynamic representations. Mulligan, Mitchelmore, Outhred and Bobis (1996), for example, developed task-based interviews that encouraged Grade 2 children to represent their solutions by modelling, drawing, and symbolising their visual images.

Visual representations are integral to the interrogation and interpretation of statistical data. Shaw (1997) found that students were more likely to say that the display indicated skewness when the data were skewed to the left than when displayed in a right-skewed orientation, especially if there was little skewness. This happened for both histogram and boxplot representations.

Hannah (1998) compared teachers' preference for using visual methods to solve problems with their use and support of visual methods in their classroom. A questionnaire, based on Mathematics Processing Instrument (Suwarsono, 1982), was posted to teachers at 35 secondary schools. A *mathematics visuality* score was generated for responses from 22 teachers in 17 schools. Four of these teachers, two with high visuality scores and two with low visuality scores, agreed to participate in extensive case studies to determine their use and support of visual methods in the classroom. Observations of the teaching methods employed by the teachers involved in the case studies indicated that the four teachers, despite their own preference for solving problems in a particular manner, encouraged students to draw diagrams to solve problems. Moreover, the four teachers appeared to be quite accepting of different approaches in the way students represented problems. Hannah concluded that the four teacher's personal preference for solving tasks in a particular manner did not influence the extent to which they presented material to the students in their classes. A study by Hai (1999) showed how the use of visual imagery, imagination, and visual reasoning can enhance the cultivation of mathematical creativity. Using a large sample from six Malaysian primary schools and a range of standardised and self-

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developed instruments, the pretest and posttest results of both an experimental group using non-traditional visual methods and a control group were compared. In the experimental group a diagram of geometric shapes with internal lines was shown and students were asked to visualise and draw different arrangements of the shape. Hai tabulated a large number of *t*-tests to suggest that the experimental group performed better on all mathematics tests including geometry and figural and verbal creativity tests whereas the control groups did not show significant gains on any test. Significant differences were not found between the gender groups.

Lowrie (1997) investigated the type of imagery Year 6 students used when attempting mathematics problems over a two-year period. The students used several types of imagery that were identified by Presmeg (1986). Concrete (pictorial) imagery was used by all children. Such imagery was found to be particularly useful in novel or difficult problem-solving situations. Pattern and memory imagery allowed students to develop rich schematic understandings. Such imagery contained propositions, event schema (students recalled "when I did this") and visual recollections that were rolled together to form an holistic type of imagery. These memories allowed students to access relevant knowledge for a variety of problem-solving situations. Dynamic (moving) imagery appeared to be a powerful form of imagery often used for specific tasks. Such imagery encouraged *what-if* thinking when promoted through activities that required students to use their imagination. It was suggested that students should be encouraged to evoke a range of different forms of imagery depending on the type of problem encountered.

Lowrie and Hill (1996) developed a dynamic model that explored the relationship between visual and nonvisual processing in the problem-solving process. The model proposed that four distinctive features: (a) task difficulty, (b) problem representation, (c) metacognitive thought, and (d) the ability to apply knowledge to other situations. These features influenced how students interpret, represent, and go about solving problems. Specifically, the model investigated the way in which problem representation, general-and-specific knowledge, learning preference, personal beliefs, and attitudes toward mathematics affect problem-solving performance. Additionally, they argued that the manner in which a problem solver represented a problem, in either a visual or nonvisual manner, was not only influenced by learning preference but also task difficulty and knowledge in a particular domain.

Conjecturing, Solving, Proving, and the Role of Diagrams and Other Representations

The importance of using diagrams has been investigated by a number of researchers during the last four years. Diezmann's investigations of diagrams (Diezmann, 1997, 1998, 1999a, 1999b; Diezmann & Watters, 1996a, 1996b) extended to the use of diagrammatic *data maps* to visually represent and analyse students' interview performances (1996, 1999b). Data maps were developed to provide a holistic representation of the data and to facilitate visual reasoning. In Diezmann's (1997) study on the effect of instruction, these maps were used to compare an individual's performance before and after instruction by identifying key differences in performance, such as the strategies that were used in the solution process, whether or not the teacher scaffolded the student's problem solving, and success on the task (Diezmann, 1996). Additionally, these maps were employed to compare the performances of various participants and pinpoint similarities and differences in their use of diagrams during problem solving. The data maps were subsequently refined and proved to be a useful tool for fine-grained data analysis to complement traditional text-based analysis (Diezmann, 1998, 1999b).

Frequency and autonomy of diagram use, and success rate were reported as easy to compare in contrast to the quality of the diagrams and the associated reasoning (Diezmann, 1999b). The quality of students' diagrams and reasoning from the diagram were compared by first producing a series of levels of performance in diagram generation and diagram use for each of the general purpose diagrams, and then comparing students pre- and post-instruction levels of performance. These levels of performance were produced from the range of students' performances on the pre- and post-instruction tasks. These levels are described and accompanied by students' work samples and responses for each of the general-purpose diagrams. Levels of diagram quality are particularly helpful for assessing part-whole diagrams, which can have enormous surface variation (1999a, 1999b). Part-whole diagrams are conceptually-oriented diagrams, unlike networks, matrices and hierarchies which are spatially-oriented diagrams. The assessment of conceptually-oriented diagrams versus spatially-oriented diagrams is discussed by Diezmann (1999a) who argues that formative assessment is crucial in improving students' generation of diagrams.

Diezmann (1996, 1997, 1998, 1999a, 1999b) conducted two case studies that evaluated the effect of instruction on students' generations of diagrams while solving novel problems. The instruction consisted of 12 half-hour lessons in diagram use. In these studies, 12 Year-5 students' pre- and post-instruction performances on isomorphic tasks during individual interviews were compared. Diezmann (1997) found that there were quantitative and qualitative changes in the diagrams that were produced by the students. There was an increase in the number of diagrams that were spontaneously drawn and a decrease in the number of students who drew a diagram after prompting. However, not all diagrams that were produced were accurate representations of the problem structure. The qualitative changes that were reported included a more accurate representation of elements of the problem structure—represented as nodes—and the movement between these nodes. The depiction of movement included both the direction and the amount of movement. After instruction all 12 students were able to identify the direction of movement, in comparison to only four students prior to instruction. The number of students who correctly depicted the amount of

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movement increased from one to six. There were, however, some students whose performance appeared to decline on a deductive reasoning task (Diezmann, 1998). It was argued that fewer students were successful on this post-instruction task because they had not used the diagrammatic strategy effectively. Students who were able to use a diagram proficiently were successful.

Diezmann (1999a) repeated the previous studies (1997, 1998) in order to test a theory of instruction in diagram use. The theory consists of a core of content related to diagram use and an instructional model. The six components of the instructional model relate to the teacher, the learner, the instructional tasks, management issues, classroom interaction, and participant structures. The theory was tested by evaluating students' pre- and post-instruction performance on the use of the four general purpose diagrams, namely, networks, matrices, hierarchies and part-whole diagrams. In this study, instruction in diagram use resulted in an overall improvement in (a) the frequency and autonomy of diagram use, (b) the quality of diagrams that were generated, (c) the appropriateness of the reasoning with the diagrams, and (d) the success rates for tasks in which a diagram had been used (Diezmann, 1999b).

Individual students' abilities to use diagrams effectively may be moderated by their idiosyncratic profiles of spatial intelligence and the development of spatial intelligence within the classroom (Diezmann, 1999b; Diezmann & Watters, 1996a, 1996b). Diezmann and Watters (1996a, 1996b) reported that even a gifted student with a strength in spatial intelligence may not spontaneously use diagrams and may have difficulty generating diagrams that would facilitate problem solving. They argued that the teacher assumes a key role in the development of spatial ability. The teacher can support the student through a cognitive apprenticeship which includes the provision of feedback that challenges inappropriate choices of strategy and reinforces confidence in the choice of a spatial strategy. Arguments for the key role of the teacher are reminiscent of Lowrie's (1998) study and a number of studies under the section on the contexts for learning.

Outhred and Shaw's (1999) study focused on the extent to which students in introductory statistics courses use diagrams when solving problems. They found that students who drew a diagram as part of the solution process were far more successful than students who did not draw diagrams. A total of 781 students, across two universities, were given an assessment task comprising three statistical problems and one more general mathematics problem. A small sample of the students were interviewed (Shaw & Outhred, 1999) and asked why they had not drawn diagrams to solve the tasks. Interestingly, there was a strong indication that the students were not sure if drawing a diagram was actually worth the effort it took to draw it. Outhred and Shaw commented that greater emphasis should be given to integrating the use of diagrams into the teaching of statistics courses. Moreover, they argued that open-ended tasks that required students to interpret the same data from different perspectives may also encourage diagram use.

Diagrams in trigonometry are an important teaching tool. Kendal and Stacey (1996) showed that with a range of schools and teachers, the ratio method was more effective than the unit method for trigonometry on triangles. Some of the difficulties arose from the visual difficulties such as rotating triangles for use with the unit circle. They concluded that while the unit circle was a valuable way of introducing trigonometric ratios, it was better to introduce the mnenomics for the right-angled triangle and allow students to use this method for this type of question.

Lowrie (1996a, 1996b) investigated the impact a visual-spatial learning program had on the problem-solving strategies of primary school children (aged 11–13). The training program, taught for 20 weeks, consisted of a number of learning activities intended to provide students with the opportunity to experience and pose open-ended problems in a range of key learning areas including mathematics, science, technology and visual arts. The experiences were used as a focus to challenge and extend the children's existing concepts, images and skills. By emphasising the use of visual imagery, it was assumed that students would be exposed to alternate methods to solving problems, and would value its use as a means of problem solving. The results suggest that the training program improved the students' capacity to solve spatial problems, although students did not use an increased number of nonvisual approaches when solving similar problems despite being exposed to a training program that promoted visual thinking. It is suggested that visual problem-solving methods are utilised best in novel or complex learning situations. Nonvisual problem-solving approaches were usually the most efficient in instances where students could quickly identify and access the relevant knowledge required to complete a specific task. It was found that students will often use visual processing in the initial stages of problem solving, and then move toward more analytic, nonvisual strategies when relevant patterns have been identified, or when a more informed understanding of the problem has been established. Lowrie (1998b) traced the methods three talented Year 6 students used to solve routine and non-routine problems over the course of a school year. One of the students tended to solve problems in a visual manner; the second preferred a more verbal/nonvisual approach; and the third tended to use both visual and nonvisual strategies. The students worked individually and cooperatively on a variety of mathematics problems including activities on the computer.

In a study of nine kindergarten children's drawings of word problems, Outhred and Sardelich (1998) found that physical representations and drawings were indicative of the ability of young students to solve quite difficult word problems. Over the two-month period, drawings developed from having no structure to showing their solution strategies (Outhred & Sardelich, 1997). Using theoretical perspectives about representation, the researchers suggested that drawn representations give children a means of recording their thinking and explaining the solution process. Drawings assist students to focus their attention on similarities between two structures preparing them for the introduction of number sentences. When students wrote down number sentences to represent the problem, the strength of their knowledge was evident in their ability to explain these understandings in different ways.

Owens (1999a) also found that students used physical materials to develop ideas, check ideas, and create aesthetically pleasing paper sculptures. In this study 14 architecture students from Papua New Guinea (PNG) recalled how they carried out a design project. A complex range of cognitive and affective processes was illustrated by their comments including the influence of their PNG heritage to strengthen their use of mathematics and design, and the influence of school on their mathematical ideas. In particular, ideas about symmetry, halving, equal lengths, shapes and balance were used. It was, however, the need to solve the problem and create a design that stimulated a wide diversity of thinking processes.

This complexity of thinking and its influence on responsiveness in problem solving had emerged in earlier studies by Owens (e.g., 1996a, 1996b). A comprehensive report of the effects of selective attention, visual imagery, and heuristics (such as monitoring progress) was published in 1998 (Owens & Clements, 1998). This grounded-theory study involved the sampling of students in different situations in order to develop a story-line regarding the students' responsiveness that resulted from their thinking and in turn influenced the context of learning. The study also gave details of students' visual and other problem-solving processes. Selective attention, that is noticing aspects of physical and mental representations, was discussed as an important aspect in the development of concepts such as angles.

Developing and interpreting schema for geometrical concepts is suggested as critical for geometric problem solving (Chinnappan, 1998). Chinnappan (1998, 1999) has investigated why students with similar numbers of schema in fact achieve different success rates. In his (1998) study, Chinnappan compared the number of different types of schema activated by 15 high achievers and 15 low achievers, and found a difference only in the number of relevant-to-solution schema rather than in the overall number of schema used. A fine-grained analysis of two students' work indicated that the high achieving student used schematic connections in later attempts at a difficult problem The number of links between schema, for example area and number, was considered significant (Chinnappan, 1999). An initial investigation of a proposed model with a novice teacher indicated that a lack of connection between concepts will also reduce the effectiveness of a teacher of geometry (Chinnappan, Nason, & Lawson, 1996).

Interestingly, Nunokawa (1997a, b) illustrated how paper folding and drawings reflected the process for solving problems about a pentagon and a tetrahedron. The use of conceptual knowledge, relationships between ideas, new links between ideas and trial ideas were evident in the drawings. Without the drawings certain conjectures would not have been made in the process of solving the problems.

Contexts for Learning

Student Achievement and Evaluation Studies

During the period 1996–1999, the Third International Mathematics and Science Study (TIMMS) was conducted. Participating countries selected representative samples of students at certain grades and presented them with items. Complimenting the paper-and-pencil restricted answer test, a smaller number of students were given extended tasks. Considerable effort was made to ensure these items were fair across countries and translations. Surveys were also made of mathematics and science teaching, teachers, lesson time, curriculum content, and textbooks. The analysis and presentation of the TIMSS results for Australia was carried out at the Australian Council for Educational Research (Lokan, Ford, & Greenwood, 1996, 1997). These studies indicated that 10% more Australian primary students (upper grade was 74%, lower grade was 65%) achieved correct answers to the geometry questions than the international average (upper grade was 64%, lower grade was 56%). Females did marginally, but not significantly, better than males in both grades. On no item did Australian students do more poorly than the international average. There were some state differences with percentages for correct answers ranging from 69% to 81%. The question on compass points showed a marked improvement between lower primary and upper primary. Only 40% of students in lower primary were able to give the correct number of edges of a drawn cube suggesting that students did it visually rather than by knowing. Although space items were relatively well done compared to other items in middle primary school, they were a relative weakness at lower secondary level. Interestingly, a fairly low 59% of items were considered covered by the curriculum in lower primary. Most of the items that were not covered were in geometry as it was not emphasised adequately in curriculum documents (Lokan, et al., 1997 p. 136). Lower secondary school results showed that students were able to visualise a rotated 3D object without too much difficult. However, items on an unexpected 2D rotation, angles, angles and parallel lines, and congruent triangles were not well done.

The summary of teaching as represented by student perspectives indicated a traditional teaching style at both primary and lower secondary levels. It was common for teachers to explain how to do problems, have students copy notes from the board, and have students work from worksheets or textbooks on their own. These results did not differentiate the space strand. Correlations of achievement with different teaching practices such as giving homework, using small groups, using computers tended to be negative if practiced in most lessons, particularly in the lower grade of primary school. However, occasional use in lower secondary indicated high links between these practices and student achievement (Lokan et al., 1997, 1998).

Internationally there was a long tail of low percentages in geometry (Lokan et al., 1997). The Philippines did not participate in the study, but an investigation of 6th grade students in some relatively well-supplied schools across the country concluded that the geometry questions were relatively well done (Mercado, 1999). The percentages correct for each item were generally at least 10% above the upper 25th international percentile except for three harder items. By contrast, items in other strands were lower than the international averages.

Social Interaction in the Learning of Spatial and Measurement Mathematics

Two early childhood researchers, Macmillan and Rogers, have continued to consider the social context of learning in spatial mathematics. Macmillan (1998) considered the activities of preschool children during play sessions and coded a range of comments in terms of Bishop's (1988) activities that entailed mathematical thinking. Macmillan presented an extensive post-structuralist and theoretical perspective for her study. She coded events that occurred while the children were imagining the events surrounding the fire truck. Macmillan not only referred to possible motivational discourses but also to comments that indicated preschool students' use of mathematical language and understandings when involved in counting, measuring, locating, designing, explaining, and playing activities. For example, students referred to "a really big" or "bigger than ever house". They referred to positions such as "go back in," "backwards" and they questioned asking for explanations "How do you get out?" Macmillan also developed a theoretical model showing that responsive and restrictive socio-regulative interactions could motivate interpersonal motivations in the mathematics learning situation. In Owens' (1996) study on angles, responsiveness in the learning situation was also seen as important.

Rogers (1999) carefully analysed video footage taken over a six week period in an early childhood centre in a large rural NSW town catering for 85 children aged three to five year. Actions, communication between children or with adults, and self-talk during block play were analysed. The themes of (a) block play extending children's construction of knowledge (b) communication and reasoning, and (c) social interaction of children were illustrated by tables of commonly used words, notes regarding non-verbal indicators, and transcripts. Much of the data was similar to the findings of Macmillan. The tables and transcripts indicated that early spatial and measurement understandings included (a) position language of degree, e.g., halfway, near; (b) shape and line names and classification characteristics e.g. "like a window", (c) enjoyment at seeing and making spatial patterns, and (d) turns and corners. In measurement, children's actions illustrated informal measures and finding midpoints, and comments were on (a) strength and balance, size, comparisons decided by sight or direct matching (e.g., not equal, twice/half as big); (b) patterns of area; and (c) time and speed recognition.

Roger's (1998) report included a diagrammatic representation of the interactions between problem solving, reasoning and communication, and mathematical processes. These processes were (a) visualisation in estimation, preconceived planning, projection and refinement; (b) experimentation in concrete problem solving, and in creating balanced, symmetrical, and aesthetically pleasing structures, and (c) application to new structures, purpose, product-real and product-imagined. These interactions were illustrated by a vignette of the discussion between children on the building of a structure resembling a Greek temple. Not only was interaction between children important but adults' modelling, acceptance, positive responses, and questioning helped students to feel confident, to cooperate, and to express mathematical ideas and purposes. Rogers concluded by claiming that block play, building and cleaning up activities prepare children for school mathematics. Further she says block play encourages children to learn through sharing of knowledge and skills with peers, experimenting, practising new discoveries or techniques, and applying what they had learned to different situations. Rogers also noted that, once the girls began to participate in block play, there was no gender differences in the themes for play given above, nor was there a difference in whether blocks were used in imaginative play or other play.

Technology and Interactions

Studies by Yelland (1995, 1996) and Yelland and Masters (1997, 1998) have examined the collaborations of pairs of young children as they worked on tasks that were technologically based. Specifically, they have explored young children's spontaneous and scaffolded problem solving in a number of spatial and measurement tasks. Results have indicated that encouraging young children to work collaboratively, scaffolding their learning via modelling, probing questions, and discussion facilitated problem solving and the more frequent use of higher order thinking skills (Yelland, 1996; Yelland and Masters, 1998). The studies have highlighted the importance of distinguishing between cognitive, affective, and technical scaffolding. When the young children were supported in this way the nature of their collaborations and interactions were characterised by more conflict resolution which in turn lead to more effective use of metastrategic processes (Davidson & Sternberg, 1985).

Extensive comments on a number of studies (e.g., Brown, 1997; Dix, 1998; Engebretsen, 1997; & Vincent, 1998) using dynamic geometry software can be found in the chapter on "Technology-Assisted Mathematics Education".

Social Class Influences

Language related to the context of learning was a specific issue raised by Zevenbergen (1997). She used a geometry example to explain that social class influenced the flow of discussion in a classroom. This is a salient reminder that theoretical perspectives on learning geometry which emphasise language aspects (see, for example, Owens, 1999b), and classroom approaches that encourage whole class discussion should also take account of the context of learning. It has, however, been noted by Owens (1999b) that classification and language development may not necessarily progress at the same rate as other areas such as the recognition of shapes in other orientations or parts of shapes.

Alternative Lesson Venues

The extensions of geometry into trigonometry and coordinate geometry are considerable. One small study by a teacher in Indonesia using outdoor examples for applying trigonometry, indicated that this was a profitable pursuit for providing context for learning. However, the researcher noted that neither students nor teachers wished to pursue such approaches and could not see that mathematics was indeed relevant to the outside world. Nevertheless, students were motivated during the activities (Pambudi, 1999). Another quasi-experimental study (non-matching control and experimental groups) showed that activity-based plane geometry lessons and traditional lessons both improved students learning but that the former were more effective (Ragual, 1999).

Home and Work Context

Considerable discussion has occurred about the use of traditional mathematics in schools. Roberts (1997) was able to describe the mathematical skills used by a Maningrida family in preparing and making didgeridoos, selecting directions, recognising tell-tale signs of animal presence, discussing shape and colour details of fishing lures, and recognising differences in appearance of the ground and river related to climate and seasons. Roberts cautioned that details of Indigenous mathematics should be discussed with the traditional owners before it is used in schools.

An historical study and inquiry of Kankana-Ey farmers and workers in the Philippines by Rapanut (1999) indicated that not only counting but measures were traditionally determined to assist in the farming, trading, timekeeping, and building processes. Upex (1999) detailed how geography and mathematics can be integrated. From the analysis, a full range of topics were shown to be in both geography and mathematics textbooks for the same primary grade of Brunei Darussalam. In particular, an example of the links with measurement in a river study was given. Geography has become a multidisciplinary study involving quantities to assist analysis and understanding, and it is therefore a valuable context for teaching mathematics.

Zevenbergen (1998) showed the value of estimation skills for box-and-framers when positioning and bending concrete reinforcing rods. From her study of workplace mathematics, Zevenbergen developed the concept of *saturation* of mathematics in a context. The box-and-framers were considered to be involved in a medium saturation situation unlike nurses needing to calculate drug doses that involved ratios. While motivation was seen as important in encouraging accuracy in both cases, it seemed that in neither case was there a full cognisance of the mathematics being used.

Studies on the Uses of Mathematics and Implications for Teaching

A number of studies have considered the uses of Space mathematics and its implications for teaching. Bolt (1999) studied the uses of dynamic geometry in a range of toys and other common moveable objects. For example, the use of the parallelogram in the swing with a platform seat and two pairs of uprights can be modelled to illustrate that the platform seat always stays parallel to the ground although a person follows a circular locus. Owens (1997b, 1998a) showed a range of uses of mathematics in architecture and other designs. Quigley (1999) has provided a cohesive list of factors important in Islamic repeated patterns and illustrated how symmetries and matrices are used to generate the various patterns. White provided a series of papers on tessellations appropriate for the school classroom (1998, 1999).

An interesting examination of mathematics, especially in the geometric area, that is suitable for primary and secondary teachers was undertaken by Hodgson (1997). He presented ideas that allow preservice teachers and their subsequent students insights that are visually accessible. He noted the value of linking geometric and algebraic representations (e.g., in star polygons), Pythagoras' theorem, or rectangular areas. He also considered the kaleidescope and self-similarity.

A number of papers explored the generation of different designs and 2D representations of 3D objects using computers (e.g., Majewski, 1998). Generating such intricate designs can be a fascination to students and a source for learning mathematical relationships. Majewski discussed two particular programs that can be used to generate webpages that can be accessed by students.

By contrast, physical 3D models have been used in Japanese television to enthral viewers with mathematics and to change negative attitudes to mathematics as found in the TIMMS study (Akiyama, Hirano, & Sakai, 1999). The study produced models that provide many relationships between numerical and spatial mathematics. Models illustrated relationships in area and volume including area under a curve, Pythagoras' theorem, and volume without gradations. The models can be used by students or used in demonstrations to explore relationships.

Conclusion

During the writing of this review, Figure 1 became a summary or framework for this area of research. One of the most important features of Figure 1 is the interplay between content and process with respect to research conducted in space and measurement. Over the past four years there have been many studies that have investigated the way in which students (from preschool to tertiary) have utilised mathematical processes when engaged in activities associated with space and/or measurement. These processes have included visualising, (spatial) reasoning, manipulating, solving, conjecturing, proving, and representing. It seems that mathematics education researchers have used space and measurement concepts as a catalyst for moving into investigations associated with a broad range of mathematical thinking processes. It would be fair to say that there has been as much work conducted in the areas of imagery and spatial reasoning as there has been in the content areas of space and measurement during the past four years.

With respect to content areas, researchers have focused on 2D and 3D shapes, angles, geometry, and area. Moreover, a considerable amount of work conducted in these content areas has involved work with the van Hiele levels of development. However, the research on angles and area would suggest that students in the early years of primary school are beginning to develop concepts that can be stimulated by appropriate materials, and that there are alternative views on development. From a teaching and learning perspective, space and measurement concepts that were thought to be difficult for young children are being developed by them through teachers using a range of personal contexts.

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Early Arithmetical Learning and Teaching

Alistair McIntosh
University of Tasmania
<alistair.mcintosh@utas.edu.au>

Shelley Dole
University of Tasmania
<shelley.dole@utas.edu.au>

Introduction

The review of research in Early Arithmetical Learning and Teaching in the previous survey of Mathematics Education Research in Australasia (Wright, Mulligan, Stewart, & Bobis, 1996) reported that there had been "a significant amount of Australasian research in early arithmetical learning and teaching" in the previous four years, citing a total of 70 references. That this research has continued in the four years 1996–1999 is shown by the list of 75 references at the end of the current chapter.

While the research cited here ranges over a variety of topics, one might list the following topics as having generated particular interest on the part of several researchers: (a) children's early number understandings and their relationship to computational ability; (b) children's computational strategies, particularly in mental computation; (c) assessment processes at all levels; (d) decimal and common fraction concepts; (e) fresh approaches to the consideration of word problems; and (f) a continued body of work on some intervention strategies.

The studies discussed in this chapter have been grouped for clarity and continuity into sections: whole numbers, decimals, fractions, percentage, mental computation, number sense, and representation and visualisation. Many alternative classifications are possible and common themes run through several studies located in different sections, and, inevitably, differing research methodologies are intermingled within each section.

Whole Number

The studies considered in this section cover early number, early development of operations and understanding of operations, children with learning difficulties, aspects of word problems, and assessment.

Count Me In Project

Much of the published work in early number continues to arise from the Count Me In project and its extension as the Count Me In Too project in New South Wales. The influence of the project has now spread beyond New South Wales to other states and territories.

Mulligan, Bobis, and Francis (1999) provided an overview of the Count Me In Too (CMIT) project with an emphasis on its role in professional development of teachers. Wright (1998) provided a more detailed overview, particularly of the

Learning Framework in Number on which the meticulous research-based assessment procedures are based. The work is now developing from early counting knowledge and numeration to the beginnings of computational understanding. There are also several accounts of classroom work, observations, and assessments during the earlier phases of the project.

Wright (1996) described a year-long teaching experiment which involved a research team working with a kindergarten class and its teacher, and in which clear gains were documented by the assessment processes. The results of these assessment processes over a much wider group (866 Kindergarten children) are described by Stewart, Wright and Gould (1998). Bobis and Gould (1999) described assessment instruments given to two Year 1 classes, one in the CMIT project and one control group, showing clear gains for the CMIT class. Wright, Stanger, Cowper, and Dyson (1996) described detailed work with a small group of Year 1 teachers and selected Mathematics Recovery children, drawn from the lower 30% of the age cohort, and providing four case studies of individual children.

Finally Bobis and Gould (1998) reported the findings of a (1996) evaluation of the impact of the Count Me In project, in particular its impact on the professional development and attitudes of teachers.

Base Ten System, Place Value, and Base 10 Blocks

Thomas (1996) and Thomas and Mulligan (1999) described a model of levels of understanding of the base ten system, based on task-based interviews with 132 rural NSW children from grades K–6. The study revealed limited understanding on the part of many children, and also raised queries about the value and effect of the use of base-10 blocks (from the Multi-base Arithmetic Blocks [MAB]) as a model.

Price (1998) interviewed 16 Year 3 students individually about two- and three-digit numeration concepts, before and after 10 groupwork sessions. While improvement in understanding was shown after the sessions, Price concluded that the use of place-value teaching aids needs careful planning and students' understanding of multi-digit number needs to be closely monitored. We cannot assume that children are making sense of number representations in the same way that teachers do.

Jones et al. (1996) described the validation and extension of a previous framework developed by some of the authors to assess and describe children's thinking in multidigit number situations. The framework defines five levels (from Level 1: Pre-place value, to Level 5: Essential place value) in terms of four constructs (counting, partitioning, grouping, and number relationships) which are seen as central to the development of multidigit number sense. The framework was then used to guide the planning, implementation, and evaluation of mathematical instruction in multidigit number sense with two first-grade classes over a full year.

Higgins (1999) focused on 10 teachers taking part in a teacher development program for Year 3 mathematics teaching. The teachers were asked to articulate their content knowledge and pedagogical content knowledge of number, with a particular emphasis on place value. Analysis of responses drew on the framework developed by Jones (Jones et al., 1996) and described above. This knowledge was compared and contrasted with mathematics activities that the teachers used in classroom practice. Higgins conjectured that what teachers say about their practice in number is shaped by their immediate classroom context, so that any data can be seen only as a snapshot in time, rather than complete statements of knowledge.

Calculators and Early Number Work

Groves and Stacey (1998) described the Melbourne-based Calculators in Primary Mathematics project in which, commencing with kindergarten and first grade in 1990 and following the children through to fourth grade in 1993, over 1000 children and 80 teachers in six Melbourne schools explored the use of calculators in mathematics. Activities were developed which involved calculator use for topics including counting and skip counting, place value, computation and exploration of the calculator itself.

An extensive program of testing and interviews was conducted, including comparison of the performance of third- and fourth- grade children in classes in the project with a control group consisting of the last cohort of third- and fourth-grade children at the same schools which had not been part of the project. There was no evidence that children became reliant on calculators at the expense of their ability to use other methods of computation. The project children performed better than children without calculator experience on a wide range of items including mental computation, negative numbers, place value and, most markedly, decimals. No detrimental effects of calculator use were found in either the interviews or the written tests.

Stacey and Groves (1996) provided more details and examples of calculator-based activities and test items associated with the Calculators in Primary Mathematics project. Curriculum issues raised by the project include whether children have traditionally been too restricted in the range of numbers met in the early grades, what it means to understand number, and the apparent conflict between the abstract nature of the calculator and ideas of the primacy of concrete experiences for young children.

Early Number Understandings Related to Addition and Subtraction

Irwin (1996a) described three carefully constructed sets of experiments with 107 children aged from 4 to 7 years, in which their *protoquantitative* understanding of numerical relationships was tested; for example, children placed a small (counted or uncounted) number of objects in the interviewer's hand, and these were then held in the interviewer's two fists; the child then moved a button from one fist to the other. Children were asked about the effect on the total. The results suggested that children may come to school with understandings that are not utilised for up to two years of their schooling.

Peters (1997) conducted individual interviews with 128 seven-year-olds to explore their understanding of place value and how this related to the strategies that they used to solve written addition problems. Less than a quarter of the children in the study could show that the tens digit in a two-digit number was a multiple of ten. Most of the children were being taught the conventional algorithms, including the carrying procedures, but teachers may have overestimated the extent to which the children in their classes fully understood place value.

Early Number Understandings Related to Multiplication and Division

Pepper and Hunting (1998) investigated the relationship between the acquisition of counting and sharing skills by preschool children. Twenty-five preschool children aged from 4 years 10 months to 6 years 2 months participated in two interviews, one consisting of a set of counting tasks, the other a set of sharing tasks. For each of the sharing tasks, no significant relationship was found between counting and sharing competence. Children used systematic *dealing* procedures, involving no apparent use of counting skills. Children classified as good sharers exhibited differing degrees of counting competence. The authors concluded that teachers should be able to involve young children in problems of sharing even if those children have not become competent with the processes of rational counting.

Mulligan (1998) described long-term research she has undertaken on children's development of multiplication and division; this has led to a framework, incorporating six developmental levels together with assessment tasks, designed to help teachers in assessing and improving children's number knowledge.

Children with Learning Difficulties

Bashash and Outhred (1996) investigated the ability of children with moderate intellectual disabilities to compare relative numbers and the strategies they used. Ten children, aged 12 to 18 years, with moderate intellectual disabilities were given a sequence of tasks to assess next number determination, understanding of the $n + 1 > n$ rule, and number comparison skills. The results suggested that the children were more capable of developing rule-based number skills than previous research indicated; the form of the assessment tasks was seen as crucial. The authors concluded that children with moderate intellectual disabilities should benefit from an emphasis on cognitive techniques of instruction rather than on rote learning.

Pearn (1998) described a collaborative mathematics intervention program involving one primary school, which aimed to identify, then assist, children in Grade 1 *at risk* of not coping with the mathematics curriculum. Details are given of the assessment processes (designed only to assess counting levels) and a brief account is given of subsequent teaching strategies.

Wilson and Robinson (1997) described an experiment in which parent volunteer tutors were used to improve the multiplication fact knowledge of 43 primary aged students with learning difficulties. Two specific approaches were used: (a) the *Count-By* method (often called *skip counting*), and (b) the *Time Delay* method, in which the teacher gives the correct answer to a computation, at first immediately, and then with a specified time delay. The experimental group received seven weeks of structured practice from their parent in learning the multiplication facts for which they lacked mastery, involving ten minutes per day, five days per week. A control group was given tables to learn at home over the same time interval with the help of their parents. The results show that any form of parent involvement is beneficial, with the Time Delay method proving somewhat more successful and easy to implement.

Aspects of Word Problems

English (1996) explored the roles of number sense and problem-solving competence in children's computational problem posing. In this study seven classes of Grade 5 children (aged 10 years) were administered two written, whole class tests, one of number sense, one of non-standard problem solving. From these 51 children were chosen showing two different profiles: (a) strong in number but weak in problem solving (28 students) and (b) strong in both number and problem solving (23 students). These children were then presented individually with two types of tasks: to create an interesting story related to a computation (e.g., 29 + 36); and questions to probe their problem-solving preferences. The children showed little diversity in their interpretation of the symbolic computations. Many expressed a preference for computational problems that were quick-and-easy to solve. In her 1998 study (English, 1998a), English extended her study to third-grade children, including a problem-posing program between pre- and post-tests. The children again revealed a limited range of problems posed for both formal and informal contexts.

In two studies (English 1998b, 1999) English explored the role of reasoning by analogy in children's solving of word problems. She noted that reasoning by analogy in problem solving involves mapping the relational structure of a previously-solved problem onto a new problem. Both studies referred to work with a group of 57 children in fifth grade. While emphasising the importance of reasoning by analogy in many aspects of mathematical activities, English concluded that children reason by analogy in their everyday problem solving, but require guidance in applying such reasoning to more formal problems.

Maguire (1996) analysed 282 whole number word problems generated by one class teacher in Victoria who aimed to provide for his students an understanding of class/age appropriate arithmetic concepts underlying the four operations. Maguire found that far from providing a wide range, the teacher provided quite a limited range of problem types with little relevance to children's out-of-school activities. For example, very few comparison problems were included for subtraction.

Decimals

Research into decimals has contributed to the field of mathematics education on at least three fronts: (a) the development of decimal understanding and misunderstanding; (b) the nature of the decimal-number numeration system; and (c) pedagogy.

The Development of Decimal Understanding and Misunderstanding

Moloney and Stacey (1997) reported on two studies, a longitudinal study of Year 7 and Year 9 students and a cross-sectional study of Years 4–10 students, in which students' development of decimal understanding was explored. In the longitudinal study, Year 7 and Year 9 students completed a diagnostic pen-and-paper decimals test mid-year, and then a second time 12 months later when the students were in Year 8 and Year 10 respectively. Students' incorrect responses to items on the test were classified as either deriving from application of three common erroneous rules: whole-number rules (WNR), fraction rules (FR) or zero rules (ZR), or were unclassified (U). Correct responses were classified as deriving from application expert rules (ER). Results indicated little change in students' understanding of decimals over the period of the longitudinal study. Results also suggested that students' misconceptions about decimals are a significant problem, and that over 50% of Year 10 students, and more in lower grades, could not compare decimals correctly.

Following from results of the longitudinal study, the aim of the cross-sectional study was to explore the development of students' thinking across the years of school when decimals are taught. In this study, Moloney and Stacey redesigned their test instrument to enable more refined classification of students' responses. The test was administered to 379 students in Years 4–10. Results suggested that whole number misconceptions were prominent in the early years, but they tended to disappear over time; that the fraction rule misconceptions persisted, and were retained by approximately 20% of Year 10 students; that the zero rule was uncommon; and that most movement between categories was evident in Years 4–7. Moloney and Stacey suggested the value of their diagnostic test for classroom teachers was that it was relatively simple to administer, and yielded valuable information on students' understanding of such things as the relationship between place-value position, decimal to fraction conversions, equivalent fractions, identification of decimals on a number line, naming decimals between two decimal, and stating the value of certain digits in decimal representations.

Following from the finding that only 75% of Year 10 students were found to be able to compare decimals with 80% of accuracy (Moloney & Stacey, 1997), Stacey and Steinle (1998) tested 1853 students in Years 4–10 using a 25 item pen-and-paper decimal pair comparisons test. In this study, Year 5 and Year 6 students were selected for interview to illuminate how students think about decimals. Through analysis of test scores and interview data, ten patterns of thinking about decimals emerged within the broad categories identified earlier as whole number rule (WNR), fraction rule (FR), zero rule (ZR) and expert rule (ER) users. Interview data revealed that some students, categorised as ER users, were actually applying rules to enable expert performance (e.g., annexing zeros); a finding similar to that of Baturo (1997) described later in this section. As a result, Stacey and Steinle modified their classification of ER users from expert to task expert.

Steinle and Stacey (1998) presented results from a similar but larger study where 2517 students in Years 5–10 were tested on decimal comparison. In this paper, Steinle and Stacey elaborated on the 10 categories of children's thinking about decimals that emerged in an earlier study (Stacey & Steinle, 1998) to provide descriptions about students' thinking that gives rise to the misconceptions behind the errors. Steinle and Stacey reported some interesting trends in the data from the large sample: (a) there is a decrease in the number of students that choose longer decimal numbers in a given pair as larger from Years 5–10; (b) the number of students who select the shorter decimal in a given pair as larger remains constant; and (c) expertise in decimal comparison reaches a plateau at about 60% in Year 10. Through further analysis of results, Steinle and Stacey provided evidence that certain misconceptions appear to arise as a result of school instruction, and that test results are influenced by time of testing and recency of school instruction. In this paper, Steinle and Stacey described a new test of 30 items designed to match expected responses for each identified category, and redistribute students' responses according to this new scheme, and report on the value of the new test for this purpose.

Stacey and Steinle (1999) reported on a longitudinal study of 64 students over three years exploring decimal comparison ability. From initial test data, students were categorised as belonging to one of the following four categories: apparent experts (A) who gave correct responses to test items, longer-is-larger (L) students who chose the longer decimal of a pair, shorter-is-larger (S) students who chose the shorter decimal of a pair, or unclassified (U) students whose results did not match either of the other categories. Results indicated that in the long term, there is a general trend towards expertise, but that 20% of students with misconceptions remain in that category. Implications for teaching are given.

Baturo (1998) reported on research into decimal-number numeration knowledge of 112 Year 5 students where interaction between available and accessible place-value and regrouping knowledge was explored. Baturo stated that available knowledge can be directly probed, and accessible knowledge becomes evident from situations where it is applied. In this study, students' available knowledge was determined through diagnostic items and accessible knowledge was determined through error analysis of place-value and regrouping knowledge applied to addition and subtraction situations. Results of this study indicate that students' performance was better on place-value than regrouping items and that there appeared to be a strong direct relationship between available and accessible place-value knowledge. There was evidence also of students using over-practised rules (e.g., insert a right-most zero) and impoverished prerequisite whole number and fraction concepts and processes in some classes and "that correct individual performance is possible without understanding and that availability of knowledge does not mean it will be used (accessed)" (p. 96). Implications for teaching and curriculum were presented. In particular, Baturo raised the issue of the place of pen-and-paper computation in schools, and suggests that the place for such computations is as a vehicle for promoting structural principles of mathematics (e.g., set inclusion, commutative and distributive laws) that apply across whole number, fraction and algebra domains.

The Nature of the Decimal-Number Numeration System

Through analysis of the decimal number system, Baturo (1997) presented a model of cognitions and connections within the decimal number system. According to Baturo, there are three levels of knowledge that enable functioning within the decimal number system, to varying degrees, but that Level 3 knowledge enables expert functioning within the system. In Baturo's model, broadly speaking, Level 1 is baseline knowledge and Level 2 is connection knowledge. At Level 3, structural knowledge comprises additive structures, multiplicative structures and reunitisation, and is the "superstructure for integrating all levels of knowledge" (p. 88). Baturo reported on a study of Year 6 students' understanding of the multiplicative structure decimal-number numeration system. In this study, 173 students completed a pen-and-paper decimal-number numeration (tenths and hundredths only) test, with 45 students selected for individual interview. From analysis of test results, five categories of students emerged: three categories of students who operated at high, medium and low proficiency on tenths and hundredths, and two categories of students who operated at high and medium proficiency on tenths but low proficiency on hundredths. As a result of the interview data, Baturo concluded that multiplicative structure was a determining factor in differentiating high performance; that only students in the high proficiency with tenths and hundredths category could process tenths and hundredths with understanding; and that lack of multiplicative structure will inhibit processing with tenths and hundredths. This research has implications for teaching particularly when students are introduced to the next decimal place of thousandths.

Decimal Pedagogy

Baturo's (1997) in-depth analysis of the decimal-number numeration system provides a pictures of its complexity, as does research into students' understanding and misunderstandings of decimals outlined previously in this section. Taking this issue to the classroom level, Irwin (1996b) reported on a study into the development of 11 and 12 year-old students' decimal number understanding through their engagement in pair discussion. In this study, eight pairs of students of similar mathematics ability (as ranked by their classroom teacher) worked together on contextualised (real-world) and decontextualised (numerical) problems over a week. Students were pre- and post-tested, and a retention test was administered one month later. Results indicate significant differences on the pretest and retention test, with greatest improvement made by students who had worked on contextualised problems. Analysis of interactions that occurred between student pairs revealed differences in interaction, thinking and discussion between students according to ability ranking. Irwin notes that errors made by students were not random and that contextualised situations enabled students to draw on their experiential knowledge to assist with problem interpretation and understanding. As a result of this study, Irwin concluded that to understand decimal numbers, students need to reconstruct their ideas of whole numbers to include decimal fractions. Detailed descriptions of the decimal tasks used in this study are provided.

Irwin provided further analysis of students' understanding of decimal number knowledge in a subsequent paper (Irwin, 1997). As previously stated, the students in this study were presented with contextualised and decontextualised decimal situations designed particularly to cause some degree of cognitive conflict or *perturbation*. Using a Piagetian framework for classifying *reactions to perturbations*, Irwin analysed and classified students' interactions as either alpha, beta or gamma reactions, with beta reactions identified as those that contribute to greatest knowledge development. From this analysis, Irwin noted that the decimal situations led to high incidence of beta reactions, but that this occurred mainly in students who had greater initial decimal knowledge (higher ability pairs). This study suggests the value of this teaching approach for the classroom, and the benefit of allowing students to work in pairs.

Fractions

This section begins with a brief summary of a review of research into the development of rational number knowledge. Following this, an outline of research into fractions is presented. Such research includes young children's notions of fairness and fair shares, fostering young children's fractions development in the classroom, the use of analogs in fraction teaching, using computer programs to promote fraction understanding, students' understanding of the connectedness of common and decimal fractions, and number sense associated with common and decimal fractions.

Pitkethly and Hunting (1996) presented a comprehensive review of research into rational number development. In their review, they identified five sub-constructs associated with initial fraction concepts (part-whole, quotient, ratio, operator and measure) and outlined the foundation of rational number knowledge, addressing such things as fraction language, intuitive fraction knowledge, constructive mechanisms, whole number schemes, partitioning schemes, measuring schemes, and so on. They also described propelling and inhibiting mechanisms, outlining how whole number schemes can serve both to inhibit or advance rational number development. Provided in this paper is a summary of specific studies by various researchers into initial fraction concepts. The authors suggested two interpretations of findings; initial fraction concepts emerge either from application of intuitive mechanisms or from ideas of ratio and proportion. In summary of this review, the authors stated that it is possible to build experiences for young children which reflect individual sub-constructs, but they posed the question: "How can experiences be structured which reflect the complex of systems?" (p. 30). The authors concluded that further research needs to be conducted into how both the part-whole and ratio sub-constructs relate to the growth of rational number understanding.

Watson, Campbell, and Collis (1996) explored young children's (pre-Grade 1 to Grade 4) notions of fairness when sharing fractional amounts. Using real pancakes, the children in this study were asked to divide one pancake and to share it equally between three dolls. Classification of students' responses occurred in relation to the SOLO development model which proposes five modes of functioning (sensori-motor, iconic, concrete-symbolic, formal, post-formal) and three cycles of responses (unistructural, multistructural, relational) within each level. Analysis of students' responses revealed gradual sophistication by grade level with evidence of students' responses classifiable within the SOLO framework as unistructural, multistructural and relational at the iconic and concrete-symbolic modes. Watson et al. provide descriptions of such responses for each classification, thus providing educators with information about the developmental nature of fairness in continuous fraction division.

Hunting (1996) described results from several studies into young children's conceptions of division, sharing and fractions, thus providing a picture of intuitive knowledge and skills young children bring to the formal school setting. Hunting outlined some classroom activities for young children, aimed to foster the development of fraction conceptual knowledge through drawing on children's intuitive knowledge. Hunting also provided suggestions for teaching to achieve the long-term goal for fraction knowledge whereby children develop the "capacity to conceive of fractions as objects embodying numerical relationships that transcend particular instances and contexts" (p. 123). Hunting called for more dynamic and action-based environments in mathematics teaching, where students are provided with opportunities to engage in problems, tasks, and explorations to promote rich conceptual understanding.

Charles and Nason (1999) reported on young children's (Grade 3) strategies for solving partitioning problems (serving pizza, pies, licorice straps and so on to varying numbers of people) using various analogs (circular, rectangular, and length models). Charles and Nason report that 12 partitioning strategies emerged, noting that 6 have been previously reported in the literature, with 6 previously unreported. They also report that as students progressed through the activities, they gradually applied more efficient processes and made efforts to minimise cognitive load. The authors noted that for some students, their *efficient* strategies assisted in task completion, but such strategies did not assist students "abstract the particular quotient fraction construct" (p. 149). In light of this finding, they gave implications for instruction including the need for teachers to be aware of the development of students' efficient task-completion strategies and to ensure that sufficient structured experiential teaching occurs to facilitate the development of the partitive fraction construct.

Following exploration of analogs for partitive fraction understanding, Charles, Nason, and Cooper (1999) present an analysis of a range of analogs commonly used in teaching fractions. Seven different analogs were investigated—four circular regional models (pancakes, pies, pizzas, pikelets), two rectangular region models (ice-cream bars, cake bars), and one length model (licorice straps). Year 3 students' reactions to the analogs in various problem situations were analysed according to ecological validity, abstraction ability, and ease of partitioning. The positive and negative features of each analog for fraction understanding are described. Of this study, the authors conclude, "if structured mathematical analogs such as those used in this study are to be educationally effective in the teaching of the partitive quotient fraction construct, then teachers need to address the factors which limited their effectiveness" (p. 9).

Pearn (1996) reported on a study of Grade 3 children's thinking processes when solving fractional tasks (partitioning tasks, fraction tasks and a ratio task). Through this study, it became evident that students were utilising two types of strategies: (a) using known whole number facts, or (b) solving tasks through physical manipulation of available materials. Pearn reported that the fractions tasks caused difficulty because students did not understand the meaning of most fractional terms beyond ½, and lacked general conceptual fraction knowledge. Descriptions of the tasks and students' responses are provided in some detail in her chapter.

Commonly in the fraction literature, it is stated that children's whole number knowledge interferes with fraction understanding. In response to this, Hunting, Davis, and Pearn (1996) undertook a two-year teaching experiment with eight and nine year-old children to explore fraction learning and the role whole number knowledge might play in it. In this study, students interacted with a computer program (CopyCat) that provided fraction experiences for the children to experiment. The researchers provided descriptions of the teaching activities over the two years, the teaching tasks, and the CopyCat program. Focusing particularly on responses from two children working as a pair throughout the experiment, Hunting et al. provided details of the development of fraction knowledge, and the "deepening understanding of rational numbers as they engaged in problems of fraction comparison and equivalence" (p. 372). These two students were seen to draw upon their whole number knowledge to assist with fraction knowledge development. As stated by the authors, "This study underscores the importance of children having a firm foundation of whole number strategies and relationships established in early years of schooling" (p. 375). This study reveals the value of tapping into whole number knowledge through appropriate tasks to build fraction knowledge is shown.

Fraction knowledge development for two other students with lesser whole number knowledge working as a pair using the CopyCat program is reported by Hunting, Davis, and Pearn (1997). In this paper, the students' responses as they worked with CopyCat are documented, revealing how the students' limited whole number knowledge frequently prevented them from successfully operating in the fraction domain. As the teaching experiment progressed it was evident that the students were developing their fraction knowledge as well as their knowledge of whole numbers and number relationships. As stated by the authors, "Development of fraction knowledge in this setting was stimulating the creation of whole number knowledge which in turn enabled success with the fraction tasks" (p. 246). This research underscores not only the crucial nature of sound whole number knowledge for developing further rational number knowledge, but also the benefit of cooperative task learning experiences.

A study to explore Year 6 students' understanding of the connections between common and decimal fractions was conducted by Hunting, Oppenheimer, Pearn, and Nugent (1998). Drawing on prior research which suggested that students generally lack understanding of the relationship and equivalent nature of decimal and common fractions, Hunting et al. conducted individual interviews with 49 Year 6 students where students were presented with six tasks relating to fractions and decimals. Students' responses to the six tasks are summarised in the paper, and generally show a "range of different relational connections upon which students' knowledge of fractions and decimals are based" (p. 277). The researchers found that students most commonly used a referent of 100 to explain the connections between fractions and decimals. The researchers pointed out the value of providing students with opportunities to reflect and explain their thinking processes, but also noted that some students knew more than they could articulate, as was evident in this study. In this study, the interviews with students also enabled misconceptions to be identified, that is, students who could see nothing amiss in their incorrect explanations to particular tasks.

Bana, Farrell, and McIntosh (1997) reported on 8, 10, 12, and 14 year-old students' misconceptions associated with fractions and decimals through analysis of data on students' number sense collected in Australia, United Stated, Sweden and Taiwan. In this paper, the multi-choice items specifically relating to fractions and decimals are presented, together with percentage scores for each choice. The authors reported that basic misconceptions were prevalent, and these persisted even amongst 14 year-olds. As a result of this study, the authors raised the issue of requiring students to perform operations with fractions and decimals as set in school mathematics curricula when clearly many students have limited and limiting conceptual understanding of these topics.

Percent

An exploration of middle school students' (Years 8, 9 and 10) knowledge of percent was conducted by Dole, Cooper, Baturo, and Conoplia (1997). In this study, 90 students across the three Year levels completed a pen-and-paper test with items relating to (a) understanding of percent terms in the real-world (e.g., profit, loss, discount, interest), (b) conversions between fractions, decimals and percents, (c) calculation of percent exercises commonly found in mathematics textbooks, and (d) percent problem-solving tasks in real contexts, including those involving calculations of percent increases and discounts. From scores on percent calculations and problem solving items of the test, 18 students were selected for individual interview on the basis of being classified as proficient, semi-proficient or non-proficient percent problem solvers (two students from each category per year level) to probe their understanding of their solution processes. Interview data revealed a range of strategies were employed by all students, but predominantly the non-proficient students used a percent formula procedure, and would not attempt to solve any problems where the formula could not be applied. Proficient students showed: (a) strong skills in mental computation and operational relationships; (b) knowledge of conversions between percent, common and decimal fractions; (c) used benchmarking, approximation and estimation; and (d) showed some ability to analyse the problems structurally. Proficient students had a well-developed *sense of percent* that enabled them to operate successfully in this domain. However, from the initial testing of the total pool of students, only a few could be categorised and selected as proficient, suggesting that percent is a poorly understood topic by the majority of students in the middle school.

Following from research suggesting that percent is a difficult topic to teach and learn, Dole (1999) reported on a teaching experiment with an intact class of mixed ability Year 8 students where instruction in percent focused on a proportion approach to solving percent problems. In this study, students were presented with instruction on the use of a dual-scale number line to assist with interpretation, representation and solution of percent problem situations. Using this method, the students became proficient in solving percent situations, with reported class mean scores on pre-, post- and delayed post-tests (two months later) of 12%, 77% and 76% respectively. Dole also reported on evidence of the growth of proportional knowledge as a result of students using this method.

Mental computation

Computational Choices

Swan and Bana (1998, 1999) described the development of an instrument to explore the relationships between students' mental, written and calculator strategies of computation. The 1998 study describes the theoretical framework and some preliminary work carried out with middle to upper primary students in Western Australia. In the 1999 study Swan and Bana described use of the instrument to explore the computational choices made by 25 students in Years 5 to 7. The data was collected in individual interviews. While only a preliminary study, the results suggest a number of weaknesses in students' current ability to choose sensibly and access a range of computational choices.

Computational Strategies of Adults

Northcote and McIntosh (1999) investigated the extent to which adults use mental, written, calculator, and other computational strategies in their everyday lives. Using an adapted form of Wandt and Brown's (1957) study, volunteers kept records of the mathematical calculations they used over a period of up to one day. They found that 85% of all calculations involved mental computation, while only 11% involved an element of written computation and 7% involved calculator use. Estimates as opposed to exact answers were needed in 60% of all computations. Rosinski (1998) in a small survey of adults found that none recalled ever having been taught mental computation strategies, and most used adapted forms of written computation.

Children's Mental Methods

Almost all the recent research on children's mental computation methods has centred on addition and subtraction. Heirdsfield and Cooper (1997) report on three studies of mental computation in addition and subtraction by children in Years 2 to 4. The first, longitudinal, study charted the strategy use of children as they progressed from Year 2 to Year 4, through clinical interviews at six points. Significantly strategy use moved from counting (Year 2) to left-to-right strategies (Year 3) to right-to-left strategies (Year 4), indicating the effects on mental computation of teaching that emphasises written computation. All three studies indicate the value of flexible computational strategies. Cooper, Heirdsfield and Irons (1996) and Heirdsfield and Cooper (1996) reported on the same study, with useful discussion of teaching implications in the Cooper, Heirdsfield and Irons paper.

Heirdsfield (1998) reports on the *mental architecture* of two Year 3 children, one *flexible* and one *inflexible*. While the two children shared a high degree of accuracy in mental computation, the paper indicates a number of significant differences between the two. This interesting study raises some important questions about the importance of flexibility. The inflexible computer was inferior in speed and in estimation, but was otherwise equally competent. It would be fascinating to revisit these two children two or three years later.

Heirdsfield (1999) tracked two other children's mental strategies over a five-year period (Years 2–6). Although both were members of very traditional classrooms, both children developed flexibility in accessing a wide variety of strategies. While Heirdsfield concluded that children should be encouraged to invent their own computational procedures, she asserted that the answer does not lie in focusing teaching on mental strategies.

Buzeika (1999) and Heirdsfield, Cooper, and Irons (1999) discussed the question of the relationship between children's development of mental and written computation. Heirdsfield, Cooper, and Irons (1999) charted the development of one child's strategies for multiplication over six interviews spanning almost three years (Years 4–6). They concluded that the boy's mental strategies were as efficient and useful at the start of Year 4 as they were at the end of Year 6, but that his responses indicted that the teaching of pen-and-paper algorithms was having an effect on his choice of mental strategies.

Buzeika (1999) explored teachers' responses when implementing an approach that encouraged children to develop and use invented algorithms. The children developed methods that mainly used a left-to-right approach. While acknowledging the children's success and their ability to explain and justify the methods used, the teachers faced a number of challenges, including previously held beliefs, perceived curriculum requirements and social expectations regarding the place of standard algorithms for computation. She concluded that change in the New Zealand setting will depend on the focus coming from the wider educational community and being backed up with broader support for teachers and schools.

Mental Computation Abilities of Children

McIntosh (1996) surveyed three studies investigating the mental computation abilities and number sense of Western Australian school students. In the first study, clinical interviews were conducted with 12 children (four each selected by their class teachers as less able, average, and more able) in each of Years 2 to 7, to elicit their mental computation ability and range of strategies used. It was found that the more competent students used more sophisticated strategies earlier and made more active use of place value strategies. The less competent relied longer on elementary counting and use of fingers.

The second study was designed to provide three perspectives on the mental computation of students in Grades 3, 5, 7, and 9: first, a survey of the kinds of computations they prefer to do mentally; second, a measure of their attitudes to mental and written computation; and third, an assessment of their performance on mental computation items. A full account of this study was given by McIntosh, Bana, and Farrell (1995). The comparative study of mental computation in three countries (Australia, Japan, and the United States) of which this study formed a part, is described in McIntosh, Nohda, Reys, and Reys' (1995) report.

The third study used the same population of students; its aims were, first, to investigate the possibility of assessing number sense by group administered written tests; and second, to gain some information regarding the number sense of students at Grades 3, 5, 7, and 9. Group tests were based on the number sense framework described in McIntosh, Reys, and Reys' (1992) paper. The comparative study of number sense in four countries (Australia, the United States, Sweden and Taiwan) of which this study formed a part, is described in McIntosh, Reys, Reys, Bana, and Farrell's (1995) report.

Teaching Mental Computation

McIntosh (1998) described classroom practices designed to develop children's mental computation strategies and abilities. The ideas, developed from action research carried out by teachers in their classrooms following the first study, are described in McIntosh's (1996) chapter. The ideas are developed fully in McIntosh, De Nardi, and Swan's (1994) book.

Number Sense

McIntosh, Bana, and Farrell (1997) and Reys et al. (1999) described aspects of the comparative study of number sense in four countries (Australia, the United States, Sweden and Taiwan) mentioned above. McIntosh, Bana, and Farrell (1997) described the structure of the tests and provide sample items, while Reys et al (1999) provided examples of comparative data from the four countries (comprehensive details of all items and results are given in McIntosh, Reys, Reys, Bana, & Farrell, 1995). In addition to revealing deficiencies in understanding, notably in fractions and decimals, the research supported the view that there is a field called number sense and that can be investigated and assessed through group-administered written tests. However it confirmed the view that, while trends or interesting fields of inquiry can be indicated by written tests, individual oral interviews are needed to probe the understandings of individual students.

Kaminski (1997) investigated aspects of the number sense of six primary preservice teacher education students at the commencement of a semester unit in mathematics education. Eight mathematical exercises (e.g., "Of which three numbers is 1948 a sum?") were explored in individual interview sessions. Students did, to some degree, use number sense in decomposing and recomposing numbers, and linked numeration and operations in meaningful ways. However Kaminski noted that the students lacked understanding of multiple relationships in the number and operations domains. Most found mental computation difficult, and all resorted to standard written formats over mental computation when calculating. Non-standard procedures were rarely used unaided.

Representation and Visualisation

The role of visualisation and the many forms of representation used in mathematics have been a focus of increased research interest in recent years, both because of their value as research tools in tracking children's thinking and understandings, and because of a growing realisation of the role they play in children's mathematical development; a role which research is showing to be far more complex and more broad than previously realised.

Bobis (1996) described an instructional intervention research project specifically aimed at studying the effect of visualisation strategies on young children's developing number sense. Two kindergarten classes and their teachers participated in the year-long study. The children were encouraged to use visualisation strategies to mentally combine and separate numbers represented by arrangements of dots. It was found that activities which focus on the visualisation of groups of dots rather than on counting one-by-one help develop part-whole relationships, especially the decomposition of ten.

Boulton-Lewis (1998) summarised research, from an information processing perspective, of children's interpretation and use of strategies and representations for place value, subtraction and addition in the first three years of school. More detail of the place value study is reported in Boulton-Lewis (1996). Representations were defined broadly to include concrete embodiments of numbers (MAB, matchsticks or paddlepop sticks singly and in bundles of ten, counters and a metre ruler), symbols for numbers and operations, and combinations of symbols in number sentences. Drawings were not included. Conclusions included (a) that practice in exchanging unit materials for sets of ten is an important activity for children in understanding place value, and (b) if children are to use structured concrete representations such as MAB effectively, without increasing their processing load, they must know the materials so well that they can use them automatically.

Boulton-Lewis, Wilss, and Mutch (1996) described similar research in subtraction with 65 children in Years 4–6. Each student was interviewed twice, once early in the year and again about 20 weeks later. It was noted that students preferred to adopt mental strategies despite, or perhaps because of, teaching practices. It is suggested that if algorithms were treated less in the early years, with greater use of mental strategies, with and without concrete representations, then algorithms could be explored later with greater understanding. This was seen to be a potentially enjoyable way of confirming mental and other procedures.

Mulligan, Mitchelmore, Outhred, and Bobis (1996) and Mulligan, Mitchelmore, Outhred and Russell (1997) reported on the developmental and first phases respectively of a two-year longitudinal study of 120 Grade 2 children which aimed to describe the growth of numerical concepts and processes and children's representations of these. The aim was to interview each child four times over an 18 month period, focusing on seven categories of tasks: numbers in context, counting, ratio, grouping/addition, regrouping, estimation and multiplicative. The 1997 report concludes that sufficient evidence was provided to show that children's imagery influenced the way they visualised and solved a range of tasks. It was found that low achievers were likely to produce poorly organised, pictorial and iconic representations that were lacking in structure; while high achievers used dynamic imagery presented solutions in well structured (but often unconventional) ways.

Mulligan and Mitchelmore (1997) observed 60 female students four times in Grades 2 and 3 as they solved the same set of 24 multiplication and division word problems. From the correct responses 12 distinct calculation strategies were identified and grouped into categories from which the children's intuitive models of multiplication and division were inferred. It was found that the students used four main intuitive models: direct counting, repeated addition, multiplicative operation and (in division problems only) repeated subtraction. Mulligan and Mitchelmore (1996) used a set of problems based on those in the 1997 study in a teaching experiment with ten Grade 3 girls over 10 two-hour teaching sessions. The teaching sessions involved the children in pairs in solving and representing the multiplication or division problems. A range of concrete materials and pencil/paper were made available. The researcher encouraged the modelling of the problems using materials or pictures. The paper describes the range of pictorial and iconic representations produced by the children. The study showed that young children are able to represent their solution strategies to a range of multiplication and division situations without prior instruction and with minimal guidance. The authors concluded that providing appropriate experiences to encourage mathematical representations, and adequate support for the acquisition of more sophisticated and wide-ranging calculations and problem-solving strategies is necessary. They warned that imposing adult representations of multiplication and division may prevent children from creating and using meaningful representations.

Outhred (1996) described the results of a study which investigated the representations and strategies used by 115 children from Grades 1–4 as they solved a sequence of multiplication tasks including finding the area of a rectangle, and three word problems. Children were encouraged to draw solutions to the tasks and the methods used were monitored. The study showed that children's own representations of cartesian product and rectangular area problems show why the concepts underlying multiplication are hard to grasp. Outhred also notes that having students explain their drawings to the class or a smaller group may help elucidate the concepts for other pupils.

Assessment

There has been a considerable increase in interest in the assessment of children's mathematical knowledge at, or soon after, school entry. Young-Loveridge (1997) described the research and thinking which led to her development of a procedure for assessing numeracy at school entry. Her research over more than a decade has led to the development of a classroom assessment device (Checkout/Rapua) in which various numeracy tasks are embedded within a shopping game. The assessment tool is now used widely in New Zealand schools.

Reports of national testing programs can be somewhat dull affairs, of little interest outside the particular country involved. It is particularly refreshing therefore to come across one that is as illuminating as it is clear and helpful in its layout. The report of the New Zealand Mathematics Assessment Results 1997 (Flockton & Crooks, 1997) reports fully on the processes involved, individual questions with protocols and results at Years 4 and 8. The results for whole number, especially *table facts* are illuminating. However, the assessment covered all areas of the mathematics curriculum.

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Issues in Mathematics Education at the Tertiary Level

Undergraduate Mathematics Teaching and Learning

Glenda Anthony
Massey University
<g.j.anthony@massey.ac.nz>

Ruth Hubbard
Queensland University of Technology
<r.hubbard@fsc.qut.edu.au>

Philip Swedosh
University of Melbourne
<swedosh@maths.mu.oz.au>

Introduction

For a long time the teaching and learning of undergraduate mathematics has been a poor cousin within the domain of mathematics education research. However, this picture is steadily changing in response to changes that are currently influencing teaching and learning at university. These include (a) the increase in the number of students who are now attending tertiary institutions; (b) major pedagogical and curriculum changes that have taken place at pre-university level; (c) increasing differences between secondary and tertiary mathematics education regarding purposes, goals, teaching approaches and methods; (d) the rapid development of technology; and (e) demands on universities to be publicly accountable (ICMI, 1997).

The more cynical could suggest that declining enrolment numbers of mathematics majors and threats to academic positions have caused mathematicians to look inwards, to reflect on their position, asking "Are we part of the problem? Does our teaching need to change? Are our courses relevant to the needs of current students?" While there is no doubt that such pressures on the viability of mathematics teaching within the university have caused many to ponder these questions (e.g., Hendy, 1997), there is also a groundswell of mathematicians and mathematics educators, and increasingly an intersection between the two, who have been working to improve our knowledge base about effective mathematics teaching and learning at the undergraduate level.

The increasing interest in undergraduate mathematics teaching and learning is most visibly evidenced in the burgeoning conference activity at the international level. The most significant development was undoubtedly the 1998 International Commission on Mathematical Instruction (ICMI) study conference on the Teaching and Learning of Mathematics at University Level convened to "raise important issues related to the study of the teaching and learning of mathematics at university level and to stimulate discussion and research on these topics" (ICMI, 1997, p. 3). Another international conference on mathematics teaching was held in 1998 in Samos, Greece. Within Australasia, there has also been conference activity. In July 1997, a workshop was held in Auckland entitled Education in Mathematics for Undergraduates (EMU), at which delegates from several countries discussed issues fundamental to the teaching and learning of undergraduate mathematics. Later that year the first symposium on Modern Undergraduate Mathematics: "What Can We Do to Improve Learning", was held in Brisbane with numerous speakers and delegates from around the globe. Also, the New Zealand and Australian Mathematical Society meetings increasingly include mathematics education sessions within their annual conference meetings. In addition, there have been biannual conferences on Bridging Mathematics.

The research reviewed in this chapter reflects the two dominant issues central to these recent conferences: (a) the continuing dissatisfaction with student achievement in higher mathematics in general, and in calculus in particular (Harel & Trgalova, 1996), and (b) the inequitable participation by females and other socioeconomic and cultural groupings combined with the overall drift away from the highest intensive mathematics and physical science courses (Forgasz & Leder, 1998a). First, the following discussion focuses on issues related to student participation in undergraduate mathematics courses, including the effects of policy changes, changing student profiles, and perceptions of mathematics and mathematics learning and teaching. Second, the chapter reviews research studies that focus on the nature of mathematics learning at the tertiary level and the related implications for teaching and course design.

Participation and Perceptions

Policy Issues

In Australia there is increasing concern about the declining numbers and calibre of students choosing to major in mathematics (Forgasz & Swedosh, 1997). Government policy has impacted heavily on tertiary education in general and mathematics education in particular (Thomas, 1997, 1998). In recent years university mathematics departments have suffered cutbacks in staffing and resources, and in some cases this has meant that course options for students have been reduced (Ministry of Research Science & Technology, 1998; Swedosh, 1997). A move by non-mathematics departments to teach their own mathematics courses or reduce the mathematics content of their programs has resulted in a further weakening of mathematics departments. Thomas (1998) reports that in Australia, mathematics departments lost about 20% of their academic teaching staff in the period 1995–98. However, this does not show the complete picture, as in the preceding year universities had increased student numbers by as much as 20% with no corresponding increase in staff. Apart from first-year courses, current student enrolments are not high, with a perception that career prospects in the mathematical sciences are limited (Swedosh, 1998).

The situation in New Zealand is similar:

> Currently they (mathematics departments) are facing a period of university restructuring and possible retrenchment. Their support from the wider community is uncertain, and closely related to perceptions of the immediate practical usefulness of their teaching and research activities. ... The present

> reliance on service courses to justify staff numbers is a potential threat to the stability of the mathematics departments, since enrolments can be critically affected by decisions taken outside the mathematics departments themselves, and over which they have little control. (Ministry of Research Science & Technology, 1998, pp. 24, 26)

While the federal government and various state governments have published policies committed to increasing participation in post-secondary mathematics education, Swedosh (1997) commented that sadly, the actions of these bodies have seldom matched the rhetoric. A reduction of time spent on mathematics in schools and a lack of well-qualified mathematics teachers (Swedosh, 1997), combined with short-sighted government policy, have all impacted on mathematics enrolment trends (Thomas, 1997).

Student Participation

In Australia, Leder, and Forgasz (1998a) reported that while the number of students engaged in higher education has grown steadily, the number of students, both males and females, completing a higher degree in mathematics was virtually the same in 1983 as in 1993. Thus, the proportion of students doing a higher degree in mathematics, out of all those completing a higher degree, has decreased steadily.

In New Zealand, Blithe (1997) reported static figures in first-year enrolments in mathematics and statistics courses over the period 1991–1995. As in Australia, the situation is worrying:

> A disturbing feature is the lack of growth in student numbers, despite the increases in overall university enrolments … the overall growth in numbers that has taken place in the quantitative sciences … has taken place outside the mathematics and statistics majors. Moreover, the growth in staff numbers in the mathematical sciences, which until recently has largely kept pace with the numbers of effective full time students, has not been fuelled by students seeking to study the mathematical sciences in depth, but by students requiring basic instruction through mainly first-year service courses. (Ministry of Research Science & Technology, 1998, p. 25).

Likewise, Engelbrecht (1998) noted from a sample of three universities in Australia and five in New Zealand that the number of graduates as a percentage of first year enrolments has decreased in Australia from 6% in 1987 to 4% in 1997 and remained relatively constant at less than 2% in New Zealand. While this study reported considerable increases in first-year enrolments, the conclusion is consistent with the previous studies: the number of first-year students who eventually become mathematics graduates forms a very small percentage.

With regard to variance in reported data it is noteworthy that Leder, Forgasz, and Brew (1998) and Brew (1998) commented on the difficulties of obtaining an accurate long-range overview of enrolment trends. They conclude from their own efforts to track enrolments that national enrolment data, from which reductions in the numbers of students engaged in high level mathematics studies are traditionally inferred, are not as precise as generally assumed, and possibly mask information about students who do persevere with mathematics. These research studies highlighted non-traditional groups who persist with their studies: mature-age students[2] and females who have remained with mathematics for two years of undergraduate studies. There is concern that inaccurate participation and attrition data may deter students inappropriately from continuing with mathematics at the highest levels.

The most comprehensive current study of student participation is by Forgasz and Leder (Forgasz & Leder, 1998a). In 1996, 1072 students enrolled in three Australian universities completed a survey; in 1997, 811 first-year mathematics students from five universities completed a survey; and in 1997, 71 students were interviewed. Surveys and interviews gathered biographical data, self-perceptions as a learner of mathematics, perceptions of the learning environment, and information on workload, assessment, availability of help and discrimination. Results from this study are useful to more clearly identify, and update, our notion of the typical mathematics student (Forgasz, 1997b; Forgasz, 1998a). Analysis of the 1996 survey (Forgasz, 1997b) found that very few first-year students were studying part-time, 66% were recipients of Austudy, the majority had moved straight from secondary school, and about 11% of the 1072 undergraduate mathematics students were mature-age (15% of males; 5% of females). Participation at first-year was relatively similar for males (53%) and females (47%), but the proportion of women decreased from 47% to 41% in second and third years. The latter finding is inconsistent with previous work (e.g., Gaffney & Gill, 1996) and warrants future monitoring.

These statistics compare favourably with data from Blithe's (1997) study of the six largest universities in New Zealand during the period 1991–1995 in which the percentage of women in mathematics courses was static (33-35%) and less than the percentage taking statistics courses (44-48%). As in the Australian study, the percentage of female students decreased in the second and third years. Further analysis found that female students always obtained more than their share of passes and of A grades in first-year courses, but not so at second and third year. An explanation of the change between first and third year in the relative success of females in obtaining A grades would require information on whether, for instance, high performing females are continuing on to higher levels to the same extent as males. It could also be attributed to the changing nature of the course as students progress, or to the types of assessment used at each level. However, Blithe's suggestion that data from one university indicates that high performing women may indeed be dropping out of mathematics between second and third year is consistent with overseas findings that *switchers* contain a disproportionate representation of talented female students (Linn & Kessel, 1996).

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[2] Students who have not necessarily completed the formal academic pre-requisites required generally, or for specific courses, and who are at least 21 years of age when they commenced their current undergraduate course.

Overall, current data suggest that mathematics remains a field with a male majority. Although data suggesting that females make up approximately 40% of the cohort may challenge the stereotypic image of the "white middle-class male", Forgasz (1997b, 1998a) argued that this statistic should not give rise to complacency. While previous research has identified a range of psycho-social and sociocultural variables that can influence students' initial course choices and participation at university (e.g., Abbott-Chapman, Hughes, & Wyld, 1992; Forgasz, 1996b) there is a pressing need for more intensive explorations of factors contributing to the reportedly high attrition rates among undergraduates, for both males and females, in mathematics (Leder, Forgasz, & Brew, 1998).

Perceptions

In order to understand factors that influence students' decision to persist, or otherwise, with mathematics, research is increasingly exploring students' perceptions of mathematics and their learning environment: "If we have some understanding of their perspective, then we are better placed to make sense of their engagement with and reactions to educational settings" (Taylor, 1994, p. 71).

In Australia, Crawford and colleagues (Crawford, Gordon, Nicholas, & Prosser, 1998a; 1998b; 1994) provided a comprehensive analysis of first-year students' prior understanding of the nature of mathematics. Using the Conceptions of Mathematics Questionnaire they distinguished two types of student conceptions: mathematics as a fragmented body of knowledge, or a cohesive view of knowledge. As part of an investigation of the criterion-related validity of the questionnaire, they found that students' prior conceptions were systematically related to their approaches to learning mathematics, their experience of studying the subject and their performance on assessments:

> Fragmented conceptions of mathematics are associated with surface approaches to learning mathematics, perceptions of assessment as measuring reproduction and perceptions that the workload is too high. On the other hand, cohesive conceptions of mathematics are associated with deep approaches to learning mathematics, perceptions of good teaching and clear subject goals and independent learning. (Crawford et al., 1998a, p. 465)

Analysis suggests that students holding cohesive conceptions of mathematics perceive the learning environment as more satisfactory and fulfilling and achieve at a higher level than do students reporting fragmented conceptions. Crawford et al. (1998a) argued therefore that students' prior conceptions of the nature of mathematics need to be taken into account in the design and teaching of the course.

Gordon (1997) also illustrated the importance of students' conceptions of the subject matter (in this case, statistics) and their learning approaches. Most of the psychology students participating in a compulsory statistics course thought of the subject as being about understanding the prescribed content and solving the problems presented in class, unrelated to the practice of psychology and isolated from the wider world. The majority of the students had perceptions of the subject mainly in terms of accumulating knowledge for assessment purposes and adopted a surface approach to learning. The concern is that while some of these students did well in assessments it is unlikely that many of them would transfer their knowledge to other environments. Likewise, McAlevey and Stent's (1999) survey of first-year business statistics students from New Zealand found that the two items "Relating the application of statistics to a relevant business area" and "Providing motivation for the material by way of applications" were ranked only 27th and 28th respectively, out of 43 items. "This, ostensibly, does not support the trend often encountered towards the use of real data and applications-oriented approaches in quantitative courses" (p. 219). Rather, they propose that the value of a service course is derived from applications in the major subject—students are not dependent on the statistics course alone to convey relevance.

In an across-discipline tertiary-based study, Taylor (1996) found that "successful students pay a lot of attention to the context of their studies. In particular, a focus on the academic context, and the assessment demands, are critically important" (p. 233). However, he suggested that it is not enough to have deep-achieving goals, as it is the students' capacity to coordinate their goals and strategies that is significant. Successful students had highly developed strategies that were closely aligned with their motivation to achieve excellence in their learning outcomes and tailored their approach to the demands of the particular learning environment and task demand. Accordingly, successful students "tended to be active co-constructors of the environments" (p. 234).

Forgasz and Swedosh (1997) reported that about half of the surveyed undergraduate students (1072, years 1–3) perceived mathematics as challenging and useful. However, despite reports that they understood the work, received enough individual help, and were confident of passing, many students did not particularly enjoy their mathematics studies. Data from three universities (Forgasz, 1996a; 1997a) found that all students reported decreased levels in enjoyment in mathematics studies at university when compared with schools. There were however significant differences in school leavers' and mature-age students' perceptions of their learning environment. Overall, the mature-age students were more positive than the school leavers regarding the quality of teaching, access to staff, and availability of help. Moreover, Forgasz (1996a) noted that mature-age students who remained with mathematics for two years of undergraduate studies seemed more convinced of the importance of mathematics, had a greater awareness of the intrinsic value of mathematics, were more task-oriented, willing to work hard, and consistently sought and obtained help when needed.

> Their love of the discipline and appreciation for the efforts of their lecturers and tutors were evident. Their life experiences appeared to advantage them over

> their school leaver counterparts. This was apparent in their greater clarity of purpose, their independent approach to their studies, and in their recognition that persistence was necessary to overcome obstacles. (p. 206)

Forgasz suggested that mature-age students' higher motivation and the greater likelihood that they were studying mathematics by choice might partially explain the differences in the two groups' perceptions. Forgasz (1998b) and Leder (1998b) provided further insights into mature-age students' university experiences through qualitative data collection techniques of *tagging* and *beeper-activated* monitoring. The resulting detailed profiles of students' experiences indicated that mature-age students, despite being more readily adaptable to the new learning environment, may not always fit comfortably into a system which is primarily geared to the needs of school leavers.

Somewhat surprisingly, considering the relatively recent and increasing exposure to technology now experienced by school students, Leder and Forgasz (1998b) found that mature-age students and school leavers reported similar beliefs about their performance levels and competence levels with technology.

School leavers, in contrast to mature-age students in Leder and Forgasz's study, thought mathematics was less useful and that there was too much work. Also, non-English speaking background students (NESB) students agreed more strongly than those from English speaking backgrounds (ESB) that mathematics was easy, enjoyable, and interesting and that there was too much work (Forgasz, 1998a). The question of appropriate student work expectations is raised by several other studies in relation to poor performance (Anthony, 1997; Arnold & Truran, 1999; Barrington, 1997; Simpson, 1996; Williams, 1998).

The impact on mathematics learning of the traditional large lecture format (Simpson, 1996) is a continuing focus for research. Hubbard (1997b) found that students, particularly part-time students, many of whom were mature-age students, supported the view that lectures help them learn: lectures were important for providing notes to study later, conveying information about tests and assignments and indicating important material. Students in Anthony's (1997) study reported that it was the lecturers' job to tell the students what the most important bits were so that they knew what would be assessed. However, although students reported that the content was interesting, students frequently expressed dissatisfaction with the lecture format and critiqued lecturers' teaching abilities (e.g., Anthony, 1997; Gordon, Nicholas, & Crawford, 1996; Hubbard, 1997b; Leder, 1998a; and Willcoxson, 1998).

Students (from across disciplines) in Clarke's study (1998) claimed that learning is helped in large lectures when the lecturer uses practical, relevant experiences and material that is clearly presented and well structured. Cretchley (1998), in a study which asked mathematics students to rate the influence of course factors such as lectures, tutorials, homework and assignments on achievement, identified effective feedback as the most significant factor.

As well as plenty of examples and clear explanations, mathematics students in Anthony's (1997) study frequently expressed the value of a more personal approach:

> The lecturer is good because he laughs and he's very much at ease and very happy—he's quite cute you know. The whole class laughs along. You want to try harder for him because he really does care and he really goes out of his way. You want to do well for him. (Student interview)

Forgasz and Swedosh (1997) suggested that the level of perceived personalisation within the department and support for further learning may well be crucial factors in persistence (Forgasz, 1998a; Forgasz & Leder, 1998b). Less personalisation at the tertiary level, compared to that experienced in the school mathematics classroom might partially account for the decrease in enjoyment of mathematics from school to university, and may provide a partial explanation for the higher female dropout rates found for the transition from first into second year.

Discrimination may also contribute to disenchantment with tertiary mathematics: "It would appear that it is not only some lecturers, but also some male students, who hold ill-informed, gender-stereotyped views that the genetic make-up of females 'naturally' inclines them away from mathematics and towards the humanities" (Forgasz, 1996b, p. 198). Thus, females who persist with mathematics at the tertiary level need to be highly motivated and prepared to face and overcome the obstacles and adversity they may face. Forgasz and Leder (1998b) reported that incidents of perceived discrimination were found to vary across institutions in their study and appeared to be greater where there was greater ethnic diversity.

As well as considering student perceptions, the understandings lecturers hold about their students and teaching greatly influence the content and form of their teaching. Willcoxson's (1998) across-discipline study found that the way lecturers perceived their own teaching did not always bear a strong resemblance to their students' perceptions. Lecturers expressed the feeling that students in their lectures were mostly uncritical recorders of information, principally interested in gathering examination material. In a study involving ten schools in a large Australian university Burroughs-Lange (1996) comments on the pervasiveness of lecturers' negative perceptions of students' capabilities: "I've got used to assuming that the students are not highly motivated, not high quality and won't take on board difficult concepts" was typical. A study by Yusof and Tall (1999) in a Malaysian university also found that the lecturers had little confidence in their mathematics students' ability to cope with a formal mathematics course and taught them accordingly. Although these lecturers preferred students to have a range of positive attitudes to mathematics, they expected the reality to be different.

Students in Willcoxson's (1998) study reported that they *were* motivated to explore the subject matter—too frequently, however, their interest waned and frustrations arose in response to the lecturing styles which they felt encouraged only note-taking. According to McKay and Kember (1997) these passive learning approaches are not always the fault of students wanting to be spoon-fed, rather "contextual variables such as heavy workloads, teacher led instruction formats and assessments that demand regurgitation of handout material, force students to take a rote learning approach" (p. 56). Anthony (1997) noted that mathematics students, in response to a questionnaire, placed more importance than lecturers on active learning and note-taking. However, subsequent student interviews provided limited evidence of active learning in lectures. Possibly the evidence supports the supposition that students may well place more faith in note-taking, especially that which is verbatim, as a more effective strategy than is warranted. Lecturers, on the other hand, may take the view that notes provide an important source of stimuli for student study outside the lecture. "That is, learning only *starts* with the lecture; it does not end there" (Isaac, 1994, p.215). Hubbard's (1997b) documentation of students' correct application of a procedure at the end of a lecture and subsequent failure on the same task in their final examination may be indicative of students placing too much faith on their ability to follow the lecturer's examples with little ensuing attention to reflective practice designed to consolidate procedures and concepts.

Burroughs-Lange (1996) noted that although there was a willingness to experiment with teaching approaches, when describing what they actually do in practice lecturers had difficulty in identifying examples in accord with these broader beliefs, or else their failure to adopt more appropriate strategies was attributed to external influences. Only in a few instances did lecturers' concepts of their role include a sense of themselves as learners. Given their general dependence upon prior experiences, it is probable that many are insufficiently familiar with strategies that might be used to effect changes in the way teaching and learning occurs within the tertiary environment (Smith & Wood, 1998; Willcoxson, 1998). Clearly, as essential background to developing appropriate teaching strategies, we need to discover what students are actually doing in and taking away from lectures, and what students feel might help them learn more effectively. We need to be concerned about how students engage in learning and what triggers their interest.

Another major concern for both students and lecturers was the influence of factors outside the university learning environment. Language background appeared to provide partial explanations for patterns of gender difference in participation and learning outcomes and for patterns of difference by age in Leder and Forgasz's study (1998b). Additionally, the effects of working long hours in paid employment, sometimes on night shift, and increasing financial pressures which impact on suitable accommodation and study resources (Arnold & Truran, 1999), were all critical variables worthy of inclusion in future studies exploring student beliefs, attitudes and participation in mathematics courses.

It is clear from current research that many factors influence both student and lecturer perceptions. The deliberate shift in attention away from teaching and learning as independent activities towards a more systemic examination of the relationships between them and the context in which they occur (Gordon et al., 1996) is providing increasingly rich information about student participation in mathematics study.

Student Skills and Understanding

In recent years, there has been growing concern at the lack of pre-calculus skills displayed by students entering university mathematics courses (ICMI, 1997; Ministry of Research Science & Technology, 1998). The consensus is that many students are seriously hampered from progressing to more advanced mathematics because of inadequate skills (Barry & Davis, 1998; Boustead, 1996; Coady, 1995; Simpson, 1996; Swedosh, 1996; White & Pegg, 1996).

Algebraic Skills

Using pen-and-paper tests with two groups of first-year university students in New Zealand, Boustead (1997) categorised five common algebraic difficulties that students bring to university. These were understanding the range of possible solutions; understanding the properties of numbers, especially fractions; formulation of false generalisations; understanding the structure of obtaining solutions; and understanding the order of operations. Boustead expressed concern that the properties of numbers and understanding the order of operations related directly to primary and early secondary school mathematics learning. Swedosh (1996) also examined the nature and frequency of mathematical misconceptions using diagnostic tests at two Australian universities. He noted that the vast majority of misconceptions appear to be due to *external inconsistencies* that occur when a student's existing mathematical concept is incompatible with newly presented information. An inappropriate transfer of theorems that apply to one mathematical subculture into another resulted in over-generalised, incorrect or inadequate rules.

Given that larger numbers of students are taking university mathematics courses, the traditional assumption of a requisite mathematical sophistication may be neither appropriate nor realistic (Ball, Stephenson, Smith, Wood, Coupland, & Crawford, 1998). Moreover, the expectation that students have learned these concepts prior to entry, means to a large extent that the content of students' misconceptions is unlikely to be explicitly revised/taught in the traditional first-year mathematics course. To address this concern, Swedosh (1997) conducted a study designed to eliminate student misconceptions in basic algebra. Using a teaching strategy, based on Piaget's notion of cognitive conflict, misconceptions were explicitly challenged (often with the use of numerical substitution) and acknowledged, before correct concepts were retaught. Swedosh reported success in reducing, and in some cases eliminating, the frequency of student misconceptions. A post study showed substantial benefits were still evident one-year later (Swedosh & Clark, 1998). Barry and Davis (1998) reports the success of another remedial mastery programme (Essential Mathematics Scheme).

Swedosh (1997) claimed that mastery of necessary prerequisite knowledge depended not only on individuals having a thorough understanding of basic concepts but also that they had practised these skills to the point where they could use them confidently. The issue of student confidence may be especially important in the self-regulated environment of the university (Anthony, 1996b). A study by Galligan (1998) which investigated students' skill level on entry to university in relation to self-efficacy reminds us that getting the 'right answer' is not the only concern. As in many previous studies, Galligan found overall poor student performance, but additionally noted that in many cases of correct answers students expressed low confidence levels. For example, in basic algebra manipulation questions 70% of students with correct answers expressed doubt as to the correctness of their answers:

> Diagnostic testing which simply highlights students' errors and misconceptions adequately deals only with students who are wrong and not confident, and students who are right and confident. Underconfident and overconfident students also need support and their answers need to be analysed more closely as to the nature of their underconfidence and errors, which may underlie a mathematical misconception. (p. 65)

Advanced Mathematical Thinking

In an earlier review on advanced mathematical thinking, White and Pegg (1996) commented that research in this area is comparatively new in Australasia. A review of recent research suggested that this is still the case. Current studies are centred largely on students' level of conceptual knowledge related to introductory calculus and related concerns about the large number of students exhibiting surface learning approaches.

White and Mitchelmore (1996) investigated the performance on calculus application problems of a group of students involved in a concept-based course (Barnes, 1992). The students had previously experienced a traditional introductory calculus course in secondary school. The aim of their study was to find out whether students who aspire to the advanced mathematical thinking involved in calculus have an adequate concept of a variable. Results suggested that the number of students who could symbolise rates of change in complex situations increased dramatically during the calculus course. However, there was almost no increase in the number who could symbolise rates of change in complex items or in items that required modelling a situation using algebraic variables. Responses to the research items strongly suggested an underdeveloped concept of a variable was a major source of students' difficulties. In particular, students frequently treated variables as symbols to be manipulated rather than as quantities to be related. Three examples of such a manipulation focus were identified: failure to distinguish a general relationship from a specific value;

searching for symbols to which to apply known procedures regardless of what the symbols referred to; and remembering procedures solely in terms of the symbols used when they were first learnt. White and Mitchelmore concluded that many students had learned to operate with symbols without any regard to their possible contextual meaning. They described such a concept of variable which is limited to algebraic symbols as *abstract-apart* (p.92). While an abstract-apart concept might be adequate to deal with routine symbolic procedures, the limitations of such purely procedural knowledge become apparent when the symbols had a specific contextual meaning. Rather, they contend that "symbolizing a rate of change in complex problem situations requires the existence of *abstract-general* concepts because the symbols used to represent general variables and derivatives have to be related to the specific variables and rate of change that occur in that situation (necessarily at a lower level of abstraction)" (p. 92).

Within the domain of calculus, White and Mitchelmore's (1996) study illustrates the significance of encapsulation (Asiala, Brown, DeVries, Dubinsky, Mathews, & Thomas, 1996) or reification (Sfard, 1991) and the importance of thinking proceptually (Tall, 1995) in the formation of abstract-general concepts:

> Students who can use defined variables but cannot identify and define their own variables would seem to be using symbols essentially to express the process of relating one variable to another. These students are still at the condensation phase of developing their concept of a variable. On the other hand, students who can create variables to solve complex problems would seem to have reified variables. (White & Mitchelmore, 1996, p. 92–3)

White and Mitchelmore concluded that an abstract-apart concept of a variable would in most cases block future meaningful learning of calculus. Moreover, they argued that it is not realistic to provide adequate remedial activities within a calculus course for those students who have an abstract-apart concept of a variable; rather these students need an appropriate pre-calculus course.

In an investigation of first-year students' higher-level skills associated with school algebraic topics, Coady and Pegg (1996) also identified a range of student difficulties: (a) interpretation of the problem; (b) focus on the relationships within the question; (c) identification of critical aspects of relationships; and (d) integration of the critical aspects to the question and its subsequent solution. Like others before them, Coady and Pegg concluded that many students at this level had not developed adequate levels of cognitive growth in both the procedural and structural aspects of algebra necessary for more advanced study.

Many first-year calculus students' understanding of the function concept is also incomplete. Jourdan (1998) found that students expected continuity as a requirement for a functional relationship; many students judged a continuous line or curve to represent a function without any respect for the rule of correspondence. As in the previous studies, there was inadequate integration of representations: students did not make connections between the graphical and symbolic representations of a function. Needing further investigation were differences in notions of discontinuity between students who have used CAS software in their learning compared with students learning in a traditional environment

The importance of a sound conceptual framework is further demonstrated in a study involving Malaysian students (Ali, 1996). Ali noted little correlation between the number of steps taken in carrying out routine symbolic algorithms for differentiation problems and the level of attainment of the student. However, he found that successful students were more likely to use some form of conceptual preparation with problems that could be simplified by a non-algorithmic manipulation, before they used a standard algorithm.

Advanced Statistical Thinking

The problems that students experience in attempting to understand the sophisticated concepts of statistics at undergraduate level, usually in first-year subjects, have been studied by several researchers from different points of view. Williams (1998, 1999) studied students' misconceptions and poorly expressed concepts of hypothesis testing, while Finch (1998) and Finch and Cumming (1998) measured the change in students' understanding of variability and the law of large numbers resulting from the use of a computer package. Outhred and Shaw (1999) demonstrated that the use of visual representations is associated with success in statistics examinations; however many students were reluctant to use visual representations, even when urged to do so (Shaw, 1998). While most studies focused on students, Pfannkuch and Wild (1998) interviewed six professional statisticians in order to explore the nature of statistical thinking. They concluded that statistical thinking is differentiated by the following key features: the role of variation; the integration of statistical knowledge with context knowledge; and transnumeration between and within the real system and statistical system. Their proposed categorisations provide a useful starting point in characterising and capturing some aspects of statistical thinking which may need to be more central to the development of statistical literacy.

A common theme with all of these research studies is the concern about students' preparation for advanced level mathematics courses in terms of initial knowledge base, advanced thinking strategies, and learning strategies. In particular, the adequacy of adequate algebraic skills will be of increasing focus within a technology-orientated learning environment. Related questions concerning curriculum design, especially in the area of calculus and analysis, will increasingly need to be addressed in the forthcoming years.

The Way Forward

Instructional Innovation

Currently, there are many new developments in the teaching and learning of undergraduate mathematics which attempt to address some of the issues above. Numerous conference reports detail alternative approaches to calculus teaching (reflecting the reform approach of the United States), linear algebra (Ball et al., 1998; Smith, Wood, Coupland, & Stephenson, 1996), service mathematics courses and bridging programs (Miller-Reilly, 1997). While not all are intended to be research reports these papers provide valuable impetus for lively debate, reflection and sharing of good practice, which can only be good for the long-term quality of mathematics education. Conference papers also indicate that a similar reform agenda to teaching statistics is prevalent (e.g., Thomason, 1998).

Porter (1996a, 1996b), using the teacher-as-researcher research paradigm, described three consecutive implementations of a tertiary preparation course in Mathematics and Statistics designed to raise students' awareness of their learning approach. She concluded that a more positive learning experience for students is achieved by giving students opportunities and encouragement to assess their own learning as well as their notions of how they learn. She argued that to be effective, reflective practice must be a two-fold process involving both the lecturer and the students.

The introduction of technology, while previously focused on the impact of graphic calculators, is now firmly focused on the introduction of computer algebra systems (CAS) into courses (e.g., Colgan, 1998; Raj & Malone, 1997; Smith & Wood, 1998). Simpson (1996), in a 1995 survey of 38 universities from Australasia, found that half reported the use of computer packages in their courses, although many forbade the use of graphic calculators in tests and examinations. Smith and Wood (1998) note that in their university, many students now use *Mathematica* routinely (not just for assignments) in checking work and in other subjects such as physics. It has become part of the students' toolkit of mathematical skills. However, they did report gender differences in the use of computer algebra systems, in line with other findings on computer use (see Forgasz & Leder, 1998b).

Hong (1997) used computer-based instruction to see if an improvement in the understanding of the concepts associated with integration can be produced by giving students direct experience of experimentation with the associated processes. Before the computer tutorials pretests showed that students' understanding was restricted to algorithmic processes and lacked adaptable thinking. For example $\int f(x)dx$ was seen as an instruction to carry out a process or algorithm, not as the encapsulation of the limit of area sums. Hong concluded that the computer instruction improved the cognitive base for a flexible proceptual understanding of limits, areas, and the Riemann integral.

Collaborative learning, traditionally confined to the school system, is increasingly being used within the university environment, especially in bridging mathematics courses that typically have only a small number of students in a tutorial class. Morton and Oates (1997) provide an account of the problems and successes of establishing collaborative tutorials with a large second year mathematics class as an alternative to the large lecture format. These tutorials aimed to develop students' understanding through active participation, communication and articulation of mathematical ideas. Assessment within these tutorials contributed to final course grade and students were awarded 50% of their marks for oral communication and their understanding of the group's joint solution. The research indicated that for most students collaborative problem-solving tutorials can provide an effective and enjoyable alternative to the standard processes of assessment and learning. Enjoyment factors mostly concerned the practical nature of the content and the opportunity to work collaboratively. However, students were not unanimous about the fairness of the assessment procedures. Additionally, Morton and Oates noted that while many students commented positively on various aspects of group discussion, there was a lack of comment on one of the most important benefits—mainly that this was good preparation for future careers, where in all likelihood they would be part of a team.

Other studies also found that students appreciated the opportunity to work in groups; whether it be in formal cooperative groups (Dansie, 1998); projects involving technology to investigate real applied problems (Colgan, 1998); or self-study groups (Anthony, in press). Apart from the collaborative learning opportunities, it appeared the social aspect of getting to know the other students in their course was of significant benefit in establishing support structures, especially for first-year students.

Another noticeable development in conference discussions is the critical examination of the material used by students—Are textbooks, video/multimedia and CAS instructional materials meeting the needs of students and supporting appropriate learning approaches and outcomes? Several researchers (Anthony, in press; Boustead, 1999) report concern for the under-use and purpose of hard-copy text reading material:

> How much does the teacher *actually know* about what use each student makes of the text and about how well each student learns from it? … It is incongruous—we may conclude—to devote care to linking the text closely with class teaching when one is largely uniformed about (i) the extent to which individual students find the text helpful, and (ii) the learning problems of those students for whom the text is not helpful. (Sowey, 1998, p. 154)

Hubbard (1997a) questioned the types of practice exercises, typical of many benchmark tertiary texts, which support the *development-result-practice* routine.

> Frequently the practice consists of very restricted types of exercises and as a result, the students construct very restricted mental representations of the concept or procedure and fail to construct links to other knowledge. … The exercises rarely ask students to reflect on the results they obtain, generalise from them, make comparisons, observe patterns or relate the new procedures to previous knowledge. Yet these are the very activities which have the potential for creating links in the student's cognitive structure. (p. 794)

Hubbard argued that too frequently exercises were presented without any context and apart from the obvious fact that practising the exercises is supposed to enable the student to pass examinations, no mathematical or pedagogical reasons were given for the selection of particular exercises. Students needed answers to question such as, "Why am I doing this page of exercises?" and "Where in mathematics is this procedure used?" Their work might then become more meaningful. Doing exercises does not in itself produce learning—the student's mind must be actively engaged and the student must reflect on the activity (Anthony, 1996a). Hubbard (1995) provided examples of questions and activities which require students to reflect on their thinking and improve their conceptual understanding, and she also illustrated how this can be incorporated into the lecture format (Hubbard, 1997b).

Boustead (1999) examined the instructional potential of software packages such as textbooks on CD ROM (e.g., Larson & Hostetler, 1995) and multimedia programs (e.g., Quinney & Harding, 1996). If students are reluctant to use textbooks and printed text for independent study would the same material presented through educational technology increase students' willingness to read and improve their understanding of mathematical concepts? Her study followed the reading and learning behaviour of 47 first-year university mathematics students, comparing their comprehension of the same task using different learning resources. The multimedia approach had the greatest motivational appeal because of the video clips and interactive graphics. However, unlike the younger students, the more mature students were frustrated with the interactive graphics. Students using EPSILON (Monash University, 1997) and software packages maximised the use of the non-sequential nature of the resource. However, students still experienced difficulty in linking multiple representations needed to assist conceptual knowledge development. Boustead concluded that trendy technology may not automatically improve learning. However, if it is well designed from the students' learning perspective, it may well enhance their understanding of mathematical concepts and perhaps be more successful than reliance on a textbook.

Distance education is increasingly becoming a viable option with incorporation of sophisticated technologies into the learning environment. Hardman and Dorman (1988) provided an evaluation of the use of a virtual face-to-face environment using a Desktop Video Conferencing (DVC) system to teach mathematics at a distance. Not surprisingly, it was found that in order to use DVC effectively, preparation time for teaching was high. However, the face-to-face facility of the DVC enabled students to develop confidence in questioning and pursuing sequences of ideas. It also enabled the teacher to more readily convey enthusiasm and motivation for the subject matter, a significant feature in students' perceptions of success (Anthony, 1997), and the regular contact helped students to stay focused and on track with their studies.

Assessment

Changing teaching methods without due attention to assessment methods is not sufficient. It is now widely accepted that appropriate assessment methods are of major importance in encouraging students to adopt successful approaches to their mathematics learning (Crawford et al., 1994; Smith & Wood, 1998; Smith et al., 1996). Ball and colleagues responded to concerns that most current examinations are heavily biased towards tasks requiring factual knowledge, comprehension and routine use of procedures (Ball et al., 1998; Smith & Wood, 1997; Smith & Wood, 1998). They developed the MATH (Mathematical Assessment Task Hierarchy) taxonomy to classify tasks ordered by the nature of the activity required to complete each task successfully rather than in terms of difficulty. The MATH taxonomy provides a framework for the construction of tasks that ensures variety in students' mathematical experiences and encourages students to reflect on their learning through the use of justification, interpreting implications, conjectures and comparisons, and evaluation. Smith et al. (1996) and Ball et al. (1998) provided an extensive list of paradigm examples illustrating descriptors in the taxonomy for those interested in implementing alternative approaches to assessment tasks.

Pierce and Roberts (1997) and Anderson, Bloom, Mueller and Pedler (1999) noted that the incorporation of technology into assessment practices is a crucial complement to courses which integrate technology into the teaching/learning process. Pierce and Roberts concluded from their experiences of mathematics examinations in the computer laboratory and the provision of computer outputs in statistics examinations that student focus shifts towards higher order thinking and application and interpretation. Some examples of how standard test questions can be modified when students are using graphics calculators during tests are provided in Anderson et al.'s (1999) article. Again the focus is on increasing the importance of analysis and interpretation as opposed to mechanical skills. Stone (1997) provided a comprehensive evaluation of the intelligent computer tutor that is based on assessed problems with diagnostic feedback. Benefits include increased student achievement, increased lecturers' knowledge of student problems, and efficiencies in staff time.

Smith and Wood (1998) suggested that external factors including the changing nature of our student clientele, the use of technology, wider requirements of graduates, and accountability demands should provide further impetus for innovative assessment practices. Smith and Wood also warned of potential barriers: (a) the innate conservatism of many mathematicians; (b) resistance from students; (c) administrative difficulties; (d) conflicting time pressures; and (e) a lack of research which is readily accessible to non-experts to show that changes in assessment practices are educationally effective.

Research studies, however, are united in the view that changing the assessment alone will not be sufficient to orient students to deep approaches to learning—students' total experiences of learning mathematics need to be addressed. (Crawford et al., 1998a).

New Directions

While the debate about lack of essential prerequisite skills continues, a new level of debate about appropriate learning outcomes is rapidly emerging. From an international perspective Engelbrecht (1998) contended that changes in the student base mean that the majority of our students do not care about abstract mathematical theories, about rigorous mathematical proofs for every statement, about mathematics that cannot be applied directly in their field of interest—they want relevant, applicable subject content which they can apply now, not some day into the future (see also Klymchuk, 1998). Those who choose to major in mathematics are more likely to move into an increasingly wide range of occupations requiring sophisticated communication and problem-solving skills. For example, Fuller and Flanders (1998, p. 111) quoted from a recent report of the Australian Institution of Engineers that "courses should become more attuned to the actual needs of society … promote environmental, economic and global awareness, problem solving ability, engagement with information technology, self directed and life long learning, communication management and teamwork skills".

Crawford (1996, 1997) argued that we need to seriously question the continuing focus on students' capabilities to demonstrate operational knowledge of approved procedures and axioms:

> In the community the set procedures and routines are the work of machines. Employment opportunities for routine, repetitive tasks are reduced in the wake of automation. On the other hand, ever more complex transactions and other forms of social organisation have become possible through the support of information technology. More complex mathematical reasoning and understanding are needed by people in assessing their justice or desirability. (Crawford, 1996, p. 81)

In particular, the value of skill-based calculus courses has come under fire because computers and calculators now perform most (if not all) of the manipulative procedures taught in such courses (White & Mitchelmore, 1996).

Noss (1998) suggested that the main contribution of new technology for mathematical teaching and learning at the undergraduate level is that it provides us with an opportunity to reassess not simply how we teach, or even how students learn, but what it is that we teach them and why. Changes resulting from technology raise serious questions about the nature of education, the kinds of mathematical knowledge that are needed by learners of all ages (Anderson et al., 1999; Fuller & Flanders, 1998), the kinds of educational settings that can and should be made available for the 21st century, and the kinds of learning experiences and mathematical knowledge that are needed by humans in an era when machines commonly carry out the set procedures that have dominated school mathematics in the past.

> The pressure for change is not merely to teach more mathematics to more students in more fields, but to prepare students through experience of a new sort of mathematical *Activity* that has investigation and problem resolution as its goal and in which the *operations* are technologically supported. (Crawford, 1996, p. 58)

Crawford (1997) contends that emerging networked technologies could be used to support mathematical learning through alternative forms of social organisation such as mentoring by community members, consultation, and cooperative problem solving in groups (e.g., The School Links Project at the University of Sydney). Nicholas (Crawford et al., 1998a) is researching and evaluating an interactive internet-based environment for students to explore mathematics and make mathematical conjectures by participation in discussion groups and chat rooms.

Overall, there is a consensus for change in the mathematics learning environment: change which will move teaching and learning from the *deficit model* of rote-learning material to the development of problem-solving skills needed for higher order thinking in mathematics. Change, however, is still largely reactive; imposed by technology advances, competition from other providers of mathematics courses, and more attractive course designs in other disciplines. However, whatever the reason for change the increased attention, both in Australasia and internationally, to course design, assessment, and integration of technology should support more effective learning and enhance participation rates.

Conclusion

Never before has the role and performance of mathematics departments and the discipline been under such public scrutiny. Within such a climate educational research has been accorded a greater role; attention has widened from an earlier focus on transition and bridging courses to include first-year courses and beyond. Current research studies which take a more systemic approach alert us to the need to be aware of the ways students experience the total learning context rather than just focusing on the mathematical content and presentation of courses. Student perceptions of mathematics and mathematics learning, as well as the quality of courses, of teachers, of the modes of delivery, of availability of help, of the learning climate, and of school-tertiary relationships, may well be related to students' subject choice decisions and persistence.

The level of dissatisfaction with the tertiary learning environment evident among students should be of major concern to tertiary mathematics educators and departments (Jackson & King, 1998) and appears to support the increased efforts and need for all tertiary teaching to take "greater cognisance of educational theories on effective student learning" (Leder & Forgasz, 1998b, p. 317). Increasingly, there is a call for universities to make mathematics and statistics more accessible and valued by students in particular, and society in general:

> The message strongly presented is that we must teach in such a way that we create value for the subject in the minds of our students. … We have abundant evidence now, through reduced student numbers, that it is not sufficient to look inwards, to teach techniques and to hope that the value of the subject will speak for itself. (Wood & Wasimi, 1998, p. 167)

In a highly competitive global environment with escalating economic problems, Thomas (1998) warned of the perils of complacency by mathematics departments: "It is no longer safe to assume mathematics will be given special treatment. It is imperative to ensure that the wider community values the contribution of the mathematical sciences" (p. 251). Major changes need both government support (Ministry of Research Science & Technology, 1998; Swedosh, 1997) and a change in societal attitudes to mathematics—there is a need to convince society at large that considerable mathematics learning is relevant and achievable for most students.

While many of the reviewed studies draw our attention to problems, it is promising that some studies also offer possible ways forward, be it with teaching innovations, support structures for students, or curriculum reform. The increased research attention through conference activity will encourage those who work in the area of advanced mathematical thinking which is still a relatively new area of research in Australasia.

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Issues in Mathematics Teacher Education

Bob Perry
University of Western Sydney
<b.perry@uws.edu.au>

Beth Southwell
University of Western Sydney
<b.southwell@uws.edu.au>

Peter Howard
Australian Catholic University
<p.howard@mary.acu.edu.au>

Introduction

The teacher education of mathematics teachers in Australasia is facing a number of challenges, some of which arise from the nature of and forces upon teacher education in general and some of which are peculiar to mathematics. In Australasian secondary schools, there is a serious shortage of appropriately qualified mathematics teachers, just as there is in many other parts of the world. The reasons for this shortage are well known and include poor salary and advancement structures relative to those which can be obtained in industries outside education, a lessening of interest in mathematics by students and a subsequent increase in management and behavioural challenges. While there is currently not a shortage of primary (elementary) school teachers in Australasia, such a shortage is predicted. Mathematics is generally taught by the generalist classroom teacher in Australasia and all primary teachers have mathematics teacher education included, at varying levels, in their university teacher preparation courses. Many of them do not show a great affinity with this mathematics and many display as well as espouse negative beliefs and attitudes about mathematics and its learning and teaching.

Hence, at both the preservice level of teacher education and at the level of professional development, there are numerous challenges to be met. In Australasia, there is a very active group of mathematics education researchers who have devoted their energies to these challenges, and this chapter is a celebration of their work over the years 1996–1999. The chapter considers research findings about Australasian mathematics teacher education across all levels of learning—early childhood to adult—and dealing with both preservice mathematics teacher education and the professional development of practising mathematics teachers. A critical analysis of this research is grouped for convenience into the following sections within the chapter:

- beliefs, attitudes and values of practising and prospective mathematics teachers and their effect on practice;
- mathematical knowledge of practising and prospective mathematics teachers;
- preservice mathematics teacher education;
- professional development in mathematics education;

- mathematics teaching as a sociocultural construct.

The authors of this chapter have endeavoured to be as comprehensive as possible in their identification of research papers in the area of mathematics teacher education published either by researchers of Australasian origin or published in Australasian mathematics education research sources. However, they have been selective in choosing those included for mention in the body of the chapter, making judgements of relevance, importance and impact.

The authors are sure that readers will agree that the chapter does celebrate a tremendous body of work by Australasian mathematics education researchers in the important area of mathematics teacher education. We trust that such a celebration is useful to each and every reader.

Beliefs, Attitudes and Values of Practising and Prospective Mathematics Teachers and Their Effect on Practice

Previously, Australasian mathematics education research in the affective domain has tended to focus on teacher and prospective teacher attitudes. However, in the period 1996–1999, significant studies have been undertaken into the espoused beliefs of teachers and student teachers. The results of these studies highlight the role that espoused beliefs, which are identified and expressed by elementary teachers and mathematics teachers, can have in all aspects of the planning, teaching, and learning of mathematics.

Problem Solving

Anderson (1996) administered a questionnaire to 39 primary school teachers investigating their beliefs about the role of problem solving in learning mathematics. The survey gathered demographic information, asked teachers for their two favourite problems and for comment on two developed scenarios—one of a teacher who advocated problem solving and one of a teacher who was more supportive of basic skills. The results indicated that most teachers support problem solving though there needs to be clarification as to what each teacher means by *problem* and *a problem-solving approach to teaching* (Anderson 1996, p. 37). It was also found that teachers make decisions about appropriate problems based on their own beliefs, the class, individual students and the concepts being taught.

Anderson (1998) reported on a more expansive follow-up study which included 174 primary classroom teachers completing a modified survey developed from the reported 1996 study. The survey, Problem Solving in Mathematics Teaching: Teachers' Views and Teaching Practices, comprised five questions related to beliefs, teaching strategies, preferred question types and sources of problems. Teachers were asked to describe a recently used problem, explain their preference for particular problem types and identify professional development needs of the school staff regarding the implementation of problem solving teaching strategies. Teachers suggested that students need number facts and some basic mathematics before they can tackle problems. Most teachers rejected the proposal that students need to know the four operations and that problems should be left to the end of a topic though the majority identified that the language used in the problem can increase the difficulty level for the student. While a majority of teachers use more traditional problems, some appear to use open-ended and unfamiliar problems on a regular basis. A wide spread of teacher beliefs about problem solving has been identified in this study as well as evidence that a variety of question types are being used in primary mathematics classrooms.

While it is generally accepted that mathematics problem solving among student teachers is something to be encouraged, Pateman and Zilliox (1996) observed that not all participants in their study saw it as a positive experience or even one that they would want to use with their (future) school students. In spite of this, they do report positive responses from most student teachers to collaborative problem solving in a supportive environment. However, it was noted that "the image of 'information dispenser' to children is very hard to forgo" (Pateman & Zilliox, 1996, p. 5).

PreService and Inservice Effects on Beliefs

Bobis and Cusworth (1997) undertook a five year longitudinal, quantitative study into the attitudes of preservice primary teachers towards mathematics and teaching mathematics. The survey used, titled Attitudes Towards Mathematics, incorporated questions which comprised: an attitude towards teaching mathematics scale; a mathematics self-concept scale, and an attitude towards teaching mathematics with technology scale. The survey was administered three times across three years to two primary teacher education groups. The Bachelor of Education students completed the survey during their course while the Graduate Diploma in Education students completed the survey during their course and their first year of teaching. Survey results suggested that all students improved their mean scores on all three scales at each round of the survey though not always significantly. At each round the teaching mathematics scale had the most improvement. For the Graduate Diploma students, their mathematics self-concept improved significantly across the survey administrations. Findings also suggest that attitudes towards teaching mathematics with technology slowed dramatically in the first year of teaching. The results show that preservice teacher education programs can shift the attitudes and mathematics self-concepts of students to more positive ones and that these shifts can be enduring.

Schuck and Foley (1998) reported on the use of a web-based computer conferencing intervention which encouraged dialogue about mathematics between an international community of mathematics educators and a student teachers' local learning community. Two surveys were administered to the 165 students involved and data from their reflective learning journals were used to show that this intervention challenged the students to become aware of, examine and evaluate their beliefs about mathematics learning and teaching. This type of intervention requires further investigation, particularly as to how it relates and impacts upon the practicum experiences of the students. For example, journal entries identified conflicts between what student teachers read through the dialogue and what was evident in classroom practice.

Scheck's (1996a, 1996b) study, extended in Schuck (1996c, 1997a, 1997b, 1998) examined the beliefs and attitudes of first year primary student teachers with regard to the learning and teaching of mathematics. Some of the beliefs that students and teacher educators bring to their courses can act as chains constraining reform in the teaching of mathematics. The study involved collecting data across one year through peer interviews, maintaining journal entries, and reflecting on practicum experiences. Following this, students completed an open-ended questionnaire, and in-depth interviews were held with eight students and the four mathematics educators involved in the course delivery. Student chains of past experience, individual attitudes towards mathematics, limited subject-matter knowledge, perceived irrelevance, students' practicum experiences, reasons for studying in the teacher education program, and the lecturers' underlying philosophy towards mathematics can all act as constraints to the reform of mathematics teaching. Suggestions to break these chains include:

- having the student teachers act as researchers of their and others' beliefs and attitudes;
- the use of reflective journals;
- developing connections between different mathematical topics, concepts and areas;
- identifying links between mathematics and other subject areas; and
- teacher educators presenting a variety of philosophies of mathematics education.

Carroll's (1996, 1997, 1998) study investigated primary school teachers' views of mathematics teaching and learning, and examined the implications of these views for professional development. A *Teaching Mathematics Survey* was completed by 108 teachers, and mathematical life histories were collected from five of the teachers. A number of findings were described relating to (a) teacher types, (b) whether the teacher felt positively or negatively about their knowledge, and (c) affect with respect to mathematics teaching and learning. The study recommended that teacher professional development should be on-going, enabling teachers to draw on their teaching experiences, consider relevant research findings, and trial new ideas before evaluating and discussing their classroom findings. It was further recommended that the differing needs of different teacher types should be considered in the planning and implementation of professional development programs.

Two secondary preservice teachers studied by Malone (1996) were shown to share three perceptions which evolved over the first year of their teacher education program. These perceptions focused on student learning, students' active involvement in their learning, and classroom management. These perceptions changed during the year of the study as the subjects made their transition to teaching.

Computers

Norton (1999) used survey and case study procedures to determine how frequently and for what purposes secondary mathematics teachers in seven technology-rich suburban high schools used computers. The case studies involved two schools with teachers being interviewed concerning their beliefs about computer use, two lessons being observed and teachers being re-interviewed about aspects of the lesson. Following a technology intervention phase which had the researcher providing a professional development session on the use of the computer program, *Maths Helper*, and teaching a class using this software, teachers were asked to reflect on the students' learning and the effectiveness of the lesson. Finally, after data analysis, case study subjects were asked to comment upon the researcher's interpretations of their beliefs about the potential of computers. The findings showed that the teachers in this study, mostly specialist mathematics teachers, rarely used the available computer resources particularly if they taught junior classes (Years 8–10) or less able senior students. Norton suggests that this deprived these senior students of the potential to use computers to enhance conceptual understandings and to learn industry-linked skills such as using spreadsheets. Most teachers recognised the calculation potential of computers but few had views of computers as useful in developing students' understandings of mathematical concepts.

Mathematics, Learning and Teaching

Perry, Howard, and their colleagues (Conroy & Perry, 1997; Howard, Perry, & Lindsay, 1997; Perry, Foong, & Conroy, 1996; Perry, Howard, & Conroy, 1996; Perry, Howard, & Tracey, 1998, 1999) have reported extensively on teachers' use of manipulatives as well as their beliefs about mathematics, learning mathematics, and teaching mathematics. Data were collected using a specifically designed questionnaire consisting of both multiple-choice and open-ended questions. A total of 939 responses (603 primary and 336 secondary) was obtained.

As a result of an initial analysis of the collected data on primary teacher beliefs, Perry, Howard and Conroy (1996) suggested that there was a complex interconnectedness between teacher beliefs; the grouping of belief statements could help lead to the development of profiles for the respondents. A categorisation of *transmission* and *child-centred* beliefs was proposed.

Howard, Perry, and Lindsay (1997) focused on secondary teacher beliefs about the learning and teaching of mathematics. A correlational analysis was applied to the beliefs data. They concentrated on the correlations of two apparent contrasting statements, viz:

- The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.
- Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge.

Significant positive and negative correlations with other belief statements were identified. In pursuing the notion of profiling teachers the terms *transmission* and *constructivist* were used in trying to locate teachers along a beliefs continuum.

Perry, Howard, and Tracey (1998) used beliefs data obtained from the 603 primary teachers. Two factors—Transmission and Child-Centredness—were identified through a confirmatory factor analysis. Using these two factors as a basis, the researchers reported on the relationships between these results and the data obtained on the use, by these teachers, of manipulatives in mathematics learning and teaching. It was found that primary teachers who used materials:

- *for each mathematics lesson* also reported beliefs that were both child-centred and less transmission oriented than those primary teachers who did not use manipulatives for each mathematics lesson
- *when they were available* were significantly more transmission focused than those who did not;
- *as part of each lesson* were significantly less transmission focused than those who did not; and
- *as the focus of the lesson* were both significantly less transmission focused and more child centred than those who did not.

Tracey, Perry, and Howard (1998) compared teacher beliefs about the learning and teaching of mathematics on the two confirmed factors. Comparison of beliefs was undertaken for the demographic variables of school sector (government/Catholic), location (rural/urban), school level, and gender of respondents. It was found that:

- Catholic school teachers scored higher on child-centredness than government school teachers;
- there was a higher level of child-centredness among some rural teachers than those teaching in some urban areas;
- primary teachers score higher on child-centredness than secondary teachers;
- female teachers score significantly higher on child-centredness than male teachers;
- male teachers score significantly higher on transmission than female teachers; and
- there were no significant differences across school level, sector or location on the transmission factor.

The espoused beliefs of 40 Head Teachers of Mathematics in Australian secondary schools obtained through the use of a beliefs survey (Tracey, Perry & Howard, 1998) was the focus of a study reported in Perry, Howard, and Tracey (1999). The Head Teacher beliefs were compared with those of classroom mathematics teachers in the same schools and with recent mathematics education reform documents from Australia and the USA. Interviews with eight of the Head Teachers who responded to the survey provided further data for comparison. The study reinforced the use of two factors of Transmission and Child-Centredness as a categorisation of practising teachers' espoused beliefs about mathematics, mathematics learning and mathematics teaching. As well, it was found that:

- Head Teachers of mathematics scored significantly higher than other mathematics teachers on the Child-Centredness factor and significantly lower on theTransmission factor; and
- classroom mathematics teachers may feel the pressure to conform to tradition even more than their curriculum leaders in school.

Buzeika (1996) reported on the relationship between beliefs and practice as perceived by three primary classroom teachers interviewed whilst implementing a new mathematics curriculum. The teachers perceived their beliefs shifting as a result of practice. However, they acknowledged that for shifts to take place they needed to access an overriding belief that educational practice was not fixed and that children's performances could suggest new approaches and ways forward. They also believed that they may be unaware of personal beliefs affecting their practice. The teachers acknowledged that a new curriculum had increased professional dialogue regarding the teaching of mathematics but, for classroom practice to change, teacher confidence was seen as a major factor.

Teacher-Student Interpersonal Behaviour

Rickards and Fisher (1997) and Fisher and Rickards (1998) reported on the teacher use of a Questionnaire on Teacher Interaction (QTI) to assess perceptions of their own teacher-student interpersonal behaviour and how such assessments can be used to reflect on teaching practice. They addressed questions such as:

- Are students' perceptions of teacher-student interpersonal behaviour the same as their teachers?
- Is there a difference between mathematics teachers' perceptions of their actual teacher-student interpersonal behaviour and what they believe to be ideal?

The QTI, originally a model for teacher interpersonal behaviour, was modified by the researchers for the Australian context. This was the first time that the QTI had been centred on mathematics classes as previous studies had focused on science classes. The study involved 405 students from Years 8, 9 and 10 in nine schools and their 21 teachers. In applying the instrument to students and teachers, comparisons can be made about actual and ideal perceptions of student-teacher interpersonal behaviour. The study confirmed the reliability and validity of the QTI for mathematics classes. The results suggested that students' attitudes were higher in classes where the students perceived greater *leadership* and *helping/friendly* teacher behaviours and lower where *dissatisfied, admonishing* and *strict behaviours* were perceived.

Enacted and Espoused Beliefs

Through the use of an ethnographic case study methodology, Shield (1999) explored the relationships between the espoused beliefs and the classroom practices of one secondary mathematics teacher. It was found that while the teacher had clear and strongly held beliefs about mathematics and its teaching and learning the classroom practices did not always reflect these beliefs. This study begins to focus on the complexities involved in differences between espoused and enacted beliefs.

Tirta (1999) described the cross-cultural aspects of an Australian and an Indonesian secondary mathematics teachers' beliefs and their effect on his/her teaching under the constraints and supports provided by the social context of teaching. A qualitative case study approach using an ethnographic framework was adopted to gather the data. Tirta presents an in-depth description of the social context of the two teachers. Outcomes of the study supported the strong influence of a teacher's beliefs on his/her teaching practices, and added further strength to the view that teacher classroom actions are not always consistent with espoused beliefs. The cultural comparison of the social, political and educational contexts of the two teachers is informative in considering the complexities of mathematics classrooms in two different cultural settings.

An investigation of Year 1 student teachers' performance on a ratio task linked to their espoused beliefs about mathematics, mathematics learning and mathematics teaching was reported by Conroy and Perry (1997). The study involved 178 Australian and 78 Indonesian prospective primary teachers. They found that there were significant differences to be found between the national cohorts in espoused beliefs and approach to the ratio task but not correctness of solutions. However, there appeared to be little relationship between espoused beliefs, approach taken to the ratio task, and correctness of solutions either within the two national cohorts or between them. This paper followed a similar analysis by Perry, Foong, and Conroy (1996) which also considered data from a Singaporean cohort of student teachers.

Assessment

Warren and Nisbet's (1999) study explored 387 primary teachers' beliefs about the uses of assessment and their actual assessment practices. The data were collected through a questionnaire. Analysis indicated that a wide variety of assessment methods were being used. Observations, practical work, oral testing, investigations, and assigned homework were the most commonly used methods. Teachers viewed the central uses of assessment as a way of informing teachers of student problems, evaluating teaching effectiveness, and as a means of public accountability. Overall, assessment was predominantly being used to provide feedback to the teacher rather than the learner or parents. There appeared to be a relationship between teachers' beliefs about the uses of assessment and the frequency of the use of various assessment techniques.

Student Teacher Attitudes

Roberts, Cretchley, and Harman (1998) used the Fennema-Sherman Mathematics Attitudes Scale (1976) to measure changes in two groups of Year 1 Bachelor of Education (early childhood and primary education) student teachers' attitudes to mathematics in relation to confidence, effectance motivation (a measure of attitude to problem solving) and usefulness. The course unit involved in the study was not a curriculum unit, was taught by mathematicians and explored mathematical ideas and experiences. The scale was administered at the beginning and end of the semester. There were no significant changes in attitudes for the confidence or effectance motivation scales across the whole sample (135 respondents) nor when the data were examined by gender, age and course. The only significant finding was a drop in the Bachelor of Education (Early Childhood) students' perception of the usefulness of mathematics.

Values

Little research has been reported in the area of values within mathematics teacher education. Southwell (1997) provides a theoretical framework for discussing the links between teacher beliefs and values while incorporating the cultural dimensions emphasising ideological, sociological, sentimental and technological elements (White, 1999) and the three key elements of the mathematics teaching/learning act—the teacher, students and curriculum. Southwell maintains that values evident within teacher education and thus mathematics teacher education can influence society. She cites the areas of: learning to live, learning to do, learning to be, and learning to live together. A range of research methodologies including questionnaires, lived-experience anecdotes, life histories, reflective journals, and interviews were used to investigate beliefs and attitudes. The challenge is to undertake mathematics research into values which makes sense of mathematics education in a multicultural society.

Shield (1999) and Tirta (1999) explored links between espoused and enacted teacher beliefs. The interviewing of teachers and students to elicit espoused beliefs followed by classroom observing to identify enacted beliefs will be a research method that has significant implications for the professional development of teachers. Southwell's (1997) study should be a forerunner to a greater emphasis on investigating the role of people's values within the mathematics classroom.

Mathematical Knowledge of Practising and Prospective Mathematics Teachers

For the past decade at least there has been a concentration on the processes associated with teaching mathematics and much less emphasis on the actual content knowledge of teachers of mathematics at the early childhood, primary, and secondary levels. This lack of emphasis is reflected in the dearth of research in this area in the first three years of this reporting period. One exception is the study of Clarkson (1998a) which shows that many primary student teachers are not "wholly in charge of what would appear to be reasonably straight forward mathematics that they certainly will be expected to teach" (p. 174).

In 1999, several more studies have been reported. The favourite topic for research in the area of mathematical knowledge has been statistics and probability. This is understandable in light of the increased use of statistical techniques in many activities of everyday life and in the introduction of probability into the primary mathematics syllabi of several states of Australia. As this is a relatively new area for teachers to include in their teaching program, research that explores current knowledge and other aspects of teaching in the area of statistics and probability is designed to assist teachers to make the transition that is needed in new curricula.

Statistics and Probability

Watson and Moritz (1997a) recognised the growing significance of statistics and probability in school curricula and developed a profile-instrument for measuring teacher' reactions to new teaching areas such as Chance and Data. While this project was intended to be an avenue for professional development for teachers, the development process provided data on the current knowledge and beliefs of the teachers taking part. The profile was developed in several sections, including general comments, confidence, background, the teaching of chance and data, chance and data concepts, survey items and media survey items, professional development and reaction to student ideas. Several items were included in each section, and, in some sections, items were duplicated from a previous study with school students. The analysis of the responses of the 47 teachers in the trial indicated that both primary- and secondary-level teachers had some difficulties with some of the concepts involved, with the primary teachers having more difficulties than the secondary ones. Following this trial, the profile was revised and greater emphasis put on pedagogical knowledge.

Watson and Moritz (1997b) also reported an investigation into the concept of sampling with over 100 teachers who responded to a questionnaire asking what they thought of when they heard the word "sample" and to briefly say how they would teach the topic. As a result, 12 different ideas that teachers associated with sampling were listed. *Representative* was the most popular (47%), followed closely by *part of 'something' bigger* (45%). Responses in general appeared to indicate that some of the teachers in the sample were only just beginning to realise the degree of importance of the concept of sampling. Very few made reference to random sampling, and the respondents seemed to have not made the connection between sampling and the validity of statistical inference.

In the area of statistics, although with university students proceeding to a non-teaching degree, Chick (1999) investigated the statistical appropriateness of conclusions drawn by this group. She used a data cards protocol in which she supplied the subjects with a set of fictitious data concerning 16 school students. This process revealed considerable weaknesses on the part of the majority of the students, particularly in the number of unsupported conclusions and the tendency to overgeneralise.

Teachers' understanding of the concept of average and their levels of functioning on a set of questions testing this understanding were reported by Callingham (1997). She analysed the teachers' responses using the Biggs and Collis SOLO Taxonomy and found evidence of multi-modal thinking among both preservice and inservice teachers. That is, the teachers adjusted their level of thinking to the problems.

Geometry

Using van Hiele levels as her descriptors, Lawrie (1996) has shown, with a sample of 60 prospective primary teachers, that many of them who had completed senior high school geometry could not display higher than Level 2 understanding. She suggested that this could be a cause for concern.

Chinnappan, Lawson, and Nason (1999) used the notion of concept mapping to investigate teachers' knowledge of geometry. Their subjects were five novice teachers and five experienced teachers and they found it to be an effective method for the purpose. They described a particular method of analysing concept map data and applied it to a particular case. Their approach resulted in the generation of information on both quantitative and qualitative features of the teacher's knowledge-base in geometry. A related study by Chinnappan and Thomas (1999) provides a framework for the investigation of the structure of the content knowledge of teachers. This framework was trialed with an experienced teacher in New Zealand in the topic of functions with sixth form students (17 years of age). It emphasises schematic knowledge and has the potential of being an effective research and teaching tool.

Number

Six primary student teachers were tested for number sense by Kaminski (1997) using a set of examples derived from previous work of Reys (1991) and

Sowder (1989). The students' lack of confidence and reliance on written calculations suggested an underdeveloped number sense. This highlights the need for mathematics teacher education to stress the number sense aspect in their courses.

A study directly relating teachers' knowledge and students' knowledge is that carried out by Kanes (1999). A teaching episode in a Year 11 class on complex number was videotaped and the tape transcribed. Kanes then carried out a detailed epistemological analysis of the teaching episode and concluded that the teacher and the students draw on a knowledge base that is peripheral to the mathematical content base of each.

The increase in the number of studies on mathematical content knowledge that has occurred in the past year seems to indicate that mathematics education researchers are tending to give more time to analysing the mathematical content knowledge of teachers than has been the case earlier in the four years reported in this review. This in turn could suggest that greater emphasis than previously is being placed on content in school curricula. As new topics and varying emphases confront curriculum developers and classroom teachers, mathematical content knowledge of students at all levels could be the subject of further research.

Preservice Mathematics Teacher Education

Almost all the mathematics education researchers in Australasia are involved with preservice mathematics teacher education. Hence, it is no surprise to find that a considerable amount of research has been generated in this area. Of course, much of it could have been categorised into other sections of this chapter, particularly those dealing with the mathematical knowledge of teacher education students and affective issues. Nevertheless, what is reported here does deal with preservice mathematics teacher education.

Participatory Action Research—Beginning Teachers

Atweh and his group of researchers and teachers from the Queensland have been quite active over the last four years in the area of the transition of beginning teachers, especially women, into their roles as teachers of mathematics (e.g., Atweh & Burnett, 1997; Atweh, Harris, Garrett, Pitman, & Sitton, 1997; Atweh & Heirdsfield, 1998; Ginns, Heirdsfield, Atweh, & Watters, 1997; Suhrbier, Moman, Fitzgerald, & Ginns, 1997). Using a basis of participatory action research, the team investigated innovative and incisive ways supporting the transition. They reported on two action research cells, one considering assessment in mathematics education and the other inclusiveness. They focused on the issues that formed challenges for the beginning teachers, all of whom were women, the support needed by these teachers, and the appropriateness of participatory action research in providing this support. However, in spite of this support from the research participants, the beginning teachers often reported feeling isolated within their teaching situations, and stressed the importance of involving more experienced teachers, especially some with authority in the research cells.

Reflective Practice among Preservice Teachers of Mathematics

The question of the impact of preservice teacher education programs on the mathematics teaching practices of graduates of these programs has been addressed by numerous researchers. Hill (1997), for example, investigated the strength of a particular mathematics methods program to ascertain if it "significantly facilitates preservice elementary teachers' willingness to learn and teach mathematics for relational understanding" (p. 211). Through an ingenious collection of approaches, she showed, with a group of preservice teachers, that relational understanding was valued if it could be attached sensibly, by the student teachers, to their personal goals. This result reinforced for Hill the position that teacher education could be a valuable intervention.

Hill's (1997) approaches were focused around the reflective practice of the student teachers. Similarly, Frid, Redden, and Reading (1998; Redden, Frid, & Reading, 1997) investigated reflective practices among preservice teachers and how they can be developed through involvement in peer and self assessment activities within mathematics methods subjects. Working with 120 final year primary-school student teachers in a seminar presentation approach, they found that, in general, the student teachers failed to implement self-reflection in their personal strategies for teaching and assessment. Further, like Hill (1997), the issue of the student teachers valuing the goals of reflective practice was seen to be critical to their adoption of it into their practice. Since this adoption was not apparent in the Frid, Redden, and Reading study, encouraging reflective practice in preservice teacher education may be a weak as intervention strategy (see also, Bobis & Cusworth, 1997).

Sullivan and Mousley (e.g., Mousley & Sullivan, 1997; Sullivan & Mousley, 1996a, 1996b, 1996c, 1998) have also studied reflection by student teachers of mathematics as part of their work involving interactive multimedia approaches to preservice mathematics teacher education. They used pedagogical dilemmas as a central feature of their multimedia resource and have found that these have assisted student teachers to develop understandings of the complexities of mathematics classrooms and the roles that teachers play in them. By encouraging opportunities for autonomous learning among the student teachers, they have developed aspects of reflectiveness among these preservice teachers, particularly in the quality of language and thinking which was used in the interpretation of teaching events. Hence, the use of the multimedia resource enhanced student teachers' knowledge of and attention to some key characteristics of quality mathematics teaching.

Poststructural Analysis

This question of the strength of preservice teacher education programs in encouraging change in teachers' practice has also been addressed by Klein in a collection of related papers (e.g., Klein, 1996a, 1996b, 1997, 1998a, 1998b). In these papers, she used a poststructural analysis to investigate preservice teacher education practices, particularly those arising from a constructivist theory of knowledge. The active undermining of more investigative approaches to mathematics learning and teaching was revealed through patterns of subjectification among preservice teachers which Klein (1998b, p. 301) suggests "we ignore ... at our peril".

Klein's work brought mathematics education researchers face-to-face with poststructural analysis as she worked through the careful development of her students as teachers of mathematics. She introduced the notion of *teaching-as-usual* and questioned whether teacher education can expect to break preservice teachers' penchant for returning to such teaching without such a poststructural analysis. The themes of Klein's work running from her doctoral thesis (Klein, 1996a) through to her work with practising teachers (Klein, 1999) are well summarised by the following.

> If these teachers are to teach in investigatory ways, they will need to recraft their eyes to recognise the limitations of teaching-as-usual: to see how filling in worksheets, hearing tables, doing irrelevant problems where there is one correct answer, streaming, practising algorithms and formulae, position the student as always unknowing and the teacher as sole authority. Furthermore, because these classroom practices are all premised on the regulatory establishment and maintenance of constructed binaries of right/wrong answers and competent/incompetent students they effectively undermine any investigative impulse. (Klein, 1998b, p. 301)

Problem Solving

Taplin (1996, 1998) investigated the strategies used by primary-school student teachers as they endeavour to solve problems in mathematics. She found that they tended to use a narrow range of mainly verbal and numerical strategies and that this use was quite stable despite efforts to increase their flexibility of approach. These results confirm those of Conroy and Perry (1996) in their study of student teachers' solutions to ratio problems.

As the need for mathematics teachers becomes even greater in Australasia, it is to be expected that research into ways to facilitate their preservice teacher education will also proliferate. At both the primary and secondary school levels, a firm foundation of investigation has been set. This will continue to bear fruit in both the national and international arenas.

Professional Development in Mathematics Education

As the teaching force in Australasian schools ages, so the need for professional development increases. While this need is recognised, government funding for such professional development has dwindled over the period 1996–1999. Most professional development is run by the major employing authorities and is targeted towards the particular programs which they wish to introduce, although the Australian Association of Mathematics Teachers and its state-based affiliates has been increasingly active. In many cases, university mathematics teacher educators are involved in the planning, implementation, and evaluation of these programs, resulting in the opportunity to provide a research base for the activities. Some of this research is reported below.

Technology in Professional Development

As mentioned above, the development of the profile-instrument in the area of Chance and Data by Watson and Moritz (1997c) provided professional development for teachers who took part in the development of the profile and also in its use. Another very significant study in which Watson was involved concerned the development, delivery, and evaluation of professional development to teachers across Australia using information technology. This was the Learning the Unlikely at Distance Delivered as an Information Technology Enterprise which became known as LUDDITE. Various aspects of this study have been reported by those involved in it. Watson, Baxter, Olssen, and Lovitt (1996) provided information on the methodology and some of the issues that were encountered in carrying out the project. Watson (1996) presented personal reflections on the videoconferencing that became part of the study, the feedback from the sessions, and the hypertext documentation for the package. Watson (1997) reported on the project and supplied details of the three phases into which the study was divided. Watson and Moritz (1997a) described the actual package as a multimedia CD-ROM package on Chance and Data, gave an overview of the materials included in the package and reported the results of the feedback sought on the package. Watson and Baxter (1997) explored the development and research of the expanding use of technology in teaching and learning statistics. They provided a description of the development and structure of the produced materials. The model they developed to investigate the effectiveness of the professional development package was implemented and evaluated. Both qualitative and quantitative data were obtained and combined to form a profile of teacher attitude, competence and behaviour as related to the SOLO Taxonomy. The researchers make the point that teacher change needs to be monitored in both the short term and the long term. Watson (in press) provides a comprehensive report on the project with the emphasis on professional development and statistics, the rationale being that statistics is the area of mathematics in which teachers could well feel the greatest need of assistance. Watson (1998) gave a briefer account of the project with the emphasis on the statistical aspects. It is obvious that this study in professional development was a major and significant one. It was a collaboration that produced very effective materials for professional development and also had the facility to generate data that can enlighten researchers in the investigation of a variety of problems.

Another use of technology to provide professional development was that reported by Mayo and Hannah (1999). They used audiographic equipment to develop a learning network among teachers in isolated areas. The project linked four schools and a college of education. They drew on the metaphor of a loom as the institutional structures in which they were working, with weaving being a metaphor for the interactive sessions with the teachers. Four patterns emerged from the interactions. These four patterns were the links between the parts of the network that has developed, the dynamics within the group, the evolving protocols, and the links between theory and practice.

Authentic Assessment and Multimedia

Herrington, Herrington, Sparrow, and Oliver (1998) and Herrington and Herrington (1998) use the term *authentic assessment* to include a range of new approaches to assessment that are more practical, realistic and challenging than the traditional paper-and-pencil assessment tests. They describe a qualitative study that was designed to investigate the effectiveness of authentic assessment within a multimedia learning environment. The subjects were prospective primary and secondary mathematics teachers using a range of assessment strategies for K-12 mathematics. As a result of analysis of the data gathered from observation and interviews, the researchers report that the findings of their study suggest that authentic assessment can be used successfully within an interactive multimedia learning environment.

Maths in Schools: A Professional Development Project

Another professional development study that had significant effects in Victoria and perhaps in other states is the Maths in Schools program. This was established by the Mathematical Association of Victoria and involved key teachers in schools and a university mathematics education lecturer. Smith (1996) reported the development of the Maths in School study in one particular school and describes the methods that were employed by the staff to ensure the results were made public to all staff. A case study of the school was developed to provide an evaluation of the effectiveness of the program. Tytler, Smith, Grover, and Brown (1996) compared the professional development models for teachers of primary science and mathematics. Data were collected through questionnaires and interviews but the return rate for the questionnaires was less than 50%. The data when analysed did, however, indicate that the program focused on whole class issues and that the task focus involving all staff was one of the positive outcomes. Key teachers reported frustration because of differing levels of commitment among the staff and the view that the program did not constitute professional development so much as a task that would not lead to advancement. Smith, Grover, and Tytler (1997) provided a further report of the program, this time emphasising case studies of two schools using a Chance and Data curriculum. The use of the program NUD·IST not only strengthened the research aspect of the program but also enabled the researchers to synthesise the perceptions of the teachers in relation to Chance and Data. These included (a) the novelty of Chance in the curriculum, (b) the topic was not always viewed seriously, (c) teachers lack confidence in the topic, and (d) there is always more to learn. Smith (1997) returned to his earlier case study of a school and delt in detail with various forms of reporting of the program. He developed a schema linking the various methods and concluded that the method chosen depends very much on the audience for which it is intended.

Count Me In

Wright (1996) drew attention to the need for teachers to have a sound knowledge of how children form concepts in mathematics. In his open-ended study, his subjects were 33 teachers in equal numbers from three states and they were all interviewed. They were all teaching the first year of school. From transcripts of the interviews, categories were determined, modified, and used for the final analysis of the transcripts. More than half the group said that children form concepts through experiences with number, while other relatively significant ideas (20%–30%) were the role of language, prior learning, counting and materials.

The teaching and learning program Count Me In, devised by Wright and his colleagues, has been evaluated in numerous ways. Bobis and Gould (1998) reflect on the strengths of the program as professional development for teachers, suggesting that teachers generally benefited from the program, both professionally and personally.

Action Research

Bobis (1998) advocated an action research model for professional development through partnerships between schools and universities. She described the possibilities and then the example of a specific school-university partnership in which a university lecturer worked intensively with the teachers to help them undertake action research projects to improve the quality of learning taking place in their classrooms. Feedback on this project was given by one of the teachers and by the lecturer so that both sides of the interaction could be identified. She then included a useful section in which she presented a schematic representation of a model of collaborative action research and her own model of such research within a school-university context. The concept of teachers-as-researchers was recommended as a fruitful method of professional development.

Another professional development program, this time used by the Catholic Education Commission of Victoria Numeracy Strategy Project and reported by Gervasoni (1999), used an action research model. She provided case studies of three teachers who have benefited from the program. Two were teaching K/1 classes and the other one was teaching Grades 3/4. From the data collected, Gervasoni was able to build a table of key insights gained about early childhood learning.

Using a sample of five secondary teachers from the Philippines, Gallos and Herrington (1997) have shown that an action research program brought benefits to the teachers in the form of noticeable changes to their pedagogical knowledge, teaching practices and beliefs about mathematics learning and mathematics teaching. They suggested further that action research may be an answer to issues of professional development in developing countries such as the Philippines.

A Projected New Model of Professional Development

Sparrow and McIntosh (1998) reported on progress in the *Developing Number Sense in Education* (DENSE) Project that they have initiated. The project is descriptive in nature so the data gathering strategies were teacher journal writing, audiotaping of group meetings, field notes, and lesson observations by the researchers. They concluded that the research and development model of professional development that they are advocating has raised a numbers of issues such as impediments to change and unrealistic expectations, but has also provided evidence of a new awareness on the part of the teachers of their own beliefs, teaching strategies, and the learning strategies of the children.

Vocational Education

On a more vocationally oriented approach, FitzSimons (1997a), in her editorial for the report of the ICME8 Working Group on Adults Returning to Study Mathematics, reviewed the studies that were reported briefly in the Working Group including several Australian contributions. Participation at national and international conferences is a form of professional development that is not always recognised as such. FitzSimons (1997b) also explored the professional development needs of teachers in the Vocational Education and Training sector in Victoria. She made the points that it is a complex area and vulnerable in terms of political changes.

Innovation

Several others have reported studies from different perspectives. Rowley, Brew, Leder, Ryan, and South (1996) explored innovation in the senior secondary mathematics curriculum through the voice of the teacher. The study was set within the context of curriculum innovation for the Victorian Certificate of Education, the final examination for senior secondary students. Changes in assessment saw students undertaking common assessment tasks (CATs) related to an investigative project, problem solving, skills and applications and students' analytical abilities. Forty teachers from 40 schools (state, Catholic and independent) were interviewed in this study. In implementing the changes "many teachers are convinced that their classroom practice has changed, and has changed for the better" (Rowley et al, 1996, p. 8), though some teachers had opted out by teaching the middle secondary years and others were giving only superficial support to the innovations. A surprising finding was that the changes to Years 11 and 12 assessment were impacting on both assessment patterns and the classroom practices of Years 7–10. Poor experiences of inservice education, extra curricular activities and academic workloads were factors impacting upon teachers' willingness to be involved in out-of-school professional development.

Motivation

Galbraith, Haines, and Pemberton (1999) reported the construction of six attitude scales and the response data from tertiary students in London and Brisbane. These data suggested a strong link between confidence and motivation with both mathematics in general and computers in particular.

Curriculum

Brew, Leder, and Rowley (1999) surveyed approximately 500 senior mathematics teachers concerning changes in the Victorian Certificate of Education (VCE) to determine whether the changes in the senior work had brought about a similar change when the same teachers were teaching junior classes. They found that the recent modifications to the VCE continued the erosion of flexibility and choice that students previously had. They placed more emphasis on a centralised assessment system, thus providing little as a medium of professional development for teachers.

Teacher Action Theories

White (1999) advocated the use of the "Theory of Planned Behaviour" to explore teacher action theories and used this theory to assist 115 primary teachers to reflect critically on their classroom practice concerning the use of group work. He found that the majority of teachers intended to use group work and that attitude was the strongest contributor to these intentions. The high-intent group in the study believed, among other points, that group work would be unlikely to place too great a demand on resources and that group work would increase the efficient organisation of resources. As might be expected the low-intent teachers believed the opposite.

Conflict Situations

Sullivan (1999), while reporting research carried out in school classrooms, suggested that his findings have implications for teacher educators. These implications concerned the responsibility of teacher educators to confront their students with the kind of conflict situations they were most likely to meet in schools. In this way, the prospective teachers may be better equipped to handle situations as they arise in school.

Because professional development is no longer funded liberally by governments, it is essential that what funding is available is provided for the most effective programs. Consequently, research in this area of professional development contains the promise of ensuring teachers are provided with the professional development most suited to their needs.

Mathematics Teaching as a Sociocultural Construct

Many Australasian mathematics classrooms comprise students from culturally diverse backgrounds. Studies within these cross-cultural settings, though challenging, provide a wealth of opportunity to delve into the complexities involved in the learning and teaching of mathematics. There is a number of Australasian mathematics education researchers who have investigated the social and cultural contexts of mathematics teaching.

Non-English Speaking Backgroundd

A new construct, titled Non-Anglo Cultural Background (NACB) was used to report on a two-year study of Years 7 and 9 mathematics students (Bishop, Leder, Brew, & Pearn, 1997). This construct identified students who were born into a non-English speaking home, whose first language is not English and "whose observable characteristics and language usage in class suggested that they should be called NACB" (Bishop et al, 1997, p. 98). Questionnaires, videotaped lessons, and interviews with both students and parents were used to gather data. The report focused on the student views about mathematics, attributions of success and failure, classroom environment, and student perception of their level of achievement. It was found that the NACB students involved in this study were "more confident, less anxious, and more at ease with their perception of their own abilities" (Bishop et al, 1997, p. 100). The students felt that their teachers rated them higher than Anglo-Cultural Background (ACB) students and that they attributed their success to their own abilities and efforts. From this study it appears that being from a non-English culture may not necessarily be a handicap when learning mathematics. In discussing this work the authors acknowledge the importance of undertaking work in cultural aspects of mathematics but pay witness to the complex nature and inherent challenges in such mathematics education research.

Clarkson (1998b) referred to concerns in discussing cultural perspectives and the import of teachers factoring into their mathematics classroom presence what pupils actually assess as worthwhile in doing mathematics. He drew on examples from Papua New Guinea and the many immigrant children in Australian mathematics classrooms to highlight the differing values that can occur in the mathematics classroom. He reflected on the classroom role of the teacher in meeting differing cultural perspectives towards the learning of mathematics. There are differences between children in their learning of mathematics from varying backgrounds but also between children with the same cultural backgrounds. Clarkson raised the notion of valorisation in that children may choose various ways to complete a problem based on personal experiences that they considered worthwhile; for example, "Mum showed me this way"; "the Arabic algorithm is better"; and "the words are easier to understand if I translate it into Vietnamese". Clarkson wondered how teachers might respond to such situations, particularly if what teachers "value is their beliefs in action, and hence on display for their pupils" (1998b, p.134). He believes that when teachers recognise their importance in the classroom and appreciate the perspectives of the children learning mathematics "we [teachers] will to some extent be recognising that we are teaching children through the vehicle of mathematics" (Clarkson, 1998b, p. 134).

Gender

Forgasz and Leder (1996) reported a study which "aimed to examine the relationship between a set of affective variables identified as critical in explaining gender differences in mathematics learning outcomes and aspects of the mathematics classroom learning environment" (p. 154). The study was set within the theoretical framework of Fennema and Peterson's Autonomous Learning model and involved one class of Year 7 students from each of 35 Victorian coeducational post-primary schools in Part 1, and one class of Grade 7 mathematics students from each of two coeducational secondary schools in Part 2. In each class the teacher selected two male and two female students for intensive study. In Part 1 of the study, questionnaires were used that sought student responses to their views about mathematics, beliefs about their actual and desired mathematics achievement levels and perceptions of their mathematics classroom environments. Part 2 saw the same questionnaire administered with additional data being collected through self-report questionnaires, interviews with targeted students, teacher comments and videotaped records of 14 sequential lessons.

Several teaching and learning implications for classroom consideration were identified from the data analysis. Individual students who thought teachers were interested in them as individuals (Personalisation), who felt participation was promoted (Participation) and investigation skills emphasised (Investigation) were more likely to have functional beliefs about themselves as mathematics learners.

It was found that the masculine image of mathematics remained implicated in gender differences in learning outcomes. Classroom factors such as teachers' and classmates' behaviours, learning activities and their contextual settings, learning settings, assessment strategies and competition might individually, or in combination, influence student beliefs. This study, undertaken in secondary schools, points towards the need for such investigation to occur in primary-level learning environments to "provide vital clues about the factors shaping student beliefs which exhibit gender differences at high school" (Forgasz & Leder, 1996, p. 170).

Forgasz (1996) reported the first phase of a three-year study which investigated gender related factors contributing to students undertaking tertiary-level mathematics study. Semi-structured interviews with 23 students (16 male, 7 female) were conducted. The sample size restricted any generalisations though variables identified for further investigation included school attended, Year 12 mathematics background, parental educational levels, and the beliefs and attitudes of academic staff and students.

Ocean (1996) draws upon a literature overview to provide one answer to the question of why more women do not continue with mathematics at post-compulsory levels. It is suggested that the *moral climate* of the classroom may well be an influencing factor. Classrooms that are seen as supportive by women encouraged discussion, group work and cooperation while those that have a climate of the traditional—silent and rule-based—were seen as harmful. This may well be a factor that mathematics educators need to consider as a reason why women avoid traditional mathematics education.

Aboriginal Learners

Howard (1996; 1997a; 1997b; 1998) reported on an ethnographic study that focused on giving voices to Aboriginal children, their parents and their Aboriginal educators. The task for Aboriginal children in learning mathematics is daunting. Issues raised included the reality of Aboriginal children having to think in two ways, "to talk in a way that was acceptable in the school context often different to the languages they used in the home and the community" (Howard, 1998, p. 16) and that the children have to live, work and learn within two cultures. This is often in a context where teachers "do not have the knowledge or experience to work with Aboriginal children" (Howard, 1998, p. 16). Aboriginal parents, educators and children raised the issues of "language, relationships, identity and specific pedagogical aspects including fun, teacher consistency and relevant mathematical experiences as critical elements for consideration in Aboriginal children's learning of mathematics" (Howard, 1998, p. 17). The important issues of collaborative research and open conversations regarding pedagogical issues and possible solutions were addressed.

Equity Issues

Tims and Parker (1996) used four dimensions suggested by Rossman: technical (professional knowledge and skills); cultural (beliefs, values and school norms); moral (matters of justice and fairness); and political (matters of authority, power and influence). These were used to distil an operational definition of equity in the contest of reform. In their analysis, they focus on two large scale statewide mathematics education reforms implemented in Ohio and Western Australia. They conclude that unless certain crucial aspects from these four dimensions are carefully considered and acted upon the goals of equity will be lost in the reform.

Zevenbergen (1997) addressed the question of whether disadvantaged students fail mathematics or mathematics fails disadvantaged students? In doing so, she urged mathematics educators to consider the social context in which mathematics teaching and learning takes place for children living in poverty. The challenge of systemic critique was presented to challenge processes which in the past have excluded and marginalised disadvantaged students. Through such a research focus there can be a greater impact on "the social justice outcomes of education" (Zevenbergen, 1997, p. 28). A stronger emphasis on the social context of the learning and teaching of mathematics can lead to a critique of the systemic constraints which currently lead to differential outcomes for learners.

Clearly in multicultural countries such as Australia and New Zealand, cultural issues are of paramount importance to students learning mathematics. At last, it appears that the importance of this area of mathematics education is reflected in the mathematics education research being undertaken.

Conclusion

The chapter celebrates a tremendous volume of work by Australasian mathematics education researchers in the important area of mathematics teacher education. The research reported on teacher beliefs about mathematics, the mathematical knowledge of teachers, the constraints that impact upon preservice mathematics teacher education, the importance of the role of continuing professional development and the importance of acknowledging the sociocultural contexts in which learning and teaching mathematics occurs suggest that the findings emanating from these studies will impact upon the lifelong teacher education of mathematics teachers at all levels. The incorporation of these findings to enhance teacher education programs for these teachers is an essential follow-up. This is a continuing challenge for tertiary mathematics teacher educators as well as other professionals involved in these programs.

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The Context of Mathematics Education

Gender and Mathematics: Changing Perspectives

Helen J. Forgasz
Deakin University
<forgasz@deakin.edu.au>

Gilah C. Leder
La Trobe University
<g.leder@latrobe.edu.au>

Colleen Vale
Victoria University of Technology
<colleen.vale@vu.edu.au>

Introduction

Our aim in writing this chapter is to review Australasian research on gender issues in mathematics education over the period 1996–1999. Changing societal forces, more general educational concerns, and theories and perspectives from other disciplines have clearly impinged on the directions taken and the methodological approaches used by the researchers and teachers who have communicated their findings in various scholarly and professional publications. In particular, we recognise the impact of *feminist* scholarship[1] on the field of gender and mathematics and have examined the research reports through different *feminist* lenses.

We begin this chapter with an overview of the work reported in earlier MERGA publications in order to set in context contemporary research in the field. Many of the themes and headings used to cluster and discuss earlier research findings also characterise the corpus of work during the period 1996–1999. However, new themes were also evident including: technology (computers and calculators) and mathematics education, as well as issues related to the tertiary level.

An additional dimension incorporated in this chapter is a snapshot summary of media coverage of pertinent issues. As was illustrated by the 1980's advertising campaign, Maths Multiplies Your Choices (Baker, 1989), the media are particularly powerful shapers of public opinion (The Research Business Pty. Ltd., 1989). In the present emotionally charged climate in which boys' educational disadvantages have been highlighted, it seemed critical to explore how the media are conveying messages about gender issues related to mathematics education. The Melbourne newspaper, *The Age*, was searched for relevant articles published during 1996–1998 using the terms "gender" and "mathematics". Selected but representative excerpts are given in a later section.

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[1] The feminist perspectives used in this review chapter are described more fully in Leder, Forgasz, and Solar (1996), Kaiser and Rogers (1995), and Noddings (1990), and in general feminist discourses within the field of *Women's Studies*.

Overview of the Earlier MERGA Research Summaries

The first MERGA sponsored review of Australian research in mathematics appeared in 1984, in time to be launched at the Fifth International Congress on Mathematical Education (ICME-5) hosted in Adelaide. Its successful reception among the international mathematics education research community encouraged the production of regular four-year reviews to coincide with successive ICME conferences. Perusal of these volumes (Briggs, 1984; Blane & Leder, 1988; Atweh & Watson, 1992; Atweh, Owens, & Sullivan, 1996) illustrates the evolution and development of research in mathematics education generally. Inspection of the chapters concerned with gender and mathematics (Leder, 1984; Barnes, 1988; Leder & Forgasz, 1992; Barnes & Horne, 1996) confirms not only the extent to which possible gender differences in mathematics learning have persistently attracted widespread attention, but also highlights consistent and changing aspects which have engaged those involved in this area of research.

The first review. The first review (Leder, 1984) covered work published between 1977 (the year of MERGA's first conference) and 1984 and aimed to document the scope and nature of Australian work on mathematics and gender over that period. The works fell readily under the headings of literature review (15 articles), performance data (10), participation data (21), possible explanations (33) and intervention studies (21). Documenting performance and participation differences then current, exploring likely contributing factors, and assessing the effectiveness of selected intervention strategies succinctly described the scope of research and scholarly activities with a focus on gender and mathematics in the early 1980s. Gender differences in mathematics learning, documented in so many of the publications surveyed, were typically assumed to be the consequences of inadequate educational opportunities, social barriers, or biased instructional methods and materials. It was further assumed that the removal of school and curriculum barriers, and if necessary the resocialisation of females, would prove to be fruitful paths to achieve gender equity. The studies were characterised by a largely uncritical acceptance of the male norms of performance, participation, and approach to work as optimum and standards to be attained by all students. Females were assumed to have the same characteristics, motivations, and educational aspirations as males. Females were considered deficient. They were to be encouraged and helped to assimilate. Consistent with the tenets of liberal feminism, interventions were aimed at bringing the achievements of females up to those of males.

The second review. The broad categories selected by Leder (1984) were largely retained, with some further refinements, by Barnes (1988) in her review of work published from 1984 to early 1988. Once again the bulk of the research effort could be categorised as possible explanations and intervention studies; there were 24 publications in both categories out of a total of 74 works. There were relatively few attempts to document gender differences (5 studies were listed under performance data; 9 under participation data). Literature reviews (8) and bibliographies (4) completed the list of works reviewed.

Briefly, girls as a group were still perceived as deficient. With appropriate support from those in authority, they were, however, assumed capable of attaining standards set by males. The notion of *women as a problem in mathematics* (Kaiser & Rogers, 1985) still served as a catalyst for much of the work carried out by those concerned with gender and mathematics learning.

The third review. Leder and Forgasz (1992) were responsible for the third review of Australian research, published in the years 1988 to 1991, and concerned with mathematics and gender. They produced integrated summaries of research foci—rather than listings of annotated bibliographies—but again largely relied on the headings used in the earlier reviews.

The *assimilationist* and *deficit model* approaches which characterised so much of the work carried out during the 1980s still guided many of the intervention initiatives aimed at achieving gender equity. Monitoring of males' and females' participation and performance rates in mathematics subjects continued. At the same time, different voices were beginning to be heard. What factors, beyond gender, might be contributing to gender differences in mathematics education? Should we accept, uncritically, the way in which mathematics was being taught and valued? Should young women strive to become like young men or should we instead celebrate the formers' goals, ambitions, and values? What, crucially, might differentiate between a single-sex and co-educational school or class environment? Should we aim for uniformity or can different be equal? In which setting might mathematics be taught and learnt more effectively by males and females? Should we accept only those conditions and approaches favoured by males? Elsewhere (Kaiser & Rogers, 1985) such concerns have been described as indicative of attempts to make women more *central to mathematics* and of *mathematics* being *reconstructed.* Yet trends in mathematics education cannot be considered in isolation. The impact of broader societal issues cannot be ignored.

> The recessed economic climate in Australia in recent times has impinged on education in a number of ways. Funding for new initiatives, and continued support for ongoing projects have been curtailed. Programs that allow for individual differences and that focus on the process and not merely the product of learning are severely challenged by politicians' increasingly vocal advocacy of national curricula and testing programs. In this period of high youth unemployment and increased competition for tertiary places, arguments that sound mathematical qualifications lead to open educational and career

> opportunities are developing a hollow ring. Concerns have begun to be expressed about the continuing emphasis on encouraging females to aim for the still male-dominated spheres of science and mathematics. (Leder & Forgasz, 1992, p. 89)

How the mathematics education research community dealt with such challenges can be gauged from the next section.

The fourth review. The review of Australian work published between 1992 and 1995 (Barnes & Horne, 1996) covered some 150 articles. Clearly, interest in the issue of gender and mathematics continued to be high. Persistent focus on certain areas, changing directions for others, can be gauged from the main headings selected by the two reviewers: reviews of research, theoretical perspectives, participation of women and girls in mathematics, gender differences in mathematics performance, psychological studies of the learner, learning environments and classroom discourse, the impact of the wider society, and policy statements and policy studies. According to Barnes and Horne (1996):

> The last four years have seen some shifts in the focus of the research on gender and mathematics, which may result in part from the widespread public perception that much has been achieved. Compared with research reported in Leder and Forgasz (1992), there has been more analysis of participation and performance data from, for example, the monitoring of government policies. There is also a greater tendency for researchers on a variety of topics to report data by gender, although much remains to be desired in this respect. In addition there has been more research on the social context of mathematics learning, more focus on analyses and critiques of government policies, some attempts to develop new perspectives on gender equity, and less psychological explanations of gender differences. (p. 76)

The more critical attempts to analyse and deconstruct explanations for gender differences in mathematics learning, and clearer recognition that different perspectives inevitably lead to differences in the ways in which the interventions aimed at challenging inequities are framed, are noteworthy developments characteristic of this period. The attempts already noted in the third review to make women more central to mathematics and of exploring the reconstruction of mathematics accelerated and diversified. The assumptions of *liberal feminism* that discrimination and inequalities faced by women are the result of social practices and outdated laws rather than due to pervasive power structures imposed by men for men, the unquestioned acceptance of male norms, the assumption that females aspire to these standards and modes of behaviours, and its presentation of a deficit model of womanhood in which girls and women are positioned as victims with deficit aims and desires were no longer deemed sufficient or necessary explanations by some researchers. Instead, evidence emerged among some researchers of a desire to settle for nothing less than fundamental changes to society. Advocates of this approach, often classed as *radical feminists*, consider that the long term impact of traditional power relations between men and women more broadly, and in mathematics, can only be redressed through such means.

Attempts to explore the interaction between gender and other background variables—for example, socioeconomic status and cultural affiliation—began well before the years reviewed by Barnes and Horne but intensified during this period. Increasingly, attempts were made to gauge the impact on mathematics learning of both patriarchal and class domination and to recognise the multiple effects in any interventions planned to redress inequities. Thus the concerns of *social feminists* voiced in the community at large, that females from working class backgrounds are often particularly disadvantaged in the home, in the labour force, in access to leisure pursuits, affected work in mathematics education. Sincere efforts were made to mirror as comprehensively as possible the complex web of factors—personal, situational, and social—which might shed light on issues of gender and mathematics. Just which factors were actually incorporated in such a complex model was invariably influenced by the personal beliefs and theoretical orientation of the researchers undertaking the work. No longer do we simplistically assume that the planning, execution, reporting, and interpretation of research are value free.

It is against this broad background that the fifth review of work concerned with mathematics and gender should be read.

International Reviews and Special Journal Issues

On the international scene, Australasian researchers have contributed widely to research reviews, books, journals, and conference working groups specifically focused on gender issues. This section begins with an overview of these contributions. Several of the publications mentioned are reviewed in more detail later in this chapter.

The chapter in the *International Handbook of Mathematics Education* (Bishop, Clements, Keitel, Kilpatrick & Laborde, 1996) on gender issues was written by Leder, Forgasz, and Solar (1996). These authors provided an overview of recent international research and intervention programs, emphasised cultural dimensions, covered traditional research approaches to investigations of gender inequities, and examined the growing feminist literature in the field.

In 1996, the journal, *Focus on Learning Problems in Mathematics*, published a special issue entitled *Gender and Mathematics: Multiple Voices*. Articles by Barnes (1996) and Forgasz and Leder (1996a) were included. A special issue of the *Mathematics Education Research Journal* on *Gender and Learning Settings* was published in 1997. It included articles from several Australian researchers FitzSimons (1997); Leder and Forgasz (1997c); Rennie and Parker (1997) and Gill (1997). In the editorial, Forgasz (1997b) commented on emergent issues in Australasia as the gender gap narrows for mathematics learning outcomes, and there is a growing perception that boys are now the educationally disadvantaged group with respect to mathematics education.

Publications Resulting from Conferences and Working Groups within Conferences

A book edited by Hanna (1996) was an outcome of the International Commission on Mathematical Instruction (ICMI) study, Towards Gender Equity in Mathematics Education which was held in Höör, Sweden in 1993. Selected papers from the conference were expanded, including several contributions from Australasian researchers Clark (1996), Forbes (1996), Gaffney and Gill (1996), and Leder (1996a). These contributions are not reviewed here as the research was published in Grevholm and Hanna (1995) and reported by Barnes and Horne (1996). Owens (1998) summarised Australasian research on gender issues for an international conference in Sweden in 1996. Australasian researchers were also well represented in the International Organisation of Women in Mathematics Education (IOWME) sessions at the 1996 International Congress on Mathematics Education (ICME 8) conference, held in Seville, Spain. A book of extended abstracts of the papers presented at the IOWME sessions was produced (Keitel, 1996) and summaries of papers by Australasian authors are shown in Table 1. Several of the authors extended their contributions which were later published in an edited collection (Keitel, 1998).

Table 1.
Summary of papers by Australasian researchers in IOWME sessions at ICME 8 published in Keitel (1996)

Author(s), title, page numbers	Brief summary of paper
Blithe, T. M., Chamberlain, M. C., Clark, M. J., & Forbes, S. *The continuing need to monitor gender differences* (pp. 72–73)	Participation and performance data in upper secondary and early tertiary mathematics courses in New Zealand are presented.
Brew, C., Pearn, C., Leder, G., & Bishop, A. *Why do females under-rate their performance? Big fish resizing themselves in the school pond.* (p. 50–55)	Findings from a study exploring gender and cultural conflicts experienced by high mathematics achievers in Grades 7 and 9 in Australia are described.
Ellis, J., Miller-Reilly, B., & Pfannkuch, M. *Ways in which some women came to influence mathematics education in New Zealand.* (p. 67)	Comments on the important role played by an equity network in New Zealand.
Forbes, S. *Gender and assessment.* (p. 86)	Reflects on research directions to understand patterns of gender difference in mathematics performance: females

Author(s), title, page numbers	Brief summary of paper
	outperform males on internal assessments, vice-versa for external assessment.
Forgasz, H. *Gender equity issues: High school and tertiary mathematics learning environments.* (pp. 34–38)	Findings from two studies revealed that males remain advantaged by high school and tertiary mathematics learning environments.
Johnston, B. *Mind and body: Alienation, rationality and pleasure.* (pp. 24–25) [See also Johnston, 1998]	Provides the framework for a study with women about the learning of mathematics that drew on the *memory-work* method adopted by German women studying sexualisation.
Leder, G. C., Brew, C., & Rowley, G. *Gender differences in mathematics achievement here today and gone tomorrow?* (pp. 60–63)	Describes a study which examined whether student performance in Victorian Certificate of Education (VCE) mathematics courses is affected by the format of the assessment tasks and whether the nature of the task contributes to gender differences.
McMurchy-Pilkington, C. *Positioning of Maori women and mathematics: Constructed as non-doers.* (pp. 41–44)	Discusses a study of Maori women reflecting on their alienating experiences of learning mathematics. An on-going study of Maori teacher trainees is also described.
Sukthankar, N. *Gender and mathematics: A study of female students in Papua New Guinea.* (p. 93)	Argues that the cultural and social context of Papua New Guinea are factors which affect the involvement of females in science and technology studies.
Willis, S. *Perspectives on social justice, disadvantage and the mathematics curriculum: A view from Australia.* (pp. 20–21)	Describes recent personal experiences which illustrate how Australians understand the relationships between social justice, disadvantage, and the mathematics curriculum, as well as the conflicts that arise.
Willis, S., & Johnston, J. *Is it possible to base systemic curriculum reform on principles of justice?* (pp. 67–69)	The philosophy underpinning the Western Australian mathematics curriculum and the authors' support of its social justice principles are described.
Wood, L. *A longitudinal study of sex differences at the higher*	Summarises the trends in mathematics performance in the final year of high school

Author(s), title, page numbers	Brief summary of paper
school certificate in NSW Australia. (pp. 93–94)	examinations in NSW. Females outperform males at all levels except the highest.
Wood, L., & Coupland, M. *What happens when the girls beat the boys? Community reactions to improved performance of girls in final school examination in NSW, Australia.* (p. 28)	Describes the media's reactions to changes in the calculation of tertiary entrance scores from final year high school examinations in NSW. It is claimed that the cry that boys have been disadvantaged is misplaced.
Zevenbergen, R. *Gender, mathematics and the media.* (p. 29)	Press reactions to final year high school examination results in Eastern Australia are described. Common trends were discerned: boys' performance is poorer than girls' and boys are now considered the educationally disadvantaged group.

Books

Two main themes were explored in *Gender, Science and Mathematics*, edited by Parker, Rennie and Fraser (1996): (a) research examining the relationships between gender, science and mathematics, and (b) the gulf between the ideas enshrined in policies related to gender equity in science and mathematics education and their implementation. The book is divided into three sections: "Confronting Perceptions and Attitudes", "The Reality of Schools, Classrooms, Curriculum and Assessment", and "From Policy to Practice—Building on Experience". Australasian mathematics education researchers made important contributions to the book. Willis (1996) discussed four perspectives on the relationship between mathematics curricula, gender-based disadvantage, and gender justice—remedial, non-discriminatory, inclusive, socially critical—as a framework for approaching gender issues in the curriculum. Habermas' framework of three *knowledge-constitutive* interests—technical, practical and emancipatory—was used by Johnston and Dunne (1996) to categorise strands of research on gender and mathematics/science. Questions were raised about the assumed understandings of gender which underpin the purposes of the research in each category. Leder (1996a) presented findings from various data gathering methods used in her studies of mathematics classrooms. Observed gender differences in teacher-student interaction patterns and in students' perceptions of themselves as learners of mathematics were discussed in relation to the traditional pedagogical methods adopted in the observed classrooms.

A biographical dictionary of *Notable Women in Mathematics* (Morrow & Perl, 1998), describes the lives of 59 women, many still living, whose contributions to mathematics and mathematics education have been recognised.

The stories of three Australian women, ex-patriot, Leone Burton (Morrow, 1998b), Gilah Leder (Forgasz, 1998a), and Cheryl Praegar (Morrow, 1998a), are told.

Research Studies

The issues that stimulated initial attention to gender-related inequities in mathematics education included findings of gender differences favouring males in mathematics performance levels and course participation rates. Clearly working from a liberal feminist perspective, the ultimate goal of those concerned with the persistent patterns observed over the years was that females should attain the same levels as males. Early explanations for the findings were linked to research revealing gender differences favouring males in a range of critical variables within the affective domain. Long-time researchers in the field have urged future generations to continue to monitor performance and participation data patterns as well as measures of the critical affective dimensions. Changing patterns serve as indicators of greater equity in the field. A summary of contemporary findings in these traditional areas of research interest is presented next.

Participation and Performance Data

In their detailed analyses of Australasian students' performance in the Third International Mathematics and Science Study (TIMSS), Lokan, Ford and Greenwood (1996, 1997) reported findings by gender. International comparisons were also reported for overall performance on the multiple choice test component. No statistically significant gender differences in mean performance scores were found for Australian or New Zealand students among either the junior secondary students (TIMSS Population 2—predominantly 13-year olds) (Lokan et al., 1996) or the middle primary students (TIMSS Population 1—predominantly 9-year olds) (Lokan et al., 1997). Australian junior secondary girls' mean performance score was slightly higher than boys', but middle primary boys' mean score was slightly higher than girls'. In New Zealand, the gender pattern was reversed, that is, junior secondary boys performed slightly better than girls, and middle primary girls performed slightly better than boys. Among the countries for which significant gender differences were found, males scored higher than females in all cases.

Australian and New Zealand TIMSS data were also examined by mathematical content area (Lokan et al., 1996, 1997). No statistically significant gender differences were noted for either the junior secondary or middle primary cohorts in Australia. However, in New Zealand significant differences favouring girls in the middle primary cohort were found for three of the six content areas: data representation, geometry and patterns, relations and functions. The TIMSS instruments also included five mathematical performance assessment tasks that were undertaken by smaller samples of students from both the middle primary and junior secondary populations. Lokan, Ford and Greenwood (1996) were very cautious in reporting the findings which showed one statistically significant difference favouring girls for the dice activity in the Australian junior secondary sample. No significant gender differences were found for these activities in the Australian middle primary sample (Lokan et al., 1997). For the junior secondary cohort, Afrassa and Keeves (1999) re-examined Australian data from the earlier International Mathematics Studies as well as the TIMSS. They noted that there had been a shift in direction of higher mean achievement scores from boys in the first study (FIMS) to girls in the TIMSS.

Barnes (1997) reported findings from a detailed examination by gender of data from the NSW Basic Skills Testing Program (BSTP). "Only small differences between boys and girls in overall performance at primary school level" (Barnes, 1997, p. 58) were found. When the data were disaggregated, some findings were consistent with previously reported gender patterns while others challenged them. The need for girls and boys to have "equal opportunities to learn" was emphasised and the inevitability of the link between contextual settings for mathematics problems and gendered patterns of differences in life and classroom experiences was noted. A detailed gender analysis of the 1992 Victorian Certificate of Education (VCE) mathematics Common Assessment Tasks (CATs) results was conducted by Cox (1996). In general, females were found to outperform males on the two school-based CATs; the converse was the case for the more traditional examination CATs. Males studying two mathematics subjects greatly outnumbered females. When the performance data were grouped by number of mathematics subjects taken, gender comparisons revealed improved female performance. The author claimed that an understanding of participation differences was needed to appreciate fully the variations in the patterns of gender difference in performance. He concluded that females were less disadvantaged in VCE assessment than in previous mathematics courses and that their greatest disadvantage lay in their low participation rates compared with males'. Cox (1996) argued that males needed assistance to improve their written mathematical communication skills.

The format of the VCE mathematics examinations since 1992 allowed comparisons to be made of students' performance on different tasks. More specifically, comparisons could be made between the performance of groups of boys and girls on rigidly timed examination papers to be completed under supervision out of class and on longer term projects to be completed in- and out-of-class time. Leder, Brew, and Rowley (1999) reported that, on average, boys performed better than girls on the examination tasks but girls did better than boys on the longer-term, and more open-ended tasks. The nature of assessment, they concluded, can affect apparent gender differences in mathematics performance.

Findings from a longitudinal analysis of performance on the Australian Mathematics Competition (AMC) from 1983–1992 were reported by Taylor, Leder, Pollard, and Atkins (1996). Entries in the AMC had increased over the period, more so for females than for males. In 1992, there were more females than males entering the competition from Grades 7–10. Participation numbers from Grades 11 and 12 still favoured males but the gap was closing. Males continued to outperform females and the gaps in performance levels increased with student age. However, female performance on the AMC overall had shown steady improvement over the ten year period. When performance data were examined by various content-related categories, the decrease in the gap between male and female performance was evident in all categories except for questions designated as algebraic for which the opposite trend was noted.

Findings from another longitudinal study, based on survey responses from students as they progressed through Grades 7–12 (Jones, Porter, & Young, 1996), serve as a timely reminder of the links between participation and performance data and the affective domain. Jones et al. (1996) found that males and females had poor perceptions of their mathematics performance early in high school but that this improved as they progressed through school; the percentage of students with negative views reduced from a majority (over 70%) in Grade 9 to a minority in Grade 10 (around 40%). Inconsistent patterns of gender difference were also noted. A higher percentage of boys than girls in Grades 7, 9, and 11 were more negative about their perceptions of performance; the direction of the gender difference was reversed in Grades 8 and 10. Proportionally fewer females were found to have elected to take higher level studies in mathematics at Grade 12. Although performance data for the 1994 Higher School Certificate (HSC) for the South Coast region of NSW revealed that girls slightly outperformed the boys, the authors maintained that their findings challenged the view that females had made it in the fields of mathematics and science. They cautioned that other factors, including ethos of school, whether it was co-educational or single-sex, public or private, were also implicated.

Brinkworth (1999) investigated explanations for the observed decline in Grade 12 mathematics enrolments and the persistent, although slightly decreasing, gender differences in participation rates favouring males. For boys, course requirements were a strong influence in deciding to pursue mathematics; both boys and girls had rejected mathematics because of the lack of creativity and self-expression.

In summary, findings from studies published in 1996–1999 in which the participation rates and performance levels of males and females were examined suggest that there are fewer and smaller gender differences favouring males than were evident in the past. In liberal feminist terms, there now appears to be greater gender equity with respect to these outcomes of mathematics education. This is not to say, however, that equity has yet been fully achieved.

The Affective Domain

Models explaining gender differences in mathematics learning outcomes favouring males incorporate a range of affective variables including: (a) confidence, (b) perceived usefulness of mathematics, (c) attributions for success and failure, and (d) the stereotyping of *mathematics as a male domain*. In many studies reviewed in other sections of this chapter, the researchers included affective variables in the research design. The few studies in which the researchers focused primarily on issues related to mathematics and the affective domain are presented here.

Norton and Rennie (1998) used the Fennema Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) to explore for gender differences in attitudes towards mathematics (anxiety, confidence, effectance motivation, attitude towards success, and mathematics as a male domain) across different grade levels and different learning settings—coeducational and single-sex schools, public and private schools. Students' attitudes were found to be less positive in more senior grades and boys, overall, had more positive attitudes than girls. With the exception of the mathematics-as-a-male-domain scale for which girls were less stereotyped than boys, only small gender effects were noted. For all variables, grade and school type effects were also very small. The authors advocated different data collections focusing on students' voices to understand better the interactions between gender, school environment and affective dimensions.

Mathematics as a Male Domain

Forgasz, Leder, and Gardner (1999) maintained that societal attitudes about the gender-appropriateness of the field of mathematics appear to have changed. They argued that items comprising the widely used mathematics-as-a-male-domain subscale of the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) were anachronistic and that interpretations of responses to them were no longer valid, a view shared by Norton and Rennie (1998). Findings from a new instrument designed to tap the extent to which mathematics is stereotyped as a gendered domain were reported by Forgasz, Leder, and Barkatsas (1998a, 1998b). Data were gathered from Grade 7–10 students. Response patterns to most items indicated that the majority of students did not gender-stereotype mathematics. However, some consistency with previously reported perceptions of mathematics as a male domain were found. Along other dimensions, earlier findings did not hold up. Apparent shifts in beliefs from boys to girls being perceived as more competent mathematically and of girls now being seen as more likely than boys to persist with difficult mathematics problems were noted. As in the past, boys were considered the main distractors in class, girls were believed to be more likely to help others, and males were regarded as more likely than girls to pursue careers involving computers and other mathematics-related endeavours. Further examination of the data revealed that ethnic/cultural backgrounds appeared to influence the stereotyping of mathematics along some of the dimensions explored (Forgasz, Leder & Barkatsas, 1999). Data gathered in Singapore was compared to data from Australian students. Singaporean students' views were more consistent with the traditional patterns of beliefs reported in the earlier research literature (Forgasz, Kaur & Leder, 1999; Forgasz, Leder & Kaur, 1999a, 1999b). Singaporean males held the most traditionally stereotyped views.

Peard and Wiley (1997) cautiously postulated that changing female perceptions of mathematics as a male domain may partially explain the findings of their research. Their sample comprised a small number of low-ability Grade 8 and 9 mathematics students (10 male, 7 female) in a withdrawal program. The students' motivations to do mathematics and their attributions for their mathematical difficulties were explored. Although most students disliked the mathematics and were negative about it, there were no gender differences in attitudes, liking of the subject, or desire to improve. Mathematics was perceived as important and useful by all of the students. Boys were more likely than girls to attribute their difficulties to lack of effort.

Only 20 years ago, gender analyses of mathematics textbooks revealed extreme gender-stereotyping to the extent that "girls could get the idea that mathematics is mainly for boys and about male activities" (Victorian Committee on Equal Opportunity in Schools, 1977, p. vi). More recently, Forgasz (1996a, 1997a) conducted a study of the extent of gender stereotyping in one Grade 7 mathematics textbook. Students in Grades 6–9 were asked to identify the gender and nationality of the 13 cartoon characters appearing in the textbook and to give reasons for their choices. Although about equal numbers of the characters were clearly thought to be boys or girls, the boy characters appeared more frequently in the book. Forgasz (1997a) concluded that there were small lingering vestiges of inequity evident in the book but that progress had "been made in challenging gender stereotypes and in recognising the ethnic diversity of our community" (p. 136).

Affect, Mathematics and Other Subject Areas

In several studies, affective dimensions related to mathematics and other subject areas have been explored. Watt (1996) found that among students in Grades 7 and 11, more boys than girls perceived themselves as most talented in mathematics, sciences and technical subjects; for humanities and arts subjects the gender pattern was reversed. More boys than girls also planned to pursue mathematically-related careers. Students' beliefs about themselves as learners of mathematics and of Language Arts (ie. first language studies—English, in Australia], their attributions for success and failure in each subject, and the extent to which the subjects were gender-stereotyped were explored by Forgasz and Leder (1996a) and Forgasz, Leder, and Taylor (1996). In both studies the same instrument was used but the samples differed. Data from Australian Grade 9 students at one school revealed that males had more functional attribution patterns (likely to lead to future success) than females towards mathematics, and that the reverse held for English (Forgasz & Leder, 1996a). Males were more likely than females to stereotype mathematics as masculine, and males and females agreed equally that English was a female domain. Although many students considered that there was no difference between men's and women's abilities in mathematics and in English, more males than females thought that men were better than women at mathematics and about the same proportions of males and females agreed that women were better than men at English. Data from Sweden, the USA and Australia revealed similar patterns (Forgasz, Leder, & Taylor, 1996) although there were some variations by country. It was concluded that "stereotyping was more evident for LA [Language Arts] than for mathematics in all three countries [and that] ... the emphasis in recent years on gender issues in mathematics and science, but not in LA, may partially account for this finding" (Forgasz, Leder, & Taylor, 1996, p. 11).

Affect and Performance

In describing the affective and cognitive behaviours of a small group of five Grade 7 students (3 female, 2 male) as they worked on a realistic mathematics task over an extended time period, Leder and Forgasz (1997a, 1997b) aimed to infer each student's likely long-term commitment to mathematics. Other self-report data sources included questionnaires and individual interviews, with a second round of interviews when the students were in Grade 12. Gender differences were noted in the students' cognitive and affective engagement in the classroom. The boys were more involved with the computational aspects of the task; the girls paid more attention to peripheral activities such as presentation. Attitudes inferred from classroom behaviours were fairly consistent with those revealed in the self-report data and with the students' Grade 12 mathematics choices and ultimate tertiary destinations. Aware of the small sample size, the researchers were cautious in their interpretations of the data and made clear their recognition that a range of other factors may also have contributed to the longer term outcomes.

Thomas and Kota (1996) explored the correlations between various affective variables and algebra test scores among Form 3 and 4 students in New Zealand. Anxiety and self-concept were found to be positively correlated to girls' problem-solving performance, especially among the younger girls. For the older boys only, self-concept was positively related to performance. In a later study with students of the same ages, Kota (1997) divided the groups into high- and low-achievers. A gender difference in the patterns of correlations was found. For high-achieving boys there were high correlations and for low-achieving boys the correlations were low; the pattern was reversed for girls.

Interactions of Gender and Social Background Factors

There is growing recognition that attitudes and beliefs may be influenced by social background factors including socioeconomic status (SES) and cultural/ethnic influences.

Lamb's (1997) survey of students in Grades 10–12 at four high schools revealed that boys were advantaged over girls by the separation of senior level mathematics into academic and non-academic subjects. The more positive views developed towards mathematics by boys during their junior high school years were thought to influence their choices of the more academic mathematics subjects. Yet, girls were not all equally disadvantaged. Those from middle-class backgrounds tended to retain earlier confidence levels and interest in mathematics which lead them towards the most demanding mathematics options. Lamb (1997) maintained that overall gender effects had been offset by the girls' social backgrounds.

Grade 7 and Grade 9 students from four schools (two high- and two low-SES) with predominantly non-English speaking background enrolments, who may have been experiencing conflicts in mathematics in terms of gender, ethnicity or class were the focus of a study reported by Pearn, Brew, Leder, and Bishop (1996). Data sources included self-report questionnaires on attitudes towards mathematics and perceptions of the classroom, videotaped lessons, and interviews with targeted students and their parents. Earlier findings on gender differences in attitudes towards mathematics were corroborated. Some school differences were found, attitudes varied by year level, and low- and high-achieving students perceived their classrooms differently. In one school, students reported more teacher support. The authors claimed that further research on "the importance of a caring and supportive school environment, especially for students from other cultural backgrounds" (Pearn et al., 1996, p. 452) was needed.

The experience of Greek girls in a working class school was the focus of Polesel's (1997) study. Mathematics and science were perceived as the most valued curriculum areas in the school but the Greek girls compared with Greek boys (both English-Speaking Background [ESB] and Non-English-Speaking Background [NESB] Greek boys and girls included) had the lowest confidence measure. In this context, Polesel argued, gender and ethnicity combined and resulted in particularly poor outcomes for these girls, who were identified as the most disadvantaged on a range of indicators. Polesel (1997) noted that "certain characteristics of Greek culture, which value and support the education of boys at the expense of girls, combine with school-based gender stereotypes to undermine the confidence and expectations of these Greek girls, particularly in the gender sensitive domain of mathematics" (pp. 41–42).

Summary

Research on affective variables reveals some changing patterns of gendered beliefs and attitudes. Although some inconsistencies were apparent, there is growing evidence that females' beliefs are increasingly more functional than in the past. Findings from explorations of the interactions of gender with other sociocultural variables (e.g., culture/ethnicity and socioeconomic status) suggest that the latter may have greater impact than gender alone.

Teachers and Classroom-Related Factors

Classroom-related factors have been central to several studies exploring gender issues in mathematics education. Findings from interviews with nine final year preservice teachers were reported by Atweh and Burnett (1997). Most of the women had perceived themselves to be high mathematics achievers up to Grade 10. None had completed higher mathematics in their senior secondary years. Based on their own experiences, three women questioned research evidence on gender differences in mathematics learning experiences and outcomes that they had encountered while at university. The authors argued that these women had failed to recognise that the reasons behind their decisions not to persist with high school mathematics were particularly relevant to females.

Haynes (1996) examined issues affecting the provision of equitable mathematics classroom programs by beginning teachers whose training had focused on constructivist principles. Three factors were identified: (a) the teachers' beliefs about mathematics and its teaching; (b) the mathematics curriculum and its underlying philosophy; and (c) the process of socialisation into the school culture as it affected their survival as teachers.

Based on responses to the Self-Description Questionnaire Relich (1996) interviewed a small number (n=16) of male and female primary and secondary teachers, with extreme measures of high and low mathematical self-concept. The survey revealed gender differences in attitudes towards mathematics, towards the teaching of mathematics, and towards self as a significant role model among primary but not secondary teachers. This was put down to the likelihood of males having studied more higher level mathematics than females. At interview, mathematical self-concept formation, perceived role of the teacher, beliefs about mathematics and mathematics learning, and gender-related issues in the treatment of students were explored further. The teachers were fairly consistent in declaring that boys and girls were equally capable, but some alluded to differences in attitudes and learning styles. "[T]he two primary female low self-concept teachers clearly expressed fairly stereotypical views about girls' attitudes to mathematics" (Relich, 1996, p. 189). Relich (1996) concluded that some teachers with very low mathematical self-concept may provide inappropriate role models for students.

Classroom Environments and Learning Settings

Focusing on students' learning through group discussion, Gooding's (1997) study included observations of single-sex groupings of four students engaged in a card placing task. The group dynamics in the boys' and the girls' groups were found to differ. The boys' style hindered some boys' participation and learning; the girls' style, on the other hand, facilitated the learning of all group members.

An extensive study of discourse patterns among secondary school mathematics students was conducted by O'Halloran (1996). Analyses of videotapes of four Grade 10 mathematics classes revealed that boys in independent single-sex schools had the greatest participation and access to mathematical discourse; females in similar schools participated in interpersonal discourse patterns which, it was claimed, were not mathematical.

Australasian researchers have used instruments measuring classroom learning environment dimensions in mathematics classrooms in Australia and elsewhere. In primary mathematics classrooms in Singapore, Goh and Fraser (1996) found that girls generally viewed their classrooms more favourably than did boys, although performance differences had been found to favour boys. Western Australian and Tasmanian secondary students comprised the sample in the analyses conducted by Rickards, Fisher, and Fraser (1996) and Fisher, Fraser, and Rickards (1997). It was found that females perceived their teachers in a more positive way than did males, and that students from Asian backgrounds tended to perceive their teachers more positively than students from other cultural background groups.

Exploring the relationship between affective measures and students' perceptions of classroom learning environments was the main aim of a study conducted at two levels (Forgasz & Leder, 1996b). Large scale surveys were complemented with in-depth studies of two Grade 7 mathematics classrooms. Data sources included: self-report data from all students; videotaped lessons focusing primarily on two boys and two girls; and interviews with the four targeted students. Classroom observations revealed overt and some more subtle instances of gender stereotyped behaviours by teachers and students. Classroom factors which might influence beliefs and attitudes were identified from detailed analyses. Included factors were "teachers' and classmates' behaviours, the learning activities in which students engage and their contextual settings, learning settings, assessment associated with learning activities, and an emphasis on competition" (Forgasz & Leder, 1996b, p. 169). It was considered that these factors provided partial explanations for the gender differences in attitudes and beliefs which were generally consistent with previously reported findings.

Summary

Research on teachers and learning settings reveals that gender remains an issue with respect to teacher behaviours and beliefs, the learning settings in which they teach, the learning environments they develop, and the differential effects these can have on the students they teach.

Single-Sex Settings

Several studies examining single-sex classes within a secondary co-educational school milieu have been reported (Draper, 1997; Keast, 1996, 1997a, 1997b, 1998, 1999; Leder & Forgasz, 1997c, 1997d, 1997e, 1998a; Marsh, & Rowe, 1996; Norton & Rennie, 1998; Parker, & Rennie, 1997; Rennie, & Parker, 1997). These interventions took place in secondary schools, both private and public, in Victoria or Western Australia at the Grade 8, 9, or 10 level. Even though there were some differences in the purposes of the intervention programs, and in the design of these evaluations studies, a number of strikingly common findings emerged.

The goal of each of the single-sex programs was the increased participation of girls in post-compulsory secondary mathematics. In most programs this was expected to be achieved through providing a safe and secure environment for girls (Keast, 1996, 1997a, 1997b, 1998, 1999; Leder & Forgasz, 1997d, 1997e; Marsh & Rowe, 1996). However two programs also focused on inclusive pedagogy to enhance student learning (Draper, 1997; Parker & Rennie, 1997; Rennie & Parker, 1997). The studies were either longitudinal (Keast, 1996, 1997a, 1997b; Leder & Forgasz, 1997d, 1997e, 1998b; Marsh & Rowe, 1996), or concerned samples drawn from more than one school (Norton & Rennie, 1998; Parker & Rennie, 1997; Rennie & Parker, 1997). As a consequence the research designs and the research instruments varied between these studies.

Rennie and Parker (1997) reported on the perceptions of students of their learning environment. Marsh and Rowe (1996) and Keast (1997a, 1997b 1998) compared effect sizes in mathematics achievement/performance and attitudes/confidence. Rennie and Parker (1997), Parker and Rennie (1997) and Keast (1998, 1999) reported teachers' experiences of teaching single-sex classes. Keast (1999) discussed learning styles as possible explanation for contradictions between the results of an earlier experimental study and qualitative data collected. Leder and Forgasz (1997c, 1997d, 1997e, 1998b) reported on parents' and teachers' perceptions of single-sex classes in addition to gender differences in students' attitudes and their levels of enjoyment of the single-sex learning environment. In general these studies found that in single-sex settings, gender differences in student attitudes were insignificant with the exception of the "mathematics as a male domain" scale on which boys reported more gender stereotyped views than did girls, and that student preferences for single-sex settings declined over time. Rennie and Parker (1997) and Leder and Forgasz (1997e, 1998b) found that girls more than boys believed that the single-sex setting was more supportive or more enjoyable. However, Marsh and Rowe (1996) claimed that the limited support for single-sex classes seemed to be stronger for boys than for girls.

The Educational Disadvantage of Boys

The emergence of concerns for the educational disadvantage of boys was evident in the single-sex studies. Rennie and Parker (1997), Parker and Rennie (1997) and Keast (1998) reported that teachers found the single-sex boys classes to be more challenging and became concerned about developing classroom management strategies and teaching strategies that would address the needs of boys in their classes. Keast (1999) found the feminist theory of *connected and separate knowers* useful in explaining the observed behaviour of students in the single-sex classes and their apparent preference for particular types of mathematics activity. However, not all students were observed to conform to particular styles. High achieving girls preferred similar activities to boys. Teachers' reflections on gender differences in approaches to learning in the single-sex settings (Leder & Forgasz, 1998b; Keast 1998, 1999) were transferred to their mixed classes (Keast, 1998, 1999; Parker & Rennie, 1997; Rennie & Parker, 1997). Rennie and Parker (1997) and Leder and Forgasz (1997e) reported parents' concerns that single-sex classes were disadvantaging their sons. Leder and Forgasz (1997e) also found that there was a decrease in parental support for single-sex classes over time. Interestingly, some parents believed that the single-sex classes were designed to improve the outcomes for their sons and claimed benefits for their sons. The shift in focus from girls to boys in parent opinion about the purpose and benefits of single-sex classes, and the focus of teachers' classroom innovations on engaging boys in the learning of mathematics reflect the backlash against feminist inspired interventions for girls, and the growing concern of the educational outcomes of boys reported and fuelled by the media during the period of the current review.

Collectively, these studies imply that single-sex settings may not suffice as an intervention strategy for improving attitudes, participation, and achievement for female and male students. The studies collectively call into question the usefulness of single-sex settings as an intervention strategy on their own. Issues associated with gender equity in mathematics learning are complex (Leder & Forgasz, 1997c, 1997d). Simplistic solutions, such as single-sex classes per se, do not appear to have been successful in themselves in achieving equity and there have been calls for new strategies to be explored (Leder & Forgasz, 1998b; Keast 1999). Epistemology, pedagogy, and parents' perceptions remain important factors.

Tertiary Students

There has been some research on gender issues with respect to mathematics at traditional universities and in the vocational educational and training (VET) sector. With respect to the latter FitzSimons (1997) argued that mathematics in the VET sector does not meet the interests of students or employers and that gender equity is not seriously addressed.

Findings from a three-year study exploring factors influencing male and female students' decisions to study tertiary level mathematics at three Victorian universities were reported by Forgasz (1996c, 1997c, 1998b) and Forgasz and Swedosh (1997). A small pilot study had revealed that the backgrounds of male and female tertiary mathematics students differed: more males had attended co-educational schools, more females had taken two Grade 12 mathematics subjects, and males' parents had more disparate educational qualifications (Forgasz, 1997b). Only females reported gender-based discrimination in tertiary mathematics departments. Large scale survey findings indicated that fewer females than males were enrolled in tertiary mathematics courses but the ethnic profile of the students was consistent with the diversity of Australia's population (Forgasz, 1998b). A higher proportion of females than males did not persist with mathematics beyond first year. Gender differences were evident in the reasons provided by students at the University of Melbourne for studying mathematics (Forgasz & Swedosh, 1997).

Participation statistics in tertiary level mathematics courses, provided by individual institutions, were examined by gender and age (Leder & Forgasz, 1998a). More males than females continued from first- to second-year mathematics but a higher proportion of females than males continued from second- to third-year. A slightly modified version of the survey instrument used by Forgasz (1998b) was administered to a large cohort of undergraduate mathematics students from five Victorian universities (Forgasz & Leder, 1998a, 1998b). On only 2 out of 15 items about the learning environment were gender differences noted: males were more confident of passing and thought there was too much work. Enjoyment levels of mathematics at school and at university were also compared. There was a large decline in the extent to which mathematics was enjoyed at the tertiary level compared with school mathematics and the decrease was more marked for females. Compared to males, females perceived themselves to be less competent with computers and calculators, but viewed tertiary mathematics to be more interesting and challenging. Two-way interactions between gender and ethnicity showed that males from English-speaking backgrounds (ESB) held more functional beliefs (likely to lead to future success) than ESB females but that the pattern was reversed among students from non-English speaking backgrounds.

Summary

As with research on affective variables, findings from research on gender issues related to tertiary mathematics education reveal that gender alone does not explain observed inequities. Students' backgrounds including schooling, culture/ethnicity, and age, as well as the learning environment itself, all contribute to observed inequities in outcomes.

Feminist Frameworks

Three researchers have been concerned with feminist epistemology during the period of this review (Barnes, 1996, 1998; Day, 1996, 1997; Ocean, 1996a, 1996b, 1998). Each researcher made use of feminist research methods but investigated the issue from different perspectives.

Ocean (1996b) drew on feminist and philosophical theories to explore the relationship between personal ethical systems or moralities and women's participation and success in mathematics. She argued that the gender-related moralities of justice and care partially explain the degree of participation and success of female students in mathematics. She provided support for this argument by making reference to the findings of previous research. According to Ocean (1996b), discussion, group work and cooperation are compatible with the care morality and therefore the use of this pedagogy is most likely to provide positive learning environments for women. Ocean (1998) followed up previous work when she described the development of a questionnaire designed to identify separate knowers (justice morality) and connected knowers (care morality).

Feminist theory concerning power relations was the starting point for Barnes' (1998) preliminary study into how students experience learning mathematics in collaborative settings and how gender is constructed by this experience or affects the experience. The study, in which videotaped data were collected in classrooms where Grade 11 students were working in small groups, was designed to establish a set of criteria for identifying the exercise of power that could be used in future research. Barnes identified three elements to the framework: control of the flow of discourse, construction of knowledge and use of research. She found that the most important indicators of the exercise of power in small groups were the initiation of a new topic and the endorsement, rejection or challenge of a statement by another student.

The presumption that women and girls are attracted to real applications in mathematics was challenged by Day's (1996, 1997) findings. She investigated feminist epistemology for mathematics through the description of the experiences of successful female research mathematicians. The complex nature of gender identity within mathematics and the multiple subjectivities of the eight women mathematicians whom she interviewed were revealed. These women were drawn to mathematics because of the creative and intuitive aspects of the discipline. Some were attracted to the abstract concepts of mathematics and others to the applicability of mathematics to real problems with social benefit. They experienced their world as outsiders. They did not fit the norm as mathematicians because they were women, nor the norm as women because they were mathematicians. They did not share the values of the mathematics culture of elitism, competitiveness and hierarchy with their male colleagues, and their level of comfort depended on the practices of their immediate working group and the number of women in their workplace. Day (1997) concluded that a feminist epistemology of mathematics involves a community of knowers who are both rational and emotional, subjective and objective, and who use reason and intuition. She also maintained that the diversity of women's attraction to mathematics needed to be acknowledged in the broad range of strategies used to improve women's position in mathematics.

Feminist approaches give women a voice and enable male-female power relations to be analysed in order to understand how women's interests are subordinated to men. These studies established useful pathways and approaches for further study of gender and mathematics.

Technology (Including Computers)

Technology, that is, calculators, graphic calculators and computers, is receiving considerable attention from curriculum innovators and mathematics education researchers in Australasia. An indication of the level of interest in this area is that about a third of the papers (25 out of 77) published in the proceedings of the 1998 annual conference of the Mathematical Association of Victoria (MAV) (Gough & Mousley, 1998) concerned technology. Only one of these papers raised the issue of gender (Routitsky, Tobin & Stephens, 1998); three out of 12 papers written by teacher researchers were from teachers in private girls' schools.

In the period under review, several researchers have explored the issue of gender in studies concerning the use of technology in mathematics. The studies reviewed here have all reported on some aspect of gender difference concerning either the use of computers or graphic calculators in mathematics. The research designs included experimental, survey and qualitative methods and all were conducted in either secondary or tertiary settings. Only two of these studies were explicitly concerned with gender and mathematics. Girls only samples were used in two other research studies. Researchers have also been involved in reviewing research published in Australasia and internationally. This section of the review commences with a brief summary of the findings of reviews published in the period 1996–1999.

Reviews of Literature Published 1996–1999

Penglase and Arnold (1996) provided a critical review of research concerning graphic calculators over the period 1990–1995. The literature examined mostly concerned experimental studies on the effectiveness of graphic calculators. As a general finding, the authors were critical of much of the research for failing to distinguish between the effects of the graphic calculator and effects created by changes to pedagogy. Two sections of the review considered research about gender differences. The first concerned achievement, including a group of studies that examined the effects of graphic calculators on performance in visual spatial skills, and the second focused on attitudes to the use of graphic calculators. Penglase and Arnold (1996) reported that positive effects on achievement and affect had been found. However they noted that females, and high confidence females in particular, were concerned about the possibility of over-reliance and deskilling as a result of using graphic calculators in mathematics and exhibited behaviour consistent with this belief.

The second review of the literature published in the period (Vale, 1996) was specifically concerned with findings concerning gender and technology in mathematics. Literature that concerned gender differentiated styles of interaction with computers and classroom studies of gender in primary settings and secondary information technology was also reviewed. Vale (1996) concluded that there was evidence of gender differentiated outcomes in computer learning environments and raised questions for teaching practice concerning the masculine culture of computing and technologically based mathematics.

Access to Technology for Mathematics

When the Victorian Board of Studies decided to allow the use of graphic calculators in VCE mathematics examinations for 1997 there was concern about equity of access and so it was decided that the examination should not favour students with graphic calculators. The Board requested a study on student access. Routitsky and Tobin (1998) and Routitsky, Tobin, and Stephens (1998) reported the results of a survey of VCE mathematics coordinators and teachers concerning student access to graphic calculators and teachers' attitudes towards their use. They found that VCE students generally had access to graphic calculators through ownership or the use of class sets, although this varied with the mathematics subject studied. They also found that 73% of VCE mathematics teachers were in favour of the use of graphic calculators in examinations, although many were concerned about equity. Gender differences in access to graphic calculators were not investigated. However, a higher access rate for students at single-sex girls' schools and a more positive attitude to the use of graphic calculators among teachers at girls' schools were reported. The need for further research to explain this finding was advocated.

Experimental Studies in Single-sex Settings

Research on the use of technology in mathematics has been conducted in single-sex girls' schools. Vincent (1998) studied 12 secondary girls' cognitive processes and test scores and found that dynamic geometry software was an effective learning tool that would enable secondary students to advance to a higher van Hiele level of geometric understanding. The researcher also placed importance on the design of the learning tasks used in the teaching program and the use of collaborative pairs in achieving these results.

Tynan and Asp (1998) conducted an experimental study designed to test the effect on performance in symbolic manipulation of the use of a hand held computer algebra system (CAS). The study was conducted in a single-sex girls' school with two Grade 9 mathematics classes. The experimental group used a model of graphic calculators with CAS capabilities, whereas the control group used graphic calculators without this facility. Pre- and post-tests were used as the basis of analysis as well as qualitative data collected from teacher and student journals. No significant differences were found between the posttest results of the two treatment groups. The CAS group of students were more inclined to persist with algebraic methods. That is, there was an under-use rather than an over-use of CAS, perhaps reflecting a fear of reliance on technology. Their results were consistent with the findings of other research reviewed by Penglase and Arnold (1996) and Vale (1996) The factors cited by Tynan and Asp (1998) as likely to explain their results included the tediousness of using the technology, feelings of cheating, frustration with the notation system, the poor motivation of students regarding mathematics and lack of access to CAS calculators because of teachers' concerns about dependence.

Gender Difference in Attitudes to Technology in Mathematics

The explanations proposed by Tynan and Asp (1998) in their study of computer algebra systems were consistent with those of another study concerning gender difference in attitudes to the use of computers in mathematics (Vale, 1998). In this study of groups of Grade 8 and 9 students, qualitative methods were used to identify themes to explain student attitudes to computers in mathematics. Four main themes emerged: pleasure, success, relevance, and power of the technology. Vale (1998) found that although both girls and boys were positive about the use of computers there were gender differences in the reasons given. Girls were more likely than boys to qualify their support. Pleasure and relevance in terms of the enhancement of computer skills were key factors for boys, whereas girls more often than boys were concerned about issues of success and thought that the use of computers would enhance their learning of mathematics or make mathematics easier. Vale (1998) suggested that these results may have been influenced by classroom culture and the nature of the tasks, hardware, and software used by the students.

Technology as a Male Domain

Research into feminist theories of the male dominated culture of computers was evident in the literature. One study concerned classroom culture and the investigation of computer enhanced classrooms as male domains (Vale, 1997). The study concerned a Grade 10 coeducational mathematics class. Qualitative data were collected through observation and videotapes of lessons, and from interviews of four students (two boys and two girls) and the teacher. Many similarities in the behaviour and responses of students were found. There was, however, evidence of some gender differences which indicated the need for further feminist research concerning the culture of computer enhanced classes in relation to participation and engagement in mathematics. The tentative conclusions were that boys demonstrated their technical knowledge of computers and computer systems and girls appeared to view computers as a tool and to personalise their interactions with computers.

Summary of Research on Gender and Technology

The research in the field of gender and technology in mathematics education indicates that the use of technology for the learning of mathematics does have gender implications. Much of the research in this field has been conducted within a liberal feminist mould, whereby researchers have assumed that girls aspire to masculine ideals with respect to technology and the place of technology in mathematics. That gender has been included in studies indicates an awareness of gender equity issues and the need for ongoing monitoring of gender differences in participation and achievement in mathematics and in attitudes towards mathematics. Girls' schools are supporting the use of technology and research into the use of technology. This, and the reconstruction of mathematics within a technological environment in which gender differentiated approaches to the learning of mathematics are acknowledged, provide evidence that interventions to improve outcomes for girls are on-going. The feminist perspectives through which the nature of technologically based mathematics classrooms were explored indicate the continuing need for further research that examines pedagogy, the nature of tasks, and the classroom culture, in order to provide insight into how to use technology effectively for particular groups of students who may exhibit various learning characteristics.

The Media

As indicated at the outset of this chapter, a glimpse of societal attitudes to boys' and girls' expected and accepted educational pathways is portrayed through reference to selected newspaper articles (Table 2). These excerpts help depict, within the space constraints available, the environmental climate in which research and intervention initiatives are carried out. General comments far outnumber substantive reflections on curriculum content or instructional strategies.

This brief listing of print media articles concerned with mathematics and gender illustrates that themes explored in the traditional research literature are also voiced in the popular press. Space constraints in such publications, and commercial considerations, often result in a starker and more simplistic portrayal of issues than is warranted. Without doubt, in contemporary Australia, public debate about gender, mathematics, and technology persists.

Table 2
Feminist Themes in Selected Newspaper Articles[4]

Summary of newspaper article [Publication date]	Feminist perspectives[5]
Participation and performance in mathematics[6]	
An article highlighting girls' superior performance in the Victorian Certificate of Education (VCE) examination. "A higher proportion of girls scored grades above C+ in core subjects across mathematics, sciences, business and economics, humanities and information technology." Girls continue to show a preference for "softer" subjects pushing them into "lower-status, lower paid service industries". The article claims "that girls' grade dominance could be explained largely by social and economic factors." [July, 9, 1996]	Evidence of both the deficit and pluralistic models. Girls' dominance as well as deficits are listed.
A recent analysis of VCE data confirmed that on average, boys did not do as well as girls in many subjects, including English and Further Mathematics. Socioeconomic factors, rather than gender, might be linked to poor performance. "It's not about boys and girls, it's all about which boys and which girls." [Nov 7, 1997]	The social justice model. Boys are perceived as the disadvantaged group.
Data from the TIMSS study revealed that "Australian boys are outperforming girls in such science subjects as physics and general mathematics.... But in an unusual twist, Year 12 girls are doing as well as boys in advanced mathematics". Girls' relatively strong performance "could be explained by the recent emphasis on student equity programs, especially programs that encouraged girls to study the harder science subjects." [Feb 26, 1998]	The deficit model, with evidence that the special programs introduced for girls have been successful for them and to the disadvantage of boys
"Biologists have gleefully concluded that there are genuine physiological differences in men's and women's brains that explain our behaviour.... Male	The deficit model. There is little evidence of support for the pluralistic model

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[4] All articles have been taken from *The Age*. Because of widespread syndication, coverage is much broader than suggested by the resources of a single newspaper.

[5] For more detail on the feminist perspectives presented here, see: Leder, Forgasz, and Solar (1996), Kaiser and Rogers (1995), Noddings (1990), and literature in the field of *Women's Studies*.

[6] Because of space constraints, no excerpts are included from the many articles in which participation and performance in education in general were discussed

Summary of newspaper article [Publication date]	Feminist perspectives[5]
prowess at mechanics, higher mathematics, musical composition and chess playing are all signs of the male brain's superior spatial ability and single-mindedness." Various other gender related stereotypes are reinforced in the article. [April 5, 1998]	
An overview of VCE results, with an emphasis on boys' falling performance. "In Victoria, the gender gap has worsened not just in English but in further mathematics, considered the easiest of the three VCE maths subjects." The article cites similar data from other states and contains comments such as "some education experts are saying moves to make learning more accessible to girls have occurred while boys' needs have been neglected" and "the growing gap between boys' and girls' English results, and the boys' poorer performance overall has been caused partly by the 'feminisation' of education". [July 7, 1998]	The deficit model, with evidence that the special programs introduced for girls have been successful and to the disadvantage of boys
Teachers and classroom factors	
A review of Pythagoras' Trousers by Margaret Wertheim is used as a vehicle to highlight the importance of teachers in fostering positive attitudes to mathematics and science. Traditional methods of teaching these subjects are said to short change girls who are assumed to be "interested, beyond the numbers, in the people, their lives, the cultures, the relationships. In short, they are interested in the human, not the disembodied discipline". [April 23, 1996]	Consistent with the pluralistic model (as well as stage 5 of the Kaiser & Rogers, 1995, model: mathematics reconstructed)
Some behaviours are slow to change. A report just published "found there was a deeply embedded ethos of misogyny, bullying, wilfully offensive behavior and occasional racism". [June 12, 1998]	Consistent with the tenets of liberal feminism
Single-sex settings	
"Education in an all-girls school is one of the most effective affirmative-action plans parents can have for their daughters." [April 30, 1996]	The deficit model
Various studies have supposedly revealed that "school type [single-sex or co-ed] was not an important factor in improving performance levels for girls". Socioeconomic factors, rather than school type, was a better predictor of girls' success in mathematics. [Sept 25, 1997]	The social justice model

Summary of newspaper article [Publication date]	Feminist perspectives[5]
Two 10 year old students, a boy and a girl, are asked about coeducation. Each is a class captain, and each claims to like "maths the best". Their preference for the coeducational setting is echoed by others cited in the article. For long term development it is important to work "within a community as a group, co-operating with the opposite sex on a day-to-day basis—just as you would in the real world." [June 9, 1998]	The assimilationist model
Technology (including computers)	
Female chief executives continue to be rare in Information Technology: "in the real world, the pinnacle of IT remains a zone savored almost solely by males". Comparative strengths of females and males relevant for IT are also discussed. [Sept 9, 1997]	Evidence of support for the assimilationist and social justice models. Here females are sketched as those typically disadvantaged.
"Skills lacking: study" A report of an American study that found a correlation between low scores in mathematics and the higher use of computers for drill and practises exercises in mathematics by black children. No reference was made to gender in the text but the article included a photograph of secondary aged girls using laptop computers in a single-sex classroom with the caption: "Computers at school: Questions on effectiveness." [Oct 6, 1998]	Deficit model for blacks and girls, with evidence that programs have not been successful.
"New-age bullies use cyberspace to harass peers." A survey of students in an all girls' school found incidences of bullying through computer networks and e-mail. [Oct 6, 1998]	Supports a view of liberal feminism and challenges the theories of radical feminists.
"Computer games zap boys' job futures." A report on a submission to a parliamentary inquiry in which it was argued that 70% of boys only played games. Girls gave value to prospective multimedia and software employers because they worked well in groups. [Dec 2, 1998]	Deficit model with boys portrayed as needing help. Pluralistic model applied to girls' skills.

Summary

The contents of this chapter illustrate the extent to which gender issues continue to attract research attention. Persistent as well as changing beliefs about boys' and girls' educational aspirations and performance, generally, and in mathematics and technology in particular were reflected in the work surveyed. Research in mathematics education conducted and reported during the four year period under review has clearly been influenced by directions in the broader fields of feminist and educational research. The overview of earlier work, presented at the beginning of the chapter, highlighted evolving and changing concerns over two decades, more diverse theoretical perspectives, and methodologies being considered. This evolutionary process continues to be reflected in much of the more recent work reviewed. A further review of the field in another four years certainly seems warranted.

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The Politics of Mathematics Education 1996-1999

John Horwood
Victoria University of Technology
<john.horwood@vu.edu.au>

Jan Thomas
Victoria University of Technology
<jan.thomas@vu.edu.au>

Introduction

Politics may be interpreted in a broad, social perspective; or it may be interpreted in a more focused perspective as referring to those decisions and actions of policy makers that impact upon mathematics education practices. It is with this latter perspective that past reviews (Clements & Thomas, 1996; Thomas, 1992) have been concerned, and this review continues in the same vein. Nonetheless the broader political perspective is never far from the surface.

This chapter adopts a somewhat broad view of research in that it attempts to document what could be called research in the politics of mathematics education and at the same time it attempts to capture the contribution of mathematics education researchers to that politics. By this we mean the broader contribution that mathematics educators have made in attempts to influence government policies as they affect mathematics education. In doing this we also attempt to document the changes in the political climate in which the research documented in other chapters has been constrained and promoted.

Living in Challenging Times

The political climate has been superficially supportive of the mathematical sciences[7] but the rhetoric has not matched the reality and, as this chapter will document, there are a number of challenges ahead if a solid base is to be re-established. Research in mathematics education does not occur in isolation from the discipline of mathematics nor from the teaching on mathematics. The health of the mathematical sciences, and therefore mathematics education research, are inter-connected. Previous reviews of the political milieu in which the mathematical sciences operate have been optimistic about the future, and the last review was written just after a general election in Australia when it appeared both major parties were making a strong commitment to education (Clements & Thomas, 1996). Within a year this optimism was being challenged. Thomas, (1997a) wrote:

> The health of the mathematical sciences is dependent on the state of the individual parts although weaknesses in one area can sometimes be compensated for by strengths in others. For example, good teachers will

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[7] *Mathematical sciences* is used as an inclusive term for the mathematics disciplines and mathematics education teaching and research.

> overcome difficulties with a poor curriculum and good teaching in one sector can overcome poor teaching in another. It would appear that weaknesses are now becoming apparent in many areas. It is questionable whether a few pockets of excellence can sustain the vibrant and vigorous discipline needed in the future if Australia is to be part of a region building its economy on science and technology (p. 505).

Four years later the situation in Australia can only be described as a deepening crisis. The reasons are complex, inter-related and largely political.

"Crisis" is an emotive word but its use is justified by the failure of the government to ensure that the fundamental building blocks of the mathematical sciences such as an adequate supply of well-qualified mathematics teachers are maintained. For many MERGA members this is an equity issue—it is not good enough for relatively wealthy nations like Australia and New Zealand to provide access to advanced mathematics only to a small percentage of the population, and there is a very real danger that this will happen. Both Australia and New Zealand need teachers who are not only well qualified mathematically but can teach students from diverse linguistic backgrounds with sensitivity and respect for cultural background. The issue of teacher supply is closely linked to other factors affecting the mathematical sciences.

The period of this review has seen continued expansion of the university sector, which has not been accompanied by commensurate funding. There have been some exceptions to this expansion, including education faculties that have contracted because student load had shifted to courses perceived as more popular or, in the case of some universities, more prestigious. A study by Dobson and Calderon (1999) for the Australian Council of Deans of Science shows a reduction in education faculty staff in excess of any other discipline area. This has been compounded by difficulties in attracting students into mathematics teaching which means that mathematics educators have been particularly affected. In the same period there was a decline in education students studying science[*] and science students studying education. The study also documents the increase in the use of sessional teaching staff. As a result, fewer tenured, full-time mathematics educators teach more students and have increased administrative loads including the supervision of sessional staff. Further, sessional staff become disillusioned about ever finding an on-going position and seek other positions, not necessarily in the tertiary sector. The workload on full-time tenured staff in maintaining teaching quality makes it increasingly difficult to also maintain the role of an active researcher.

This is, however, not the only constraint on mathematics education research. During the same period there has been a considerable reduction in the government bureaucracy, and a move to consultancy and tendering of projects and reports which were once managed by public servants in quite collaborative arrangements with educators in schools and universities. The new arrangements are controlled more than collaborative and are often subject to restrictions on the publication of results. Thus researchers are tied up with projects where the intellectual property is with the funding body and outcomes may never be public. The control of mathematics education research through these kinds of mechanisms is a major concern and may become worse.

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[*] ""Science"" includes mathematics and computing.

A recent Green Paper (Kemp, 1999a)[9] on research funding in Australia placed greater emphasis on industry collaboration and indicated that consultancies and tenders would be considered as a measure of research output when allocating research funds. At the same time the paper played down the role of publications in identifying research output. Responses to the discussion paper from both MERGA and the Australian Mathematical Society (AustMS) identified many common concerns with the threat to basic research, both in the discipline area and to mathematics education, although the reasons are somewhat different. Basic research in mathematics has the difficulty of demonstrating its relevance if it is not clearly tied to applications. Basic research in mathematics education may be directed in particular ways that are perceived to be more applicable and relevant by the bureaucrats but not by the researchers involved.

The White Paper appeared at the end of 1999 (Kemp, 1999b) and, while some of the concerns of the mathematical sciences community were addressed, no new funding is mooted and a strong emphasis on industry collaboration remains. The emphasis on individual institutions rather than disciplines as one of the *quality* measures may act against mathematics education as the small number of mathematics educators in any one institution means there is a need for cross-institutional activity. The use of completion rates for research students also mitigates against mathematics education as many postgraduate students are teachers who struggle to complete in the minimum time.

Thus many Australian mathematics education researchers are finding it increasingly difficult to maintain the quality and quantity of research output of previous years due to a number of factors which relate to the political milieu in which they operate. Other issues compromise the excellence of the mathematical sciences more generally. These include the state of mathematics itself in the tertiary sector, some issues relating to recent curriculum and assessment in schools, and major problems with the supply of teachers already noted.

In summary, this chapter will demonstrate the resilience of research in mathematics education in Australasia because, as the other chapters demonstrate, good research does continue to be carried out and MERGA members continue to make significant contributions nationally and internationally. This research happens in spite of the political climate rather than with its support. At the same time MERGA members continue to be active in trying to change the political climate in many ways that are documented in the following sections. This activity includes research but also actions that promote discussion and the dissemination of research ideas.

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[9] A *"Green Paper"* is a document for discussion. A *White Paper* is a government policy and final position statement.

In the following sections, a summary of the major political issues currently facing the mathematical sciences in Australasia begins with the ever-contested area of school curriculum, followed by discussion of the related areas of the Third International Mathematics and Science Study (TIMSS), numeracy benchmarks, and vocational education and training. Teacher supply is then examined in greater detail, together with the related area of university mathematics and the effect of some funding changes. Mathematics education research is then considered in the context of these related and connected issues. The chapter concludes with a summary of some of the contributions of MERGA members to the politics of mathematics education and some comments on the challenges that face research in mathematics education.

Political Issues in the Mathematical Sciences

Schools

Although attempts to introduce a National Curriculum had been resisted successfully by the combined actions of mathematicians and mathematics educators, the drive to introduce a National Curriculum, along with "government determination to decrease the role of Australasian academics in matters concerning school curricula, and to increase centralised curriculum decision-making powers of education bureaucrats" (p. 96), was a major issue in the previous review of the politics of mathematics education (Clements & Thomas, 1996). The National Curriculum may have foundered, but the intervention of government, and the prominent role in centralised curriculum decision-making by state education bureaucrats, has continued. The concern of the 1960s and 1970s with both physical and personnel resources have given way in the 1980s and 1990s to concern with the outcomes of schooling. Throughout the 1990s, curriculum has emerged as a major area of overt government concern and ministers of education have taken an active interest in the development of curriculum.

The history of control of a curriculum, and the emerging role of government in the process of control, is the major concern of Horwood (1997a). This work is concerned with the development of the mathematics curriculum within Victoria since colonisation. It examines the early control by the university professoriate, traces the transitions in control to mathematics educators in the 1960s, and documents the importance of the role played by government and educational bureaucrats in the late 1980s. The effect of different control groups on the nature of the mathematics curriculum is examined in the context of developments within the senior secondary mathematics curriculum in Victoria.

Curriculum is a complex social construction and different actors within the mathematics education environment have different impacts on the nature of the development of mathematics curriculum. Its background, however, is historical. It is interesting to note that the effects of the New Maths movement in the 1960s had similar consequences in New Zealand (Haynes, 1996) and Australia (Horwood, 1997b). The major political consequence was the emergence of *mathematics educators as a significant force in the determination of curriculum.* However, the prominence that mathematics educators enjoyed during this era was short lived, both in Australia and New Zealand. Haynes identifies the 1990s as another period of significant change in New Zealand mathematics curriculum, and points to the fact that most of the impetus for change in this era arises from government intervention and demands for reform. Haynes concludes with a plea for further research in order to trace the influences of mathematics educators on curriculum, thus better to provide support for future mathematics curriculum change. This plea resonates with the work of Horwood (1998) who attempts to develop a model that will provide an explanation for the transitions of mathematics curriculum in terms of the groups that control the curriculum. This model builds on the work of Weber (Albrow, 1970) and draws on the notion of professionalisation, and associated expertise, to explain the emergence of new actors in the curriculum scene. Much of the history of mathematics curriculum in Australia can be conceptualised in terms of the expertise of mathematicians, of mathematics educators, and of mathematics education bureaucrats. It is the emergence of this latter expertise that may well account for the successful intervention of government into mathematics curriculum.

Government concern with curriculum can be traced to its concern with changing workplaces and increasing unemployment (Haynes, 1996; Horwood, 1997a); and the increased involvement of bureaucracy, at least within Australia, relates to a major transformation of the public service that took place throughout the eighties. These reforms took the form, initially, of a restructuring of the management style of the organisation. The new management style was characterised by

> the establishment of an elite corps of bureaucrats who were highly paid, motivated in part by performance appraisal schemes, financial bonuses and performance improvement plans and with a performance measurement scale related to pay levels. An essential feature was their recruitment from outside the existing public service, preferably from industry and commerce (Bessant, 1995, p. 60).

One of the consequences of this reform was a change in the style of operation of the departments within the public service. Traditionally, the public service provided an efficient and disinterested administrative arm of government. This did not mean that it was not involved in the formulation of policy; rather that it acted in a less explicit way to prepare briefs for the Minister based on expert advice. Much of the advice that was collected and passed onto the Minister had its origins in sources beyond the public sector. The changed emphasis that the reforms engendered brought with it a change of culture, as public sector management sought to exercise greater and more explicit control over the generation of policy. An immediate consequence was the marginalisation of outside influences. Policy input by professional bodies was minimised. Control of curriculum thus became the preserve of professional bureaucrats with little or no input from the professional bodies (Clements & Ellerton, 1996).

The shift in the focus of the control of education, from educationalist to managerialist, has seen the emergence of concerns for the privatisation of education, along with a desire for overt accountability of the processes of education (Clements, 1998; 1999; Horwood, 1999). This has led to increased central control over curriculum development, implementation, and assessment. One consequence of these trends has been the adoption of the notion of outcomes-based education (OBE), and the advocacy of the adoption of school-based management programs. These movements proceed in the face of substantial overseas research that suggests that such policies do not lead to any significant school improvement (Clements, 1998; see also Stigler & Hiebert, 1999).

The rhetoric is pervasive. Both the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991), and key documents prepared by the Australian States, in particular the subject specific documents, are written in terms of OBE concepts and associated accountability mechanisms (Clements, 1999). The consequences of such an approach to curriculum is most surely felt within the classroom, where the emphasis on performance on external tests translates into lesson structures that are designed to improve student results at the expense of student learning.

That this is the fate of the Victorian Certificate of Education (VCE), an innovation of the early 1990s that enjoyed considerable acclaim among teachers and mathematics educators alike, is evidenced in the research of Rowley, Brew, and Leder (1998). In a study of the experiences of mathematics teachers with the VCE, the authors found that the introduction of the VCE had resulted in considerable change in the approach of teachers to the teaching of mathematics at both senior and junior secondary levels. Changes demanded in the teaching of mathematics at senior level had percolated down to the junior curriculum. While general support among teachers remains for the content and flexibility of the current VCE curriculum, there was evidence of increased concern with the nature of the assessment process, and a clear lack of confidence in the reliability as a verification procedure of the externally imposed General Achievement Test. (GAT). This test is now administered to all Year 12 students and used as an instrument of validation on a school-by-school basis.

While teachers were positive about the initial changes occasioned by the introduction of the VCE, to which they had substantial input and influence (Horwood, 1997a), they were less confident about recent changes introduced by a centralised and administratively controlled curriculum board. A substantial number of teachers considered recent changes to the VCE, in particular the trend towards the reduction of school-based assessment and its replacement by increased external assessment, as a retrograde step leading inevitably to a retreat to a more traditional approach towards the teaching of mathematics. Although the heavy workload imposed on students and teachers alike by school-based assessment procedures remains a problem to be addressed, the administrative solution of reducing teacher involvement in favour of increased external control was not seen by teachers as an acceptable alternative (Brew, Leder, & Rowley, 1999).

Nonetheless, the pressures remain to cede control of the curriculum to educational administrators (Horwood, 1999; Thomas, 1999). What appears to be overlooked in this process is the consequent redefinition of the meaning of education in general, and of mathematics education in particular. Administrative accountability is built around assessment, so that increasing emphasis within the public arena is placed on the notion of testing, and on the performance of students on the pencil-and-paper tests that are employed. Examples include reactions to the recent TIMSS study, and the introduction of state-wide mathematics performance tests that commenced in various states in the 1990s and are administered to all Year 3 and Year 5 students. These are now being extended to Year 7, and possibly Year 9, and linked to National Numeracy *benchmarks*.

Although the tests are portrayed as diagnostic, the reactions of government and bureaucracy alike ensure that such tests carry more weight than other modes of assessment used by classroom teachers. The move towards increased bureaucratic control is accompanied by a diminution of the role of the professional, and the marginalisation of the professional bodies. Curriculum becomes reconstructed in administrative terms and becomes the preserve of the educational administrator.

This does not mean that there was no input from professional bodies. Any input, however, was constrained by the conditions laid down by the bureaucracy. For example, although key groups of academics were employed by the curriculum committee to develop the *Victorian Curriculum Frameworks* (Board of Studies, 1995), these groups operated within the constraints determined by the bureaucracy. The emphasis of the bureaucracy was on core curricula. This move to standardise the mathematics curriculum is examined in Clements and Ellerton (1996), who relate these moves within Australia to a combination of forces. They highlight the drive towards accountability in Australian education systems that emerged during the 1980s and culminated in the production of the national *Mathematics Profile* (Curriculum Corporation, 1994).

Ellerton and Clements (1997) see a natural extension of the demand for comparability of schools and school systems, which have dominated education discussion throughout the 1990s, in the demand for a core curriculum and standardised pencil-and-paper tests. This drive towards core curriculum is also enhanced by events taking place overseas. Comparisons between the outcomes of curricula in the United States of America and of the United Kingdom are seen as influential in determining curriculum directions of Australian decision makers, a view that was reinforced by the reactions to the results of the recent TIMSS study

(Lokan & Doig, 1997; Stacey, 1997).

The TIMSS Results

Reaction to Australia's performance in the TIMSS study are difficult to explain and were reported in a similar manner for the three cohorts—primary, junior secondary, and final years of schooling. The most significant cohort was the junior secondary, 13–14 year olds, for a number of reasons: (a) more countries participated, (b) differences in starting ages should have been minimised, and (c) participation in mathematics could reasonably be expected of the whole cohort. The Australian results were reported in three bands—*higher than Australia, not different from Australia,* and *lower than Australia* (Lokan, Ford, & Greenwood, 1996). New Zealand was in the lower-than-Australia group (Garden, 1996). For the 14 year olds, there were eight countries in the higher-than-Australia group. Australian students were 16th and New Zealand students 24th overall. For the 13 year olds, there were seven countries in the higher-than-Australia group; Australian students were 17th and New Zealand 25th overall.

The results were widely reported as some kind of major success for Australia. By taking the results for the 13 year olds and putting all the not different from Australia together, it was claimed that Australia was 8th. There appeared to be a deliberate attempt not to draw attention to the differences between the highest scoring nations and Australia, but rather to concentrate on how well Australia did relative to other English-speaking nations which were clustered around bottom of the not-different-from-Australia group and the top of the lower-than-Australia group. The success of countries like Singapore was dismissed as rote learning, although many more of the Australian students thought success in mathematics was linked to memorising than those in Singapore. In the international report (Beaton et al., 1996) was the percentage of students believing that it is important to memorise the textbooks or notes to do well in mathematics. The figures of 67% for Australia, 72% for New Zealand and 32% for Singapore are indeed intriguing, especially as this was one of the few areas with widely differing results dealing with attitudes and beliefs. This information received little or no attention.

There was one finding that did receive some attention. In response to an optional question (not used by all participating countries), between 50 and 60 per cent of Australian and New Zealand teachers said they would change their career if they had the opportunity. These percentages are high compared to other countries, ranging between 10 and 25 per cent (Lokan, Ford, & Greenwood, 1996). However, while concern was expressed, there appears to have been no follow-up studies or actions. One reason for this worrying result could be that many of the teachers of mathematics at this level are not teaching it by choice. On-going shortages of mathematics teachers mean that many teachers are drafted into teaching mathematics at the junior secondary level. A study in South Australia (Brinkworth, Coates, Harradine, & Jarrett, 1999) showed how widespread this was in that State; a shortage which is duplicated across Australia and New Zealand. However, the primary teachers also reported high levels of dissatisfaction, so the problem is widespread in Australia and New Zealand (Lokan, Ford, & Greenwood, 1997). The recommendations of a major government inquiry into the status of teachers (Senate Employment, Education and Training References Committee, 1998), into which many MERGA members had input, may have helped address this, but appears to have been largely ignored while State and Federal governments argue about whose responsibility it is to act on them.

Following the release of the first TIMSS results, the Australian Council for Educational Research (ACER), which had responsibility for the TIMSS study in Australia, hosted a conference *Raising Australian Standards in Mathematics and Science: Insights from TIMSS* (ACER, 1999). The program, with some exceptions, indicated that the speakers would not focus on an analysis of whether the Australian results could or should match those of higher performing countries, nor was there genuine discussion about what might be done to address issues such as the dissatisfaction of teachers. Thomas (1997b) released an unsolicited paper in an attempt to articulate some of the issues and concerns of the mathematical sciences community at the time. While this received considerable media coverage and was nicely complemented in a paper by Stacey (1999), which identified some of the areas where Australian performance was unsatisfactory, there is little evidence that any of this has been taken seriously by the various bureaucrats in charge of curriculum.

The paper by Thomas was subsequently reported by the head of the ACER as trying to contrive a crisis (McGaw, 1998). In fact the word "crisis" was used only once in the context of "fundamental problems that underpin what is rapidly turning into a crisis in the mathematical sciences in Australia", and was an attempt to draw attention to emerging problems. According to McGaw "the discussion has become politicised—not in the party political sense, but in the sense of other agendas getting in the way of productive discussion". This is a very strange statement as anything like the TIMSS study will always be political. It suggests that issues like the supply of teachers in maintaining performance in Australia, let alone improving it, were seen as unimportant. The point from the mathematical sciences community was that Australian results could be improved—the focus of the conference—but that this was going to be difficult to do unless a number of related and interconnected issues were addressed.

It is this role of ACER in promoting the TIMSS results in the most positive ways possible that remains a mystery. There has been constant emphasis on talking up Australian results rather than careful analysis of what the results may say about how to improve mathematics education. From being concerned that the focus would just be on the league table of results, MERGA members have found themselves in the position of having to try to draw attention to any aspect of the results. There seems to have been less reaction to the TIMSS results in New Zealand, but similar frustration that the results have produced little or no government response directed at analysis and improvement.

Numeracy Benchmarks

As reported in the last review of research (Clements & Thomas, 1996), the concept of benchmarking was becoming increasingly important. However, the way in which benchmarks have been defined in Australian education is at odds with industry definitions of excellence, as they seek to identify students at risk rather than identify excellence and international best practice that could have built on the TIMSS study data. The use of the term *numeracy* rather than *mathematics* appears to have its origins in the federal ministry, and a serious concern is the consistent use by the Minister for Education that infers it is *basic skills*. Thus at the opening of an international mathematics festival in Melbourne he spoke of "basic numeracy skills" and "foundation skills".[10]

Documents emanating from the federal bureaucracy, and also from the Australian Association of Mathematics Teachers (1998), are attempting to define numeracy in a way that distances it from mathematics. In doing so the power and beauty of mathematics is being lost, along with notions of "modern mathematical science, is in its own right a supreme creation of the human intellect; ... critical for economic competitiveness and a basis for investigations in many fields" (National Committee for Mathematics, 1996, p. ix). More importantly, the notion of *mathematics for all* is being lost. Thomas (in press a) has argued that nations risk becoming divided by issues of access to mathematical knowledge if a privileged minority have access to high levels of mathematics through a limited number of schools with well qualified teachers, and low level benchmarks determine what is adequate for the majority.

Discussion of the numeracy benchmarks is accompanied by rhetoric of *numeracy across the curriculum*, which in itself is a laudable aim. However the history of the language-across-the-curriculum movement should be remembered. English teachers were found to be ill-prepared when it came to assisting their colleagues in identifying what they could do to teach the language of their discipline. In Australia some progress was made when English as a Second Language teachers started working with classroom teachers. In the case of numeracy many teachers with little mathematical knowledge will be expected to reinforce the mathematical ideas inherent in their discipline areas. Yet many of them will be far less confident of their ability to do this than they were in the language area. Most of them were confident users of English but the same cannot be said of mathematics. Watson (1998) drew attention to the mathematical aspects of risk being ignored in discussion of risk in health and physical education curriculum and the mathematical aspects of many subjects have tended to be down-played in recent years rather than emphasised.

Thomas (1998) discussed the implications of the numeracy benchmarks for schools and the background to the project to that stage. She predicted that changing governments would have little effect, and that there would be a push for national testing when problems arose from trying to use existing state regimes. She noted that "sometimes the AMSC[11] and other discipline based societies managed to delay the inevitable or get it done somewhat better" (p. 493). This has certainly been the case, but the insistence on national comparability using rather low-level benchmarks continues.

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[10] Press statement, January 10, 2000

Vocational Education and Training

The trend towards greater bureaucratic control and the imposition of an outcome-based-education curriculum model is clearly evident in the area of Vocational Education and Training (VET) and the Adult, Community, and Further Education sector. In November 1996, the Federal Minister for Schools, Vocational Education and Training initiated a program to contribute to the expansion of vocational education. This program, and its consequent developments, were designed to ensure that control of vocational education should be vested in the business and industry community. As a consequence the efforts of mathematics educators and mathematicians to provide significant input into the development of mathematics units within this sector have been in vain. The actions of government have been premised on the widely-held perception that economic problems are the result, not so much of the failure of government policy or industry decision, but of the failure of the educational system. As a consequence major changes to educational policies and operations have been introduced in order to align the goals of education to the perceived needs of business and industry. In the Vocational Education and Training sector this has resulted in a national curriculum, in the development of an open training market whereby all training providers are forced to tender competitively in an open market place, and in the introduction of competency-based education and training as a mandatory requirement of all nationally accredited courses (FitzSimons, 1996; 1998a; 1998b).

Specifying content for school and vocational education and training in support of general competency standards downgrades the notion of the teaching of mathematical thinking skills (FitzSimons, 1999b). Thus the nature of the mathematical process is redefined, and obfuscates the need to re-examine the educational requirements of current and future school populations. An emphasis on competency flows from the desire to render education both measurable and accountable to administrative procedures. This notion relates to the redefinition of the control of education in terms of technical administrators, and also to the increasing preoccupation with a notion of education as consumer driven. Yet, as FitzSimons (1999a, 1999b) has pointed out, the belief that vocational education can equip students with the skills required in the workplace is fallacious—it would be impossible to teach content appropriate to all vocations.

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[11] Australian Mathematical Sciences Council

The student body in the VET sector has widely differing needs and aspirations, ranging from basic numeracy to the needs of students articulating to tertiary courses such as engineering. Delivery of such courses requires teachers with quite different backgrounds, excellent pedagogical skills and secure employment, in order to encourage curriculum delivery that is reflective and evolving to meet diverse needs. The extensive use of short-term contracts, sessional staff and packaged curriculum is not encouraging the development of a VET mathematics teaching profession.

A report completed in 1995 suggested that "for many TAFE[12] graduates entering university first year mathematics subjects often represent a hidden barrier … TAFE graduates therefore may not be prepared to take full advantage of these credit transfer arrangements" (Pantlin & Marr, 1995, p. 1). This project produced an agreed structure and content for TAFE students articulating into engineering as well as a structure and content for students seeking credit transfer for the equivalent of first year mathematics in an engineering degree. The Australian Mathematical Sciences Council's (AMSC) nominee to the committee was Bill McKee from the School of Mathematics at the University of New South Wales. He believed that if TAFE colleges implemented the recommended courses, students would get a proper grounding, but doubted whether the TAFE colleges had the capacity to teach them (personal communication, June 12, 1996). There is no evidence that this report was implemented, and considerable anecdotal evidence that nothing has been done to improve the delivery of mathematics in TAFE colleges. The amalgamation of some TAFE colleges into universities would appear to open doors, but requires the management of these dual sector institutions to ensure that there are well-qualified mathematics teachers in both sectors. There is little evidence that this is happening at the current time.

Teacher Supply

The reasons for teacher shortages are complex, and not just to do with comparative salaries. This is one deeply political area where, in Australia, the Federal government tends to claim it is a State responsibility, and the States to claim it is a Federal issue. While some States have introduced some incentives, this has tended to be on a very small scale. The notion that it might be a Commonwealth responsibility needing national strategies currently appears to be ignored. A study by Galbraith (1999) on forecasting teacher supply and demand demonstrated how complex the issues are. He noted:

> Prediction at a level of precision, given the plethora of variables is beyond the capabilities of any forecasting tool. Further a confluence of social and economic coincidences at any point in time could give unwarranted credence to a flawed

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[12] Technical and Further Education

formula, with dire future consequences (p. 71).

Galbraith noted that short-term emergency measures should be accompanied by strong messages to graduates that, with good teaching qualifications, they will always be in demand. This message was certainly not conveyed to prospective teachers in Victoria and some other states in recent years as teacher employment and university places in education courses were reduced. Figures produced by Preston (1998) for the Australian Council of Deans of Education on predicted shortfalls are frightening, especially as they show shortfalls across primary and secondary sectors, and the situation in mathematics is likely to be far worse. This is supported by Forgasz, Horne, Pearn, and Siemon (1997) who examined the backgrounds of Victorian teachers and the time devoted to mathematics in the teachers' schools.

Reduction in the number of places in education faculties, and knowledge of the shortfall, has seen increased demand which has improved the quality of beginning teachers in undergraduate courses. At the same time it would also appear that, in some States, a large number of graduates missed out on graduate education courses in both primary and secondary education for the 2000 academic year. This illogical situation has arisen because universities can not afford to increase places in expensive courses, and any course which has a field or laboratory component is relatively expensive. In the case of the Graduate Diploma in Secondary Education at the authors' institution, school placement is a fixed cost of just over $1000 per student. Each year this is a greater proportion of the reduced income the School of Education gets per student so other parts of the program are compromised.

The situation in regard to mathematics content in undergraduate education programs is even worse. Reduction in class contact hours means that many are not getting nearly enough content especially as mathematics prerequisites are non-existent in some institutions. Many mathematics educators in Australia are following the introduction of exit testing in core subjects for graduating teachers in some parts of America with interest. After waiting over ten years for the Speedy report (1989) on this issue to be taken seriously, and always being in a minority in education faculties, this kind of sledge-hammer may be the only way to stop primary teachers being allowed to graduate with less than satisfactory mathematical knowledge.

The New Zealand experience seems to be somewhat different to Australia's as teacher shortages have been acknowledged and some actions taken to try and remedy the situation. This has included active recruitment programs in Australia. One effort that appears to be having an impact on student enrolment for the postgraduate preservice diploma courses is the TEACH NZ scholarships scheme. These are offered to potential mathematics or physics teachers, and Maori and rural teachers. The money covers their fees in their year of study ($4000), with a $3000 salary supplement for each of the first two years they are teaching.

The New Zealand Ministry of Education funded a project which investigated the effectiveness of mathematics graduates entering teacher education, and experienced teachers of subjects other than mathematics, on a specially designed course to become mathematics teachers (Paterson, Horring, & Barton, 1998; Horring, Paterson, & Barton, 1999). The success of the project with the latter group has implications for improving the supply of mathematics teachers through appropriate courses for existing teachers, but at this stage it appears it is deemed too expensive to extend.

University Mathematics

The health of mathematics education research is intimately related to the health of all aspects of mathematics, including university mathematics. The report (Dobson & Calderon, 1999) for the Deans of Science mentioned earlier had its genesis in a DETYA[13] (Borthwick & Murphy, 1998) report on *Supply and Demand for Scientists and Engineers*. To quote from the press release that accompanied the release of the report, "This paper looks at whether the supply and quality of scientists and engineers is adequate for national labour market requirements … Despite concerns … there is some evidence of an over-supply of new science graduates". However, the paper was somewhat lacking in detail, especially regarding the figures related to discipline specific areas. Problems with DETYA figures are not new, especially as they relate to enrolments in mathematics (see, e.g., Cobbin, 1995; Leder, Forgasz, & Brew, 1998).

The Deans of Science report sought to get behind the DETYA figures. What it found was radical change in emphasis in course enrolments from mathematics, chemistry, physics, and biology to computer science, biology, and psychology, with chemistry barely holding its own and physics going backwards. The writers praised the DETYA higher education statistics, but found that staff and student collections cannot be conveniently linked "so that even a statistic as basic as the 'student:staff ratio' is difficult to calculate". What the data do show is that science enrolments went up by about 58% in the period of the study while mathematics went up 1%. This would seem to indicate an increasing number of science graduates who have little or no mathematics in their degrees. It would follow that they have little in the way of physical sciences either and that many science graduates actually have a very weak background in what the Deans labelled the *enabling sciences*—mathematics, physics, and chemistry.

Having noted the difficulty with staff:student ratios, the writers still note that "DETYA has a stringent data collection methodology" and "it is unlikely that the results from ad hoc surveys of departments will have the same veracity". This referred specifically to data collection by mathematicians and physicists in the universities. Data relating to student enrolments and staffing in mathematics departments was first collected by Peter Kloeden for the National Committee of Mathematics after a review of the advanced mathematical sciences (National Committee for Mathematics, 1996). It showed that the mathematical base in Australia was fragile. It indicated a further weakening, particularly in academic staff numbers. A review of the state of mathematical sciences in New Zealand found similar results to those for Australia (Ministry of Research, Science & Technology, 1998).

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[13] Formerly Department of Employment, Education and Training and Youth Affairs (DEETYA) and prior to that Department of Employment, Education and Training (DEET).

Since then the Australian economy has remained strong but funding for universities has been reduced. The Heads of Mathematical Sciences Departments convened as a group in January 1998 and are now meeting annually. Data collected by the group showed an alarming decline in staff of about 20% over three years (1995–1998) although student numbers had remained about the same at first year level with small declines in second and third year. There are clearly flaws in data collected in this way but there is little doubt that it gives a clearer picture of university mathematics than anything else being collected. DETYA figures cannot give a break down of enrolments in the various year levels, nor do they follow staff numbers. At this stage it is getting increasingly difficult for any university to maintain an honours year of any breadth whereas in the past it was not unheard of for teachers of mathematics to have studied mathematics for four years or more.

Funding Mechanisms

Both Australian and New Zealand governments have moved more of the cost of tertiary education to students. Both approaches present various problems for education in general, and mathematics education in particular. In Australia students pay a Higher Education Contribution Scheme (HECS) based on three bands which are meant to reflect the cost of courses and likely incomes of graduates. Students have a choice of up-front payments which are encouraged by generous discounts or deferred payment collected via taxation once they reach a certain threshold salary level. Mathematics is in the middle band and education in the lowest. The result of this is that a mathematics graduates start teaching on the same salary as teachers who have completed all of their study at the lower level, but with more debt. In New Zealand, student loans mean many students finish their university degrees with debts. In both countries avoiding debt by emigrating means that this becomes a more attractive option. Debt also makes teaching unattractive as, although starting salaries for teachers are quite good, a mathematics graduate has other options likely to lead to better promotional opportunities and the ability to repay debt sooner.

Problems in both mathematics and education have surfaced in regard to postgraduate course-work degrees in Australia. While initial teacher qualifications for graduates are still HECS based courses, the majority of others now attract up-front fees. This has greatly reduced the number of postgraduates in education and teacher participation in such courses. Similarly, a healthy development in course-work degrees in mathematics, which were providing another avenue meeting industry needs for particular mathematical skills, has just about disappeared. As research degrees are often given HECS exemption, this also means that students do not necessarily make very sensible choices about whether their needs are met by a research or a course-work degree. The net result is that current funding arrangements are a disincentive for mathematics graduates to become teachers and for existing teachers to either up-grade their mathematical knowledge or to undertake further studies in education.

Research in Mathematics Education

Control of Research in Mathematics Education

The political environment in which mathematics education research has been conducted in the past four years is thus one which has witnessed the increasing intrusion of direct government intervention into the field of curriculum at all levels of the Australian educational environment. As mentioned in the introductory section, this can impact on the work of mathematics education researchers in ways that compromise intellectual property and where researchers may be answering to a committee singularly lacking in mathematics education research expertise. To illustrate this an example from Victoria will be used but this is not an isolated case as the rhetoric and constraints are an integral part of much of the work being done.

The example is the Project Brief for the Middle Years Numeracy Research Project: 5–8+ (Department of Education, 1999). The context of the project was stated as:

> Recent research and policy decisions at national and state levels have identified literacy, numeracy and science as key priorities for learning. These key priorities have been strengthened through the establishment of explicit standards and, increasingly, the specifications of targets to be met by schools. Ministers for Education in States and Territories agreed in March 1997 to a new national goal "that every child leaving primary school is numerate and able to read, write and spell at an appropriate level". A Victorian Government target states that by the year 2010 Victorian students will be in the top 5 in the world in Mathematics.

The underpinning of this is that of improvement by setting targets and measuring them. While it is called a research project, the description in the tender made clear the preferred methodology. More importantly it noted that:

> The report remains the property of the DoE, the CECV and the AISV[14] through the Successful Interventions Steering Committee and cannot be distributed in any format without the express permission of the DoE, the CECV and the AISV.

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[14] DoE is Department of Education, CECV is the Catholic Education Commission of Victoria and AISV is the Association of Independent Schools of Victoria.

Further, under the management structure:

> A Project Coordinator within the Curriculum Development Section of the DoE will be appointed. The Successful Interventions Steering Committee will receive reports and provide advice to the project. The Successful Interventions Steering Committee consists of representatives from the DoE, the CECV, the AISV, the Association of School Councils in Victoria, a Principal representative and the relevant subject associations.

It was this kind of controlled so-called research that the Green Paper (Kemp, 1999a) on research seemed to be suggesting was not to be distinguished from competitive grants achieved by peer review. While this notion may have been weakened (Kemp, 1999b), it is still the case that many MERGA members are currently involved in this kind of work. This is not to say that this work is not valuable, as opportunities to drive these projects in more productive ways certainly exist. However, it also means that many MERGA members have little time for other research which is not seen as immediately applicable, and reporting of the work they are doing may not be possible particularly when it is most relevant.

Overall, in universities, the number of students has risen and staff numbers have fallen. The result is that busy academics are becoming increasingly isolated from their colleagues. While the annual MERGA conference continues with a healthy number of participants, regional network meetings are becoming increasingly rare.

Research in the Politics of Mathematics Education

In one sense, little seems to have changed since 1992. At that time Thomas (1992) noted that "At this stage there is little documented research in the current politics of mathematics education" (p. 30), and the research that has since appeared has done little to alter that statement. It is surprising that an area that seems to have such a significant impact on the nature of mathematics education practices should draw such scant immediate attention from the mathematics education fraternity. However, the political nature of the area may well account for the reluctance of many to become involved in what is potentially risky research. There is evidence to suggest that government departments are prepared to intervene actively in academic processes where they perceive a threat to their own actions (Thomas, in press a). In reality, it may well be that the challenges faced by the mathematics education fraternity are greater now than they have been in the recent past.

The issues surrounding tendering processes and related matters need to be set in the context of diminishing academic freedom to speak out on controversial matters in both Australia and New Zealand. This is best illustrated by Ellingsen (1999) whose article in the weekend magazine supplement of two of Australia's leading papers, *The Age* and *The Sydney Morning Herald*, noted, "And as those academics who protest soon learn, loyalty to the firm means keeping your mouth shut—or else." At about the same time it was reported that the academic union in New Zealand had instigated an inquiry into academic freedom. Colebatch (1999), an economics editor with one of Australia's leading newspapers noted: "The politicians and economists who give us propaganda—the incomplete picture—are doing this country wrong … But how can we as a nation solve problems that our leaders refuse to admit?" It is a sentiment many mathematics educators would share, but articulating the problems is increasingly problematic. An article by Thomas (1997c) written for teachers, suggesting that teaching some students was not easy and they needed to use their professional judgement when using government documents, was suppressed from re-publication as part of a professional development package.

Clements and Ellerton (1996) attributed much of the attitude of educational bureaucracy to colonialist tendencies which they perceive to be a recurring theme in Australian educational decisions. While this model of educational history is challenged by Truran (1997), who examined the introduction of probability into Australian schools in order to introduce an alternative broad-spectrum ecological model, it is nonetheless indisputable that overseas factors, for whatever reason, have continued to exert influence on the thinking of Australian educators. The concern of Clements and Ellerton was that this influence is too great and, in addition, is heeded at the expense of local educational expertise. They emphasised their concerns by detailing examples where the work of Australian educationists were ignored, whereas the writings of overseas contributors was highly valued.

Examination of current global tendencies in educational influence shows a much more dangerous situation developing described by Apple (1995, p. 333) as the "rapid growth in the adoption of reductive procedures of standardization and rationalization". Gardiner (1998) has noted that the situation in regard to the mathematics curriculum had been complicated by two rival philosophies of mathematics (*traditionalists* and *revisionists*) facing a third force which he called *burgeoning bureaucracy*, and which he described as more powerful and less principled than either of the other two forces. He cautioned about the use of labels, and suggested that these forces be seen as three opposing tendencies—traditionalist, revisionist, and bureaucratic—which may coexist within individuals. In Gardiner's view traditionalists may try to find new ways of doing things but are determined not to reject key elements, revisionists believe more strongly that the old ways have failed, and bureaucrats come from either or both camps but they, or their superiors, believe that educational effectiveness comes from central control and formal accountability.

Thomas (in press a) has noted that it is now possible to discern the movement of bureaucrats between nations and agencies. Hence a central figure involved in the Australian national curriculum moved to a key role in the OECD. Another is involved in a major project in America. Within Australasia there is a greater tendency to see senior bureaucrats take senior administrative, not academic, positions in universities. The growth of this third force helps explain why there can be a change of government and, like in England, very little changes in education. As Horwood (1999) has noted, "The decision as to who constitutes the expert, and whether advice is accepted, now appears to rest with the bureaucracy" (p. 273).

Little has changed in Australia or New Zealand during recent changes of government except perhaps a softening of some of the language used. Thomas (in press a) notes that Clements (1996) argued that one of the greatest achievements of mathematics educators over the past two decades was to bring centre-stage the fact that all forms of mathematics education are surrounded by cultural constraints and forces. Barton (1999), in addressing the way *ethnomathematics* has been used in political ways in curriculum development, also notes the distinctive mathematics programs that have been developed in small communities and that ethnomathematical ideas have the potential to enrich mathematics. To a very great extent, much of this achievement is now at odds with many of the developments in mathematics education which in the push for *international benchmarks* take scant regard of cultural constraints.

Some distinction needs to be made between the situation in regard to Indigenous peoples in Australia and New Zealand. It is clear that Maori culture and the development of mathematical discourse in Maori continue to be a vibrant area of research and development (e.g., Barton, Fairhall, & Trinick, 1998), but this is not the case in Australia where bilingual programs for Indigenous people were recently abandoned (e.g., Lo Bianco, 1999; Tomlinson, 1999). For non-indigenous minority language groups, issues surrounding language and culture in mathematics learning are largely ignored in curriculum policy in both countries.

Concerns with the creation of greater inequity through outcomes-based approaches, and the enormous gaps that still exist in assessment outcomes for students from different socioeconomic backgrounds (e.g., Teese, Davies, Charlton & Polesel, 1995), have been the subject of a number of papers. Neyland (1998) suggests that once the rhetoric has been stripped away the grim reality of outcomes-based curriculum is revealed and "we are left with an education system which has lost its way, having been uprooted from its base of social democratic values, and reorganised to protect the interests of the wealthy and powerful" (p. 74). Zevenbergen (1997) expressed similar concerns, stressing that teaching and learning do not occur in a social vacuum and government agendas need to be challenged. Both Thomas (in press b) and Zevenbergen (1999) have expressed concerns with equity in the tertiary sector, and the problems associated with equitable delivery when dealing with students from very different backgrounds with reduced resources.

The Debate that Never Was and Is Now Urgent

Industry funding of scientific and medical research is already leading to new distrust of the objectivity of researchers working in areas such as genetically modified foods. In mathematics education research, industry funding is commonly from government, and control of research through such funding is seldom openly debated. When raised, it can provoke division within the research community between those who are, or have been, involved with government contracts and those who try to raise questions about this kind of research. Ellerton (1995) argued:

> We can allow our research priorities to be dictated by funding agencies and political considerations even though our research may do little more than ratify the status quo. Such compromises made to achieve funding or recognition can, in fact, keep researchers away from doing the very investigations that are needed to clarify our thinking in the realm of mathematics education and thereby improve the teaching and learning of mathematics (p. 93).

A subsequent letter in response by Willis (1996) indicated that further debate on this issue was likely to provoke divisions within the mathematics education research community in Australia and the discussion stopped.

Some years later examples of bureaucratic interference in educational research are beginning to be documented. Kamler, Reid, and Santoro (1999) wrote of their experiences with the Victorian Department of Education resulting in nine months of delays, and substantive changes to, a research project that required access to teachers. They concluded that controversial questions "may be silenced and less likely to gain approval than those that bolster the status quo" (p. 64). There have always been issues about particular kinds of research being preferred by funding agencies. Bureaucratic interference in the actual research to the extent documented by Kamler, Reid and Santoro is new in Australia and the debate that Ellerton tried to start is now urgent.

These kinds of developments have serious implications for the credibility of mathematics education research. So-called research publications commissioned by government to support a particular policy will do little to support the credibility of educational research in general if it is subsequently revealed that selected reporting was done. As the tender document quoted earlier indicated, this may be out of the hands of the actual researchers. If educational research, including mathematics education research, is to be taken seriously then the potential uses and abuses of such commissioned research must be debated and it must be made public. The so-called *Math Wars* in the United States of America is something MERGA members would prefer to avoid but in many ways that very public debate is healthier than the silence on many issues in other countries at this time.

Research and Political Activism

Defining what is research in the politics of mathematics education is difficult in any context. Many people are involved in research which is not about politics as such but assists enormously in policy and political forums. This section documents some of the work of MERGA members in this kind of research. It can only be a selection of that work as much goes on behind the scenes and not all of it is documented. In spite of the many constraints, MERGA members have continued to be active participants in trying to influence the health of the mathematical sciences, and in both Australia and New Zealand there is involvement in many government activities.

Particularly in Australia there is active involvement in political lobbying. In November 1999, some MERGA members were involved in a Science Meets Parliament day, a major event organised by the Federation of Australian Scientific and Technological Societies (FASTS) of which MERGA is a member through its membership of the AMSC. About 170 scientists met with about two-thirds of the members of parliament to promote the cause of science. Educational issues turned out to be the most popular topic of conversation and there is little doubt that the hard work of MERGA members in constantly drawing attention to issues like teacher supply had a major impact on this.[15] Supporting documents used on the day (Mousley, 1999) showed the dramatic changes in students' enrolments in Year 12 mathematics (Malone, Dekkers, de Laeter, in press), the decline in graduating secondary mathematics teachers in Victoria (unpublished, Thomas), and the decline in staff in university mathematics departments (Heads of Department data).

The falling enrolment in Year 12 advanced mathematics courses has been linked to a failure to give students real reasons for studying mathematics. Brinkworth and Truran (1998) reported that students "have a distorted view of mathematicians and users of mathematics, whom they see as somewhat cloistered individuals ... not very concerned with social issues" (p. 56). Hodge (1999) quoted the president of the AAMT as saying that careers teachers actively discourage students from the profession and that the association was working to reverse the view of mathematics as a rote-learning subject. While this is a laudable aim it is only part of the problem. The AustMS has long been of the view that careers teachers can not be relied on to promote the importance of mathematics in careers but that mathematics teachers with enough good up-to-date information should be able to do this task.

A number of MERGA members are actively involved in working with the discipline-based societies in trying to achieve this. One of the authors (Thomas) assisted staff at the National Science Academy of the Australian Academy of Science in a proposal for support for mathematics topics on a web site managed by them concerned with science in the news. Government support was obtained but matching grants were very difficult to achieve until the Heads of Mathematical Sciences group assisted and at last count nearly all had contributed.[16] A re-write of the entry for mathematician in the Job Guide which goes to all Year 10 students was coordinated by one of the authors (Thomas) through this group and the AustMS Education Standing Committee.

Other collaborative ventures with other societies, and the resulting networks, assist greatly in both promoting mathematics in the broader community as well as being invaluable in the broader political lobbying field. MERGA and AustMS shared responses to the Green Paper (Kemp, 1999a) and were able to reinforce one another's responses.[17] Many MERGA members have had extensive involvement in the Benchmarks and this was particularly so for the Year 7 ones. Following concern about the way the Years 3 and 5 benchmarks had been created, and about their poor quality, meetings were sought with the then Minister responsible, Senator Chris Ellison. When the processes did not improve a letter was sent from the AMSC noting that MERGA members were bearing the brunt of the pressure for almost instant responses and that this pressure was "rapidly turning the whole project into a farce".[18] This letter had the remarkable result that AMSC representatives were summoned to Canberra to meet with both Ellison and the senior Minister, David Kemp. It had two major results, an initial two day meeting to consider the Year 7 Benchmarks and on-going involvement of MERGA in the process, and the commissioning of ACER to consider the Benchmarks in light of curriculum in other countries. This confirmed that the Years 3 and 5 Benchmarks were of a very minimum standard (Lokan, O'Connor, Doig, Lindsey, & Masters, 1998). However, at this stage the Year 7 Benchmarks appear to be accepted as a much better product provided they are seen as a minimum standard.

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[15] For more details see http://www.usyd.edu.au/fasts/

[16] http://www.science.org.au/nova/

The energy and commitment of many MERGA members to projects such as these and to events in the individual states and New Zealand, show the important ways they are contributing to the politics of mathematics education. It also has to be recognised that sometimes this only happens if some rather aggressive lobbying is used to make sure the expertise of members is not overlooked by bureaucrats and governments who find it easier to ignore such advice and involvement.

Concluding Comments

One of the authors (Horwood) is a mathematician whose research interests are in mathematics education. The other (Thomas) is a mathematics educator who is also employed by the AustMS to work on policy and promotional issues. In writing this chapter we have tried to demonstrate that the health of mathematics education research is intimately tied to that of the mathematical sciences more generally and especially to the political milieu in which we all operate. We believe that this is becoming increasingly difficult to influence as a global network of powerful bureaucrats develops who use a language of standards and accountability that has strong resonance with governments. The chapter on "Language Factors in Mathematics Learning and Teaching" expands this issue. There is a very real danger that the issues that have been seen as important to MERGA members that relate to equity, social justice, and respect for difference will be lost in this globalised view of mathematics education. In particular, the failure to ensure an adequate supply of properly qualified mathematics teachers in a world where access to a good mathematics education opens the door to the more prestigious careers, may turn mathematics once again into the gate-keeper of privilege.

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[17] Copies of the documents referred to here are held by the authors

[18] Letter to Minister Ellison from AMSC, 28 October, 1997

We do indeed live in challenging times and the biggest challenge is to break the stranglehold that bureaucrats and government advisers have on the politics of mathematics education. We need to recognise that many, but certainly not all, of these bureaucrats and advisers are themselves frustrated by a system that requires them to operate in ways which are often more about media perceptions than dealing with the real issues. The mathematical sciences in Australia and New Zealand need bipartisan support and long-term planning, not ad hoc decisions for short-term political gain. This is an equity issue as such planning must reflect the multicultural nature of our societies. The solutions to the problems in both of our countries lies with the mathematical sciences communities, not with the bureaucrats, and certainly not with a globalised curriculum based on outcomes. For this reason MERGA members must continue to try to be heard.

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About Australasia (with Commonly Used Abbreviations)

Australia

Indigenous Australians have lived here for at least 40 000 years, probably 100 000 years. Australia was invaded in 1888 by the British but there had been numerous seafaring groups entering Australia, especially in the north west before this date. Since colonisation, there has been a constant flow of immigrants from many different countries although, for a time in the middle 1900s, non-European immigrants were discouraged. Australia is proudly a multicultural nation. Australia's immigration is continuing with about 40% of children in schools being immigrants or children of immigrants. Australia's Indigenous population consists of those people who identify with their Indigenous culture and can identify hereditary links. There are many distinct Aboriginal and Torres Strait Islander cultures. Most of Australia's population lives down the eastern and south eastern seaboard, mostly in the capital cities and nearby cities. Australia consists of six states and two territories. Their names, abbreviations, capital cities, other comments, and some universities are given below.

- *Australian Capital Territory*, ACT, Canberra. It is a small enclave inside NSW about 200 Km from Sydney
  - ANU-Australian National University.
  - University of Canberra.
  - ACU-Australian Catholic University.
- *New South Wales*, NSW, Sydney. It is the central eastern state with the largest population.
  - SU-Uni. of Sydney.
  - UTS-University of Technology Sydney.
  - UNSW-University of NSW.
  - UWS-University of Western Sydney.
  - MU-Macquarie University.
  - ACU.

  Out of Sydney:
  - University of Newcastle.
  - University of Wollongong.
  - UNE- University of New England in Armidale.
  - CSU-Charles Sturt University in many country cities.
- *Northern Territory*, NT, Darwin. It is the sparsely populated central northern, tropical and desert area with large numbers of lease-hold cattle stations and large Indigenously owned lands.
  - NTU-Northern Territory University.
  - Bachelor College.

- *Queensland*, Qld, Brisbane. It is the northern half of eastern Australia with numerous Indigenous cultures.
  - UQ-University of Queensland.
  - QUT- Queensland University of Technology.
  - GU-Griffith University.
  - JCU- James Cook University in northern cities.
  - USQ- University Southern Queensland.
  - ACU
- *South Australia*, SA, Adelaide. It is the southern central state with large areas of desert and a number of Indigenous cultures.
  - Flinders University,
  - UniSA-University of South Australia.
- *Tasmania*, Tas, Hobart. It is the southern island state.
  - UTas- University of Tasmania.
- *Victoria*, Vic, Melbourne. It is a well populated south eastern state.
  - University of Melbourne.
  - RMIT.
  - Monash University.
  - ACU.
  - Deakin University.
- *Western Australia*, WA Perth. It is the western half of Australia, mostly desert and its population includes numerous Indigenous cultures.
  - UWA-University of Western Australia.
  - ECU-Edith Cowan University.
  - Curtin University.

All the above universities have schools of education and are maintaining masters and doctoral studies in education.

Each state has its own education system and Board of Study approving curriculum. The State schools come under Departments of Education or Departments of Education and Training (DET). Several names are also used to cover VET, Vocational Education and Training. There are also systemic Catholic Education schools and a growing number of independent schools, often Church-related. Education is compulsory to a set leaving age. Each state has its own final-year school assessment procedures and examinations. For example, in NSW, this is the Higher School Certificate (HSC) and in Victoria it is the Victorian Certificate of Education (VCE).

Universities are largely funded ;by the Federal government through its Department of Education, Training, and Youth Affairs (DETYA).

The umbrella teachers association is the Australian Association of Mathematics Teachers (AAMT). Each state has at least one teacher organisation. For example, the Mathematics Association of Victoria (MAV), Mathematics Association of NSW (MANSW), and Queensland Association of Mathematics Teachers (QAMT).

There is also a grouping of Mathematical Societies called the Australian Mathematical Societies Committee (AMSC) which includes MERGA, the Australian Mathematics Society (AusMS) and Australian Statistics Society. AMSC belongs to the group Federation of Scientific and Technological Societies (FASTS).

New Zealand—Aotearoa

Aotearoa is the Maori name for New Zealand (NZ), it means *the land of the long white cloud* which is how the early Maori explorer Kupe described it as he approached by canoe from Hawaiki over one thousand years ago. The Indigenous culture is Maori, the dominant culture is British. Many Pacific Islanders, in particular from the Polynesian triangle have settled in New Zealand. Comparatively recently New Zealand has had many more immigrants from Asian countries. In spite of this and the fact that some teachers have 25 nationalities in a class of 30 students, New Zealand does not appear to be as multicultural as Australia.

The main mathematics teachers association is the New Zealand Association of Mathematics Teachers (NZAMT) and this brings together about 20 maths associations from the various regions.

New Zealand has six university cities and in each of these there were Colleges of Education (Teachers College) although two have merged with the university. The following cities provide the main campus for the following universities or colleges.

- *Auckland:*
  University of Auckland,
  Auckland University of Technology,
  Auckland College of Education
- *Hamilton:*
  University of Waikato
- *Palmerston North*:
  Massey University
- *Wellington:*
  Victoria University of Wellington,
  Wellington College of Education
- *Christchurch:*
  Canterbury University,
  Lincoln University,
  Christchurch College of Education
- *Dunedin:*
  Otago University,
  Dunedin College of Education

Nine of these twelve institutions teach some mathematics education as part of preservice teacher education. (There are also a number of other minor providers of teacher education.)

The number teaching mathematics education at masters level is growing with at least eight offering masters courses this year. Currently at least five, the University of Auckland, Waikato, Massey, Victoria and Otago have people studying for doctorates in mathematics education although none of these have departments that specialise only in mathematics education. In Auckland and Otago doctoral study is closely linked to mathematics, at Massey and Victoria the link is with education, while at Waikato the links are with education and with science and technology education through an inter-school centre.

Other Countries in the Region

Many of these countries reside in the region called *Oceania*. It is also called the Pacific Islands or Western Pacific. The countries include Fiji, Vanuatu, Tonga, Micronesia, and Samoa. Many of these countries are serviced by the University of South Pacific centred in Suva, Fiji, with campuses in the different countries. Many students from these countries have studied in New Zealand.

Papua New Guinea is also an island nation based on the eastern half of New Guinea. It has over 800 languages. Its capital is Port Moresby. Its main educational institutions are the University of Goroka and a number of Teachers Colleges. It has an indigenous languages elementary school system followed by primary education to Year 8 in order to encourage more students to gain a basic education. Very few students move on to secondary education and a significant number of children do not finish primary school.

South East Asian countries are also grouped in the Australasia region. The Asian and Oceania countries are under-represented in MERGA, often because these countries are non-industrialised. MERGA provides special membership rates for individuals from non-industrialised countries.

Compiled by Kay Owens and Andy Begg

Contributors

Glenda Anthony

Glenda Anthony is a Senior Lecturer in mathematics education at Massey University, Palmerston North Campus. She is currently Head of the Department of Technology, Science and Mathematics Education, having previously been a lecturer in the Mathematics Department where she taught undergraduate calculus, aviation mathematics, and mathematics education courses. Her research interests include students' learning strategies, undergraduate teaching, and learning in mathematics, and more recently numeracy issues in schools and cognitively guided instruction.

Gary Asp

Gary Asp is a Lecturer in the Department of Science and Mathematics Education at the University of Melbourne. He has been involved in the preservice training of primary and secondary teachers, and in their continuing professional development through postgraduate study and inservice courses, for a number of years. Over the past ten, he has helped to develop and conduct a number of research studies and inservice presentations concerned with the use of computers and calculators for the teaching and learning of mathematics, and about the teaching and learning of mathematics.

Philip Clarkson

Philip Clarkson teaches education at the Australian Catholic University. He has been interested in the interplay between learning mathematics and language for many years, particularly with the issues surrounding bilingualism. He has a continuing research project working with bilingual immigrant children learning mathematics in Australia, and he is co-directing a research project dealing with the values mathematics teachers teach, and directing an evaluative project of the preservice teacher education courses and teaching in Papua New Guinea.

Ken Clements

Between 1964 and 1973 Ken Clements worked as a teacher in four secondary schools in Victoria. He became a mathematics education lecturer at Monash University (1974-1982), but in 1982 returned to school teaching. After a 2-year stint in India (1984-1985), Ken worked with infant teachers on an action research project in Melbourne before moving to Deakin University in 1987. Subsequently, he became Professor of Education at the University of Newcastle and, in 1997, Professor of Education at Universiti Brunei Darussalam (where he is currently working). Ken, who has published widely, was President of the Mathematics

Education Research Group of Australasia (MERGA) (1987-1993). He has been elected honorary life member of both MERGA and the Mathematical Association of Victoria (MAV).

Shelley Dole

Shelley Dole is a Lecturer at the University of Tasmania, and lectures in early childhood, primary, and middle school mathematics education in undergraduate and postgraduate courses. Her research interests include percent and proportional reasoning, alternative assessment in maths, teacher professional development, error patterns, and children's learned difficulties in mathematics, and more recently children's mental computation and number sense abilities.

Nerida Ellerton

Since 1997 Nerida Ellerton has been Professor and Dean of Education at the University of Southern Queensland. Before that she was Professor of Education at Edith Cowan University (1993-1997) and Lecturer, Senior Lecturer and Associate Professor at Deakin University. At Deakin she was Director of the National Centre for Research and Development in Mathematics Education (1991-1992).

Nerida has published numerous refereed journal articles and has written or edited 12 books. Between 1993 and 1997 she was editor of the *Mathematics Education Research Journal*. She was coauthor of two chapters in the (1996) *International Handbook of Mathematics Education*, including the chapter on language factors. She also coauthored the (1996) UNESCO book, *Mathematics Education Research: Past, Present and Future*.

Helen Forgasz

Dr Helen Forgasz is a Lecturer in mathematics education at Deakin University. Prior to her current appointment, she held a three-year Postdoctoral Research Fellowship, awarded to her by the Australian Research Council. Helen's research interests include: gender issues in mathematics education; the affective domain; tertiary mathematics education; and mature age students. She has published widely in scholarly and professional journals and books. Helen is Vice-President (Conferences) of MERGA, Vice-President of the Mathematical Association of Victoria [MAV] (Policy and Research), and a member of the editorial board of MERJ.

John Horwood

John Horwood is a Senior Lecturer in Computer Science at Victoria University of Technology with prior experience in teaching mathematics in secondary schools and university. His active research interests are in the sociology of mathematics and computing education, with particular emphasis on control of the secondary curriculum. His current research interests also include the impact of multimedia in education, and the effects on the teaching of mathematics of recent advances in information technology.

Peter Howard

Peter Howard is Senior Lecturer within the School of Education, NSW, at the Australian Catholic University. He is Director, Field Based Learning working on the development of strategic partnerships between the School of Education and school, community and business agencies. Through his teaching and consultancy work Peter has a background in mathematics curriculum development and program evaluation. His mathematics education research interests include the nature of people's beliefs about mathematics, teaching mathematics and learning mathematics. This work involves collaborative research with mathematics educators in several South East Asian countries. Peter has particular interest in issues related to the learning and teaching of mathematics by Aboriginal students at school and tertiary levels.

Ruth Hubbard

Ruth Hubbard is nominally retired but is still involved with teaching mathematics and statistics at the Queensland University of Technology and the University of Queensland and with statistical consulting. Ruth is on the editorial board of two mathematics education journals and reviews papers for a statistics education journal. She was one of the initiators and organisers of the two recent Delta Conferences on tertiary mathematics education. Her research interests include the use of technology, bridging mathematics, and student-centred approaches to teaching.

Gilah Leder

Gilah Leder is Director of the recently established Institute for Advanced Study at La Trobe University – Bundoora and a professor in the Institute of Education at the same institution. Her teaching and research interests embrace gender issues, the interaction between teaching, learning and assessment of mathematics, affect, and exceptionality. She has published widely in each of these areas. Gilah serves on various editorial boards and educational and scientific committees, is past President of the Mathematics Research Group of Australasia and current President of the International Group for the Psychology of Mathematics Education (PME).

Tom Lowrie

Tom Lowrie is a lecturer in mathematics education and information technology at Charles Sturt University, Wagga Wagga. He was a primary school teacher for many years before finishing his doctoral studies on the way students use visual imagery to solve mathematics problems.

Barry McCrae

Barry McCrae is a Senior Lecturer in the Department of Science and Mathematics Education at The University of Melbourne. He is an experienced secondary teacher who has been involved with the preservice and post-initial training of primary and secondary teachers for many years. Although his main focus is now mathematics education, he was a pioneer of computer education in Victorian schools and teacher training. His main research interests concern the role of technology, especially computer algebra systems and dynamic geometry software, in mathematics learning, teaching and assessment. For most of his career, he has had a key role in the external assessment of Year 12 mathematics in Victoria.

Alistair McIntosh

Alistair McIntosh is Associate Professor in mathematics education at the University of Tasmania in Launceston. Until 1999 he was Director of the Australian Institute for Research in Primary Mathematics Education at Edith Cowan University in Perth, Western Australia. He is a former Secretary of MERGA. His research and professional development interests include number sense, mental and written computation, and children's mathematical thinking.

Judy Mousley

Judy has taught in the undergraduate and postgraduate teacher education programs at Deakin University since 1982. Prior to that she was a primary and secondary teacher for 15 years. Her current higher degree students are working in the areas of philosophies of mathematics education, curriculum change, use of technologies in education, and higher education policy. Her PhD topic is "Teachers' construction of mathematical understanding". Judy has published widely on CDs, journals, videos and the internet. She is Vice President (Teaching) of MERGA, ex-Vice President of PME, President Elect of the AMSC, and an active member of a range of other professional organisations.

Steven Nisbet

Steven Nisbet is a Senior Lecturer in mathematics education at Griffith University in Brisbane. He is a former teacher of mathematics, science and music. He currently teaches at both primary and secondary levels in preservice and inservice courses. His research interests include the teaching of problem solving, children's statistical thinking, and the professional development of teachers. His doctoral studies were on children's matching of melodies and graphs with respect to auditory and visual contour. Steven is a former MERGA Vice President for Publications.

Kay Owens

Kay Owens is a Senior Lecturer teaching mathematics education and educational research at the University of Western Sydney. She has taught students at primary, secondary, and tertiary level, mostly mathematics and health education. Her research has mainly focused on Space mathematics, and how students learn from being responsive during problem solving in classrooms. She also has an interest in ethnomathematics, especially in relation to Papua New Guinea cultures, and in evaluation of preservice teacher education.

Bob Perry

Bob Perry is Associate Professor of Education at the University of Western Sydney. Originally educated as a secondary mathematics teacher, he has been involved in mathematics teacher education since 1972. Bob has research interests in the beliefs of teachers and teacher education students about mathematics, mathematics learning, and mathematics teaching; statistical thinking among children; constructivist approaches to mathematics teaching and learning and mathematics teacher education and the use of manipulatives in these approaches. He is a foundation member of MERGA and Associate Editor of its journal. Bob's research interests extend beyond mathematics education to include a major project on children's transition to school.

Beth Southwell

Beth Southwell is an Associate Professor in education at the University of Western Sydney. She has been a mathematics educator in Sydney, Laos, and several other countries as a visiting professor. She was a mathematics teacher for many years before working in education. She is a Fellow of the Australian College of Education, a life-member of MERGA and several other professional organisations. Besides being the editor of MANSW's *Square One,* and an editor of *MERJ,* she serves on numerous executives for a range of organisations. Her research interests are in values and beliefs about mathematics, learning and teaching, and teacher education.

Philip Swedosh

Philip Swedosh has a PhD in Mathematics Education and currently lectures in the Department of Mathematics and Statistics at the University of Melbourne where he teaches a range of undergraduate courses. Philip is the Victorian Director of the Australian Mathematical Olympiad Committee and also the

Director of the Mathematics Camp run jointly by the Department of Education and the Mathematical Association of Victoria. He is heavily involved in secondary mathematics, being on the Setting Panel and the Examining Panel for Specialist Mathematics. His research interests include tertiary mathematics education, mathematical misconceptions, the secondary-tertiary interface, and issues relating to the promotion of mathematics.

Jan Thomas

Jan Thomas is a Senior Lecturer in mathematics education at Victoria University of Technology. She is Vice-President of the Federation of Australian Scientific and Technology Societies and works part-time as Executive Officer for the Australian Mathematical Society. Her research interests are in the politics and policy of the mathematical sciences in general and linguistic and cultural effects on the learning of mathematics, specially for non-English speaking background students.

Colleen Vale

Colleen Vale is a Lecturer in the School of Education at Victoria University of Technology. She is currently completing her doctorate at La Trobe University. Her thesis is entitled Gender and Computers in Secondary Mathematics Classrooms. Colleen's other research interests include mathematics in the middle years of schooling.

Elizabeth Warren

Elizabeth Warren is a Senior Lecturer in mathematics at the Australian Catholic University, McAuley Campus, Brisbane. She lectures at both the primary and secondary preservice courses. Her research interests include the development of pre-algebraic and algebraic thinking, the effects of beliefs and values on teaching and learning mathematics, and models that support effective professional development in mathematics. Her current work is focusing on the use of technology as a support for developing mathematical thinking.

Content of MERGA Journals 1996–1999

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Submissions to Journals

Mathematics Education Research Journal

MERJ is an international refereed journal which provides a forum for the publication of research on the teaching and learning of mathematics at all levels. Submissions of high quality papers should normally be of general interest to an international readership, but papers exploring specifically Australasian issues are welcome. Both full-length articles and shorter reports may be submitted. No preference is given to any particular research methodology.

Manuscripts should conform to the style guidelines specified in Chapters 1-3 of the Publication Manual of the American Psychological Association, 4th edition (1994) with minor modifications for Australasian usage as specified in the MERGA Publication Guidelines available from mergapub@tpgi.com.au The submitted paper should be in the form ready for printing (e.g., tables and figures inserted in text and italics used and no underlining). All direct and indirect references to the authors should be eliminated and replaced by author. The manuscript should be emailed to mike.mitchelmore@mq.edu.au before July 2000 or afterwards to one of the associate editors, b.barton@auckland.ac.nz with a cover letter saying the paper has not been submitted elsewhere and giving full details of the authors and references.

Each manuscript is handled by one of the associate editors. The manuscript is reviewed blind by one of the Editorial Board members and two other referees. Working from the referees reports the associate editor and editor jointly decide on publication. They may decide to ask for specific corrections or they may give suggestions for revisions for rejected papers which might make them publishable.

Books for review should be sent to Marj Horne, Australian Catholic University, PO Box, 213, Oakleigh, Vic. 3166, Australia.

Mathematics Teacher Education and Development

This international refereed regular publication provides a stimulating collection of articles with a focus on mathematics teacher education in the broader sense. Quite a diversity of high quality articles will be included: research reports, papers addressing general issues and perspectives, successful pathways, with a strong theoretical basis and an evaluation, discussions on recent and potential policy decisions, curriculum developments, evaluations of programs, particular practices, research methods, philosophical debates and think pieces, and so on.

Manuscripts should conform to the style guidelines specified in Chapters 1-3 of the Publication Manual of the American Psychological Association, 4th edition (1994) with minor modifications for Australasian usage as specified in the MERGA Publication Guidelines available from mergapub@tpgi.com.au The submitted paper should be in the form ready for printing (e.g., tables and figures inserted in text and italics used and no underlining). All direct and indirect references to the authors should be eliminated and replaced by author. The manuscript should be emailed to g.stillman@mailbox.gu.edu.au with a cover letter saying the paper has not been submitted elsewhere and giving full details of the authors and references.

Each manuscript is handled by one of the editors. The manuscript is reviewed blind by one of the Editorial Board members and two other referees. Working from the referees reports the editors jointly decide on publication. They may decide to ask for specific corrections or they may give suggestions for revisions for rejected papers which might make them publishable.

Books for review should be sent to Mary Klein, School of Education, James Cook University, PO Box 6811, Cairns, Qld 4870.

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